Title: Cape Cod Credibility (CCC)
Topic: Other

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ABSTRACT

In many instances when using the BF method, we do not have an external source for the expected loss costs. In this situation, we will frequently use a weighted average of ultimate loss costs of the preceding accident periods. The Cape Cod method suggests using weights proportional to exposure and inversely proportional to development. Unfortunately, this approach fails to recognize variability in both the historical loss estimates and loss development factors. This paper attempts to recognize this variability, via credibility formula, and apply the results as alternative weights to the traditional Cape Cod method.

KEYWORDS

Trend, Credibility, Loss Development

1. INTRODUCTION

The calculation of the expected pure premium for the CC method is as follows:

$$\tilde{E}[PP] = \frac{\sum_i \left[\left(LTD_i \text{ x } DF_i / E_i \right) \text{ x } \left(E_i / DF_i \right) \right]}{\sum_i \left(E_i / DF_i \right)}$$

$$= \frac{\sum_i LTD_i}{\sum_i \left(E_i / DF_i \right)}$$

Where:

LTD = Loss to date (adjusted for trend).

DF = Ultimate loss development factor

E = Exposure (adjusted for trend).

Summed over each accident period *i*

There are obvious cases where the weights in the above expression do not provide satisfactory results. Take the extreme case where all age to age factors are identical for any given age. Here, each accident period loss estimate should be completely known at a first report of loss. However the expression above applies weight inversely proportional to the ultimate loss development factor. In our example, each loss estimate should get equal weight. This is an example where the process risk is zero (no variability in age to age factors), and all the variability is associated with parameter risk (variability associated with different accident period loss costs). In terms of credibility, each year's ultimate loss cost is 100% credible.

2.1. Credibility

It is well known that: Total Variance = Variance of the Hypothetical Means + Expected Value of the Process Variance

Therefore, Variance of the Hypothetical Means = Total Variance - Expected Value of the Process Variance

It can be shown that[1]:

$$Z = \frac{VHM}{VHM + \frac{EPV}{N}}$$

2.2. Total Variance

In the following example, exposure will be on level premium so that we will be dealing with on level loss ratios rather than loss costs. The on level premium for each year has been held constant (by scaling actual data) so that we can more easily isolate the impact of credibility weights versus loss development weights.

The first step is to calculate weighted average loss ratio and variance from the estimated accident period loss ratios.

Weighted Average Loss ratio = $\overline{x} = \left(\sum w_i x_i\right)/\left(\sum w_i\right)$, where the w_i are initial weights set equal to one and the x_i are the accident period developed ultimate loss ratios.

Total variance can be given as:

$$\sigma^2 = \left(\left(\sum w_i\right) / \left(\left(\sum w_i\right)^2 - \sum\left(w_i^{\,2}\right)\right)\right) \sum w_i \left(x_i - \overline{\mathrm{x}}\right)^2$$

Exhibit 1 displays the initial calculations for the weighted average loss ratio and total variance. Also shown is the Cape Cod Loss Ratio where each accident period ultimate loss ratio is given weight inversely proportional to its cumulative loss development factor. It is visually apparent that applying equal weight to the 2006 accident period LR produces a much higher weighted mean than the Cape Cod approach.

Exhibit 1

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Accd Period	Accd Period Subject Premium	On-Level Factor	On-Level Subject Premium	On Level Losses	Cumulative LDF's	Developed & Trended Loss	Trended Reported LR	**Ultimate LR**
						(5)x(6)	(5)/(4)	(7)/(4)
2002	184,841,752	1.458	269,544,295	107,935,638	1.401	151,269,545	40.04%	56.12%
2003	258,499,389	1.043	269,544,295	84,122,238	1.754	147,587,305	31.21%	54.75%
2004	293,407,886	0.919	269,544,295	67,551,133	2.618	176,869,827	25.06%	65.62%
2005	303,248,288	0.889	269,544,295	44,663,392	4.176	186,535,040	16.57%	69.20%
2006	290,398,667	0.928	269,544,295	5,247,632	47.973	251,744,418	1.95%	93.40%
Total/Avg			1,347,721,477	309,520,034		914,006,136		**67.82%**

$(\sum wixi)/(\sum wi)$	**Weighted Mean:**	**67.82%**
$((\sum wi)/((\sum wi)^2 - \sum(wi^2))) \sum wi(xi - \hat{\mu})^2$	**Total Variance:**	**0.02421**
	Cape Cod:	**59.63%**

2.3. Expected Process Variance

The following is just one approach for estimating process variance. We assume the age to age factors are lognormally distributed and are independent across evaluations. The parameters of the age to age loss development factors at each age i are:

$$\tilde{\mathrm{u}}_{\mathrm{i}} = \frac{\sum_k \ln x_k}{n}$$

$$\tilde{s}_i^{\,2} = \frac{\sum_k \left(\ln x_k - \tilde{\mathrm{u}}\right)^2}{n}$$

x_k are the individual observed loss development factors at age i.

For age to ultimate loss development factors, parameters are additive so:

$$\tilde{U}_i = \sum_{j>=i} \tilde{u}_j$$

$$\tilde{\sigma}_i^2 = \sum_{j>=i} \tilde{s}_i$$

Here the subscript i is consistent with both the accident period and age of ultimate loss development under consideration.

The mean and variance of the ultimate loss development factors are:

$$\text{Mean} = e^{\tilde{U} + \tilde{\sigma}^2/2}$$

$$\text{Variance} = \left(e^{\tilde{\sigma}^2} - 1 \right) e^{2\tilde{U} + \tilde{\sigma}^2}$$

The expected process variance, $E_\theta\left[Var_{x|\theta}\left[\bar{X}_i \mid \theta_i\right]\right]$, can be shown as[1]:

$$\left[Var_{x\mid\theta}\left[\bar{X}_i \mid \theta_i\right]\right] = Var_{x\mid\theta}\left[\left(\frac{1}{N}\right)\sum_{k=1}^{N} X_{ik} \mid \theta_i\right] = \left(\frac{1}{N}\right)^2 \sum_{k=1}^{N} Var_{x\mid\theta}\left[X_{ik} \mid \theta_i\right] = \sigma^2\left(\theta_i\right)/N$$

$\sigma^2\left(\theta_i\right)$ is the variance of the loss development factor for accident period i, while N represents the number of loss development factors the average is based upon. We're not finished yet. This represents the process variance of the loss development factor. We need the process variance of the ultimate loss ratio, ULR.

Since ULR= (Reported LR) x LDF, the variance of ULR = (Reported LR)^2 x $\sigma^2\left(\theta_i\right)$

Thus the expected process variance is = (Reported LR)^2 x $\sigma^2\left(\theta_i\right)/N$

Exhibit 2 displays the result of these calculations. The calculations are performed on the last four diagonals for stability considerations. It can be seen that the row labeled "Mean" is the ultimate loss development factors utilized in Exhibit 1. The variance of the hypothetical mean, VHM, is the difference of the Total Variance ITER 1, calculated in Exhibit 1, less (RLR)^{2}xEPV/N.

Z is found as:

$$Z = \frac{VHM}{VHM + \frac{RLR^2 xEPV}{N}}$$

Each age to age factor generates its own Z. The Zs are scaled and we revisit Exhibit 1 except using these Zs as revised weights in our calculations.

Exhibit 2

Accd Period		6:18	18:30	30:42	42:54	54:66	66:78
1996		20.777	1.100	1.193	1.364	1.134	1.052
1997		7.700	1.248	1.238	1.143	1.091	1.057
1998		10.021	1.474	1.394	1.178	1.075	1.104
1999		6.551	1.282	1.377	1.262	1.140	1.111
2000		20.851	1.210	1.575	1.244	1.251	1.154
2001		16.804	1.622	1.479	1.263	1.217	
2002		11.692	1.481	1.481	1.238		
2003		14.991	1.608	1.438			
2004		8.367	1.676				
2005		11.807					
"Last"		6:18	18:30	30:42	42:54	54:66	66:78
4	$\tilde{s}_i^2$	0.0431	0.0021	0.0011	0.0001	0.0035	0.0010
	$\tilde{\sigma}_i^2$	0.052	0.009	0.007	0.006	0.006	0.002
	$\tilde{u}_i$	2.440	0.467	0.400	0.225	0.156	0.101
	$\tilde{U}_i$	3.869	1.429	0.963	0.562	0.338	0.182
	Mean	47.973	4.176	2.618	1.754	1.401	1.199
	Variance:	129.498	0.161	0.049	0.018	0.012	0.003
	N	4	4	4	4	4	4
	EPV:	129.49770	0.16059	0.04854	0.01834	0.01155	0.00340
	Reported LR (RLR)	0.01947	0.16570	0.25061	0.31209	0.40044	-
	$(RLR)^2$xEPV/N:	0.01227	0.00110	0.00076	0.00045	0.00046	-
	Total Variance ITER 1	0.02421	0.02421	0.02421	0.02421	0.02421	0.02421
	VHM:	**0.01194**	0.02311	0.02345	0.02377	0.02375	0.02421
	Z:	0.4932	0.9545	0.9685	0.9816	0.9809	1.0000
	Scaled Z:	0.1126	0.2180	0.2212	0.2242	0.2240	0.2284

2.4. Iteration

The revised "weights" in Exhibit 2 are shown below in the column (3). The weighted mean and total variance are revised using these new weights. Since the VHM is the residual of total variance and expected process variance, this affects our calculation of Z. Column (4) are our scaled Zs after this change.

We keep iterating until Zs converge. A comparison of Cape Cod Credibility (CCC) weights versus the traditional Cape Cod loss development weights is shown in columns (18) & (19) below.

Exhibit 3

(8) Trended Reported LR	(9) **Ultimate LR**	(10) "Credibility" Weights ITER1	(11) "Credibility" Weights ITER2	(17) "Credibility" Weights ITER8	(18) "Credibility" Weights ITER9	(19) LDF "weights"
(5)/(4)	(7)/(4)					
40.04%	56.12%	**0.2240**	**0.2335**	**0.2557**	**0.2558**	**0.3705**
31.21%	54.75%	**0.2242**	**0.2337**	**0.2562**	**0.2562**	**0.2960**
25.06%	65.62%	**0.2212**	**0.2295**	**0.2483**	**0.2483**	**0.1983**
16.57%	69.20%	**0.2180**	**0.2251**	**0.2398**	**0.2398**	**0.1243**
1.95%	93.40%	**0.1126**	**0.0783**	**0.0000**	**0.0000**	**0.0108**
	67.82%	100.00%	100.00%	100.00%	100.00%	100.00%

Weighted Mean:	67.82%	64.97%	63.84%	61.27%	61.27%
Total Variance:	**0.02421**	**0.01823**	**0.01594**	**0.01072**	**0.01072**
Cape Cod:	59.63%				

2.5 Reserving Loss Ratios

Without scaling, we can use the credibility directly to estimate expected loss ratio for each accident period as a weighted average of the overall loss ratio, 61.27%, and the observed ultimate loss ratio for the period. For reserving purposes, de-trend the credibility weighted result and apply the BF method.

2.6 Summary and Discussion

Relative variability of accident period loss estimates and loss development produces weights that differ from the traditional Cape Cod approach. The above example focuses on a constant exposure base. The CCC approach can be generalized to the non-constant exposure case.

Below are a couple of snapshots of other scenarios:

In the first shot, Exhibit 4, we increase the stability of the 6:18 average age to age factor by replacing the 8.367 with the 12.000. This has the effect of increasing the credibility of the most recent accident period. This makes sense since we've reduced process risk.

In the second shot, Exhibit 5, we reduce the stability of the 18:30 average age to age factor by replacing the 1.481 with the 1.100. This has the effect if reducing the credibility of all accident periods relying on this factor.

Exhibit 4

Accd Period	6:18	18:30	30:42	78:90
1996	20.777	1.100	1.193	1.063
1997	7.700	1.248	1.238	1.026
1998	10.021	1.474	1.394	1.019
1999	6.551	1.282	1.377	1.119
2000	20.851	1.210	1.575	
2001	16.804	1.622	1.479	
2002	11.692	1.481	1.481	
2003	14.991	1.608	1.438	
2004	12.000	1.676		
2005	11.807			

8.367

(8) Trended Reported LR	(9) **Ultimate LR**	(10) "Credibility" Weights ITER1	(11) "Credibility" Weights ITER2	(17) "Credibility" Weights ITER8	(18) "Credibility" Weights ITER9	(19) LDF "weights"
(5)/(4)	(7)/(4)					
40.04%	56.12%	0.2065	0.2070	0.2070	0.2070	0.3709
31.21%	54.75%	0.2066	0.2071	0.2071	0.2071	0.2963
25.06%	65.62%	0.2048	0.2052	0.2052	0.2052	0.1985
16.57%	69.20%	0.2029	0.2031	0.2031	0.2031	0.1245
1.95%	102.09%	0.1792	0.1777	0.1775	0.1775	0.0099
	69.56%	100.00%	100.00%	100.00%	100.00%	100.00%

Weighted Mean:	69.56%	68.67%	68.61%	68.61%	68.61%
Total Variance:	0.03684	0.03445	0.03427	0.03426	0.03426
Cape Cod:	59.69%				

Exhibit 5

Accd Period	6:18	18:30	30:42	78:90
1996	20.777	1.100	1.193	1.063
1997	7.700	1.248	1.238	1.026
1998	10.021	1.474	1.394	1.019
1999	6.551	1.282	1.377	1.119
2000	20.851	1.210	1.575	
2001	16.804	1.622	1.479	
2002	11.692	1.100	1.481	
2003	14.991	1.608	1.438	
2004	8.367	1.676		
2005	11.807			

1.481

(8) Trended Reported LR	(9) **Ultimate LR**	(10) "Credibility" Weights ITER1	(11) "Credibility" Weights ITER2	(17) "Credibility" Weights ITER8	(18) "Credibility" Weights ITER9	(19) LDF "weights"
(5)/(4)	(7)/(4)					
40.04%	56.12%	0.2658	0.2905	0.2863	0.2863	0.3667
31.21%	54.75%	0.2661	0.2911	0.2870	0.2870	0.2930
25.06%	65.62%	0.2609	0.2778	0.2750	0.2750	0.1963
16.57%	64.29%	0.2073	0.1406	0.1517	0.1517	0.1325
1.95%	86.86%	0.0000	0.0000	0.0000	0.0000	0.0115
	65.53%	100.00%	100.00%	100.00%	100.00%	100.00%

Weighted Mean:	65.53%	59.93%	59.51%	59.58%	59.58%
Total Variance:	0.01653	0.00732	0.00814	0.00799	0.00799
Cape Cod:	59.02%				

REFERENCES

[1] Dean, Gary [2005]: "Topics In Credibility Theory," Construction and Evaluation of Actuarial Models Study Note