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Modeling and Management of Nonlinear Dependencies – Copulas in Dynamic Financial Analysis

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Outline

1 Introduction

- Dynamic financial analysis (DFA) is a financial modeling approach that projects financial results under a variety of possible scenarios, showing how outcomes might be affected by changing internal and external conditions
- DFA is used in practice for cash flow projection and decision support (solvency monitoring, performance measurement, capital allocation,…)
- Aim of this paper:

  1. Implement copulas in a DFA framework

  2. Study the effects on the insurer's risk and return position

  3. Give helpful insights for the development of DFA tools, for regulators and for policyholders

- Simplified model of a property-liability insurer: $EC_t = EC_{t-1} + E_t$
- Earnings are given by the investment and the underwriting result. Taxes are paid contingent on positive earnings: $E_t = I_t + U_t - \max(tr \cdot (I_t + U_t), 0)$
- Investments: $I_t = r_{pt} \cdot A_{t-1}$, with $r_{pt} = \alpha_{t-1} \cdot r_{1t} + (1 - \alpha_{t-1}) \cdot r_{2t}$
- Underwriting result: $U_t = P_{t-1} - C_t - Ex_{t-1}^{P} - Ex_t^{C}$

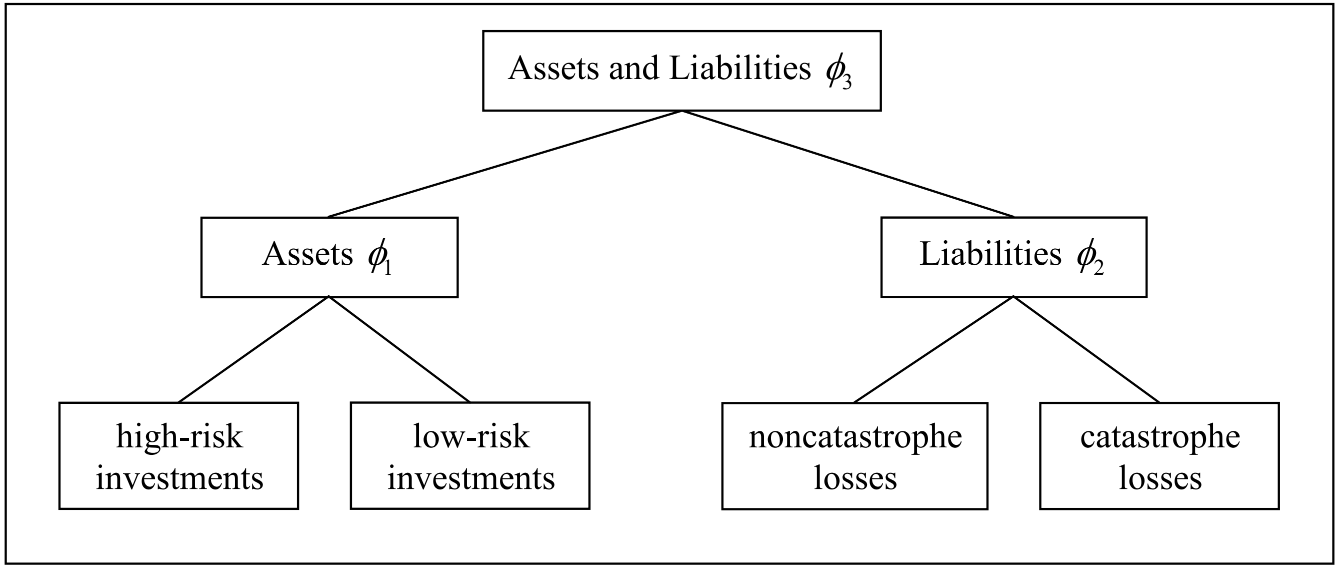
  => Premiums, claims, upfront costs, and claim settlement costs
- Premiums: $P_t = cr_t^{EC_{t-1}} \cdot \pi_t^s \cdot \beta_t \cdot MV$

  => Underwriting cycle and consumer response to changes in solvency
- At the beginning of each period management can change two variables: The portion of the risky investment (*α*) and the market share in the underwriting business (*β*)

- Correlated model elements

  - two exchangeable groups

  - separate correlations for investments, losses, and between assets and liabilities



- Copulas integrated
  - Gauss-Copula
  - *t*-Copula
  - Hierarchical Archimedean Copulas

Copula	Tail Dependence	Generator $\phi(t)$	Kendall's tau ρ_τ
C_θ^{Gumbel}	upper	$(-\ln t)^\theta$	$1-1/\theta$
$C_\theta^{Clayton}$	lower	$\frac{1}{\theta}(t^{-\theta}-1)$	$\theta/(\theta+2)$
C_θ^{Frank}	none	$-\ln(\frac{e^{-\theta t}-1}{e^{-\theta}-1})$	$1-4\theta^{-1}(1-\theta^{-1}\int_0^\theta t/(\exp(t)-1)dt)$

	Symbol	Measure	Interpretation
Return	$E(G)$	Expected gain per annum	Absolute return
Risk	$\sigma(G)$	Standard deviation of gain per annum	Total risk
	RP	Ruin probability	Downside risk
	EPD	Expected policyholder deficit	Downside risk
Perfor-mance	SR_{σ}	Sharpe ratio	Return/total risk
	SR_{RP}	Modified Sharpe ratio (RP)	Return/downside risk
	SR_{EPD}	Modified Sharpe ratio (EPD)	Return/downside risk

Model Specifications

- Time horizon: T = 5 years, equity capital in t = 0: €75 million
- Investments (α): High-risk *N*(0.1,0.2), low-risk *N*(0.05,0.05)
- Underwriting business (β): Market volume €1,000 million

  - Log-normally distributed noncatastrophe claims *LN*(0.85,0.085)

  - Pareto distributed catastrophe claims with mean parameter 0.5 and dispersion parameter 4.5

  - Underwriting cycle with cycle length of 7.76 years

  - Consumer response: 0.95 if EC < MCR·1.5
- Tax rate: 25%

Results

Dependence structure	No corr.	Gauss	t	Gumbel	Clayton	Frank
Tail dependence	none	none	upper and lower	upper	lower	none
E(G) in million €	30.81	30.39	30.39	30.47	30.10	30.45
σ(G) in million €	14.33	16.69	16.77	17.56	19.02	16.45
RP	0.07%	0.36%	0.58%	0.24%	0.87%	0.25%
EPD in million €	0.01	0.04	0.08	0.85	1.78	0.03
SR_{σ}	1.98	1.68	1.67	1.60	1.46	1.71
SR_{RP}	210.87	39.04	24.18	57.59	15.95	56.57
SR_{EPD}	22.53	3.67	1.81	0.17	0.08	4.12

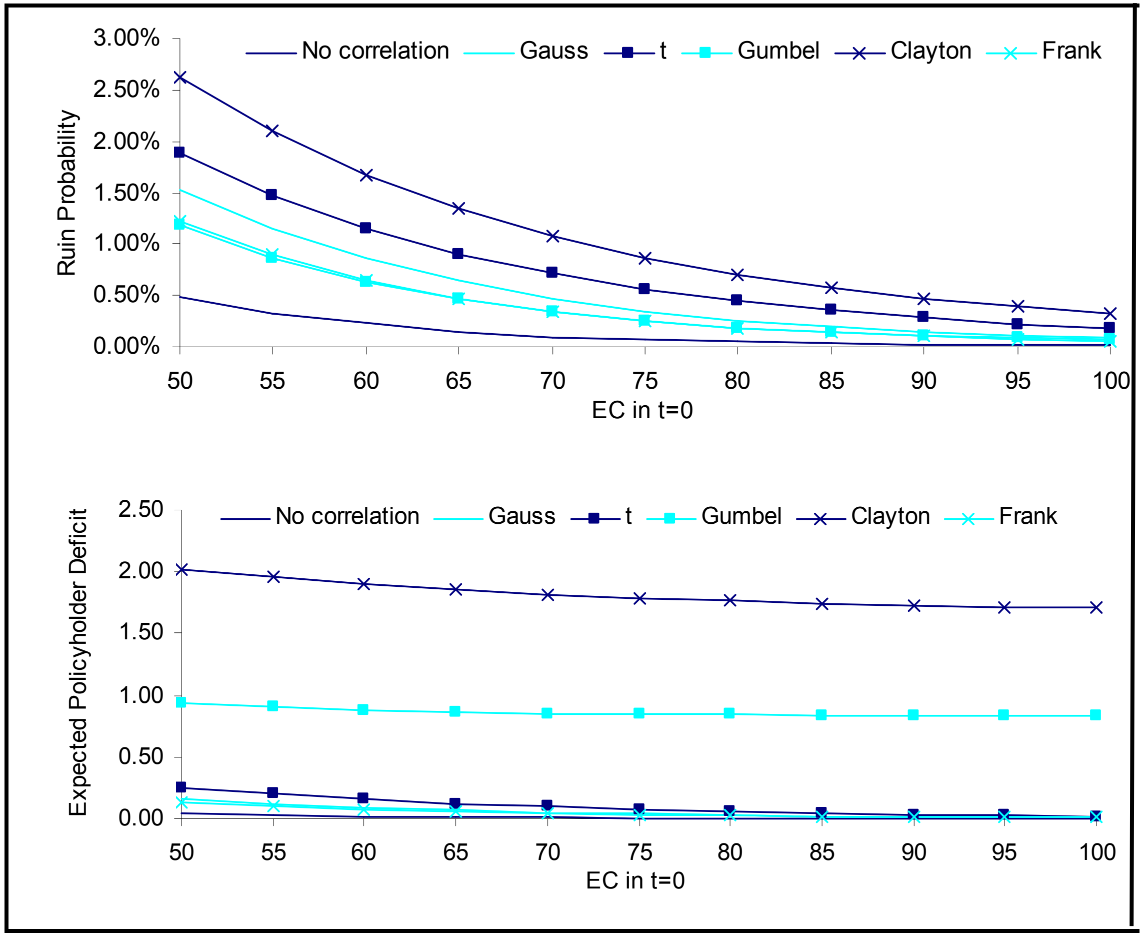
Robustness Checks

- Variation of the equity capital in t=0 (from €50 to €100 million)
- Variation of correlation settings
  - Correlation of assets between 0.1 and 0.5
  - Correlation of liabilities between 0.1 and 0.5
- Other robustness tests (not presented here)
  - Variation of the time horizon (from 1 to 10 years)
  - Variation of starting values (application of different α and β in t=0)
  - Variation of the parameter changes (for changes induced by the management, different step lengths for α and β are assumed)
  - Variation of consumer response function

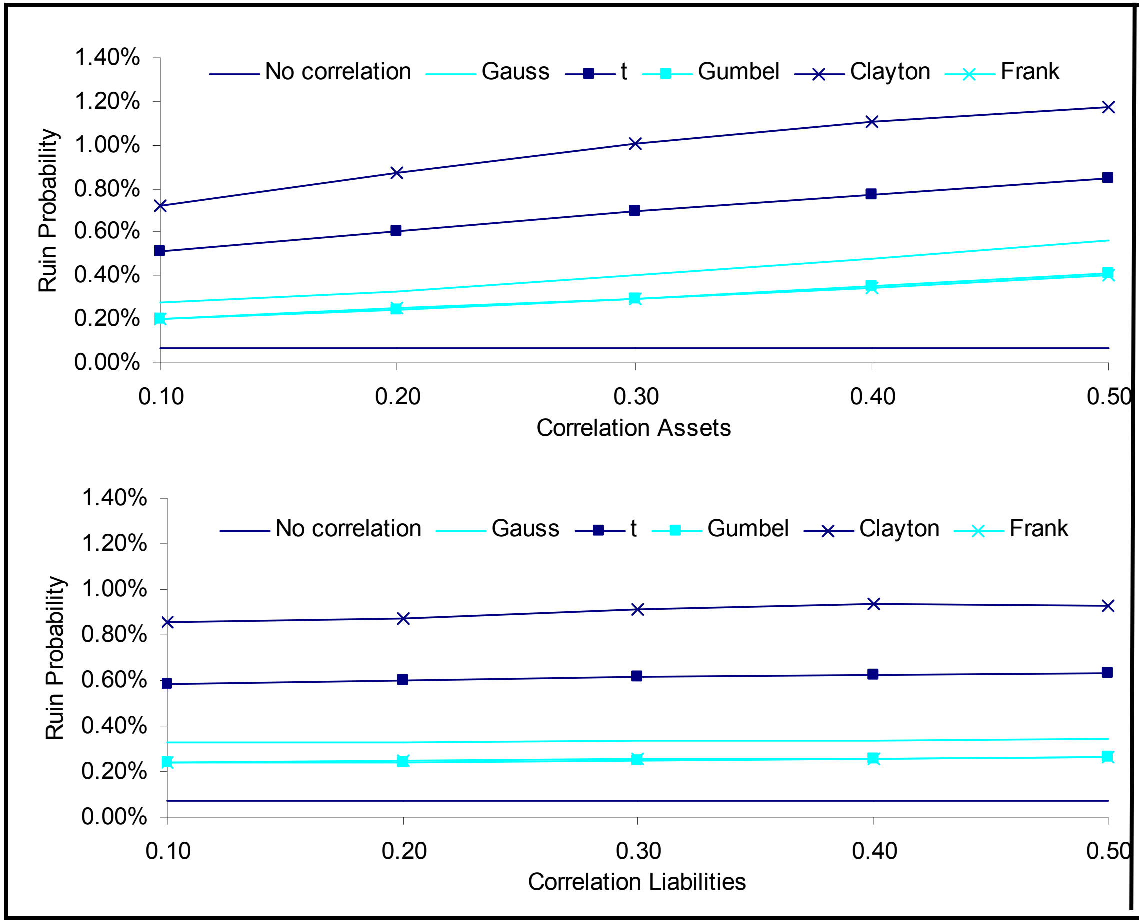
  ...

5 Simulation Study

- Variation of the equity capital in t=0 (from €50 to €100 million):



- Variation of correlation settings:



6 Management Strategies

- Implemented three management strategies

Strategy	Solvency	Reinsurance	Growth	
Target	Risk Reduction	Risk Reduction	Risk Reduction and Risk Taking	
Trigger	$EC_t < MCR_t \cdot 1.5$	Losses > €200 million	$EC_t <$ $MCR_t \cdot 1.5$	$EC_t >$ $MCR_t \cdot 1.5$
Rule	α and β 0.05 ↓	Indemnity = $\min(\max(C_t\text{-}200,0),40)$	α and β 0.05 ↓	β 0.05 ↑

6 Management Strategies

Dependence structure	No corr.	Gauss	t	Gumbel	Clayton	Frank
Tail dependence	no	no	upper and lower	upper	lower	no
Solvency strategy						
E(G) in million €	30.63	30.12	30.14	30.22	29.83	30.19
RP	0.06%	0.31%	0.52%	0.20%	0.78%	0.20%
EPD in million €	0.00	0.03	0.07	0.84	1.77	0.03
SR RP	255.82	45.46	26.87	68.82	17.53	68.14
SR EPD	30.66	4.70	2.08	0.16	0.08	5.02
Growth strategy						
E(G) in million €	35.05	34.38	34.42	34.52	34.04	34.47
RP	0.09%	0.45%	0.71%	0.31%	1.04%	0.30%
EPD in million €	0.01	0.05	0.10	0.86	1.84	0.04
SR RP	183.50	35.76	22.70	52.46	15.20	53.01
SR EPD	19.31	3.25	1.56	0.19	0.09	3.96
Reinsurance strategy						
E(G) in million €	29.51	29.12	29.14	29.18	28.89	29.16
RP	0.03%	0.22%	0.32%	0.16%	0.53%	0.16%
EPD in million €	0.00	0.02	0.03	0.83	1.70	0.02
SR RP	431.83	60.20	42.19	84.57	25.14	82.83
SR EPD	55.15	6.71	4.36	0.16	0.08	7.91

7 Conclusion

- Implementation of copulas in a DFA framework
- Effects on the insurer's risk and return position:
  - return not affected
  - ruin probability and expected policyholder deficit extremely affected
  - lower tail dependent copulas induce highest risk in our model
- Increase of equity capital reduces ruin probability, but not necessarily EPD
- Simple risk reduction strategies do not affect the risk profile when nonlinear dependencies are considered
- Reinsurance delimits ruin probability but not EPD

- Gauss-Copula

$$C_P^{Gauss}(\mathbf{u}) = \mathbf{\Phi}_P(\Phi^{-1}(u_1), \Phi^{-1}(u_2), \Phi^{-1}(u_3), \Phi^{-1}(u_4))$$

- *t*-Copula

$$C_{\nu,P}^{t}(\mathbf{u}) = \mathbf{t}_{\nu,P}(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2), t_\nu^{-1}(u_3), t_\nu^{-1}(u_4))$$

- relationship Kendall’s Tau and off-diagonal elements of correlation matrix

$$\rho_\tau(X_i, X_j) = (2/\pi)\arcsin \rho_{ij}$$

- Nonexchangeable Archimedean copula

$$C(u_1,u_2,u_3,u_4) = \phi_3^{-1}(\phi_3 \circ \underbrace{\phi_1^{-1}(\phi_1(u_1)+\phi_1(u_2)}_{high-risk\ and\ low-risk\ investments}) + \phi_3 \circ \underbrace{\phi_2^{-1}(\phi_2(u_3)+\phi_2(u_4))}_{non-catastrophe\ and\ catastrophe\ losses})$$