ENHANCING INSURER VALUE THROUGH CAPITAL, DIVIDENDS AND REINSURANCE OPTIMIZATION:

SOMETHING OLD, SOMETHING NEW

Yuriy Krvavych, *PhD – Insurance Australia Group (IAG), Sydney*

ASTIN Colloquium – *Orlando FL, June 2007*

Insurance Australia Group

Agenda

- Motivation:
  - What do financial companies optimize?
  - Reinsurance! Is it relevant to the maximization of insurance firm's economic value?
- Models of reinsurance optimization
- Results

Agenda

- Motivation:
  - What do financial companies optimize?
  - Reinsurance! Is it relevant to the maximization of insurance firm's economic value?
- Models of reinsurance optimization
- Results

Agenda

- Motivation:
  - What do financial companies optimize?
  - Reinsurance! Is it relevant to the maximization of insurance firm's economic value?
- Models of reinsurance optimization
- Results

Agenda

- Motivation:
  - What do financial companies optimize?
  - Reinsurance! Is it relevant to the maximization of insurance firm's economic value?
- Models of reinsurance optimization
- Results

Agenda

- Motivation:
  - What do financial companies optimize?
  - Reinsurance! Is it relevant to the maximization of insurance firm's economic value?
- Models of reinsurance optimization
- Results

Introduction

What do insurers optimize?

- Stock insurers resemble financial corporations: they leverage themselves by issuing risky debt, i.e. insurance policies;
- Why issue insurance debt? Insurers have competitive advantage in creating value by borrowing in insurance (not capital) market;
- Insurers are financed by their principals (shareholders);
- **Answer: Insurer economic value!**
  - Shareholders (equity) capital is used to satisfy solvency requirements imposed by a regulator;
  - Shareholders of the insurance company are well diversified in a capital market;
  - Conclusion: the main (natural) insurer's objective is to maximize the shareholders' value under solvency constraints imposed by a regulator.

What do insurers optimize?

- Stock insurers resemble financial corporations: they leverage themselves by issuing risky debt, i.e. insurance policies;
- Why issue insurance debt? Insurers have competitive advantage in creating value by borrowing in insurance (not capital) market;
- Insurers are financed by their principals (shareholders);
- **Answer: Insurer economic value!**
  - Shareholders (equity) capital is used to satisfy solvency requirements imposed by a regulator;
  - Shareholders of the insurance company are well diversified in a capital market;
  - Conclusion: the main (natural) insurer's objective is to maximize the shareholders' value under solvency constraints imposed by a regulator.

What do insurers optimize?

- Stock insurers resemble financial corporations: they leverage themselves by issuing risky debt, i.e. insurance policies;
- Why issue insurance debt? Insurers have competitive advantage in creating value by borrowing in insurance (not capital) market;
- Insurers are financed by their principals (shareholders);
- **Answer: Insurer economic value!**
  - Shareholders (equity) capital is used to satisfy solvency requirements imposed by a regulator;
  - Shareholders of the insurance company are well diversified in a capital market;
  - Conclusion: the main (natural) insurer's objective is to maximize the shareholders' value under solvency constraints imposed by a regulator.

What do insurers optimize?

- Stock insurers resemble financial corporations: they leverage themselves by issuing risky debt, i.e. insurance policies;
- Why issue insurance debt? Insurers have competitive advantage in creating value by borrowing in insurance (not capital) market;
- Insurers are financed by their principals (shareholders);
- **Answer: Insurer economic value!**
  - Shareholders (equity) capital is used to satisfy solvency requirements imposed by a regulator;
  - Shareholders of the insurance company are well diversified in a capital market;
  - Conclusion: the main (natural) insurer's objective is to maximize the shareholders' value under solvency constraints imposed by a regulator.

What do insurers optimize?

- Stock insurers resemble financial corporations: they leverage themselves by issuing risky debt, i.e. insurance policies;
- Why issue insurance debt? Insurers have competitive advantage in creating value by borrowing in insurance (not capital) market;
- Insurers are financed by their principals (shareholders);
- **Answer: Insurer economic value!**
  - Shareholders (equity) capital is used to satisfy solvency requirements imposed by a regulator;
  - Shareholders of the insurance company are well diversified in a capital market;
  - Conclusion: the main (natural) insurer's objective is to maximize the shareholders' value under solvency constraints imposed by a regulator.

What do insurers optimize?

- Stock insurers resemble financial corporations: they leverage themselves by issuing risky debt, i.e. insurance policies;
- Why issue insurance debt? Insurers have competitive advantage in creating value by borrowing in insurance (not capital) market;
- Insurers are financed by their principals (shareholders);
- **Answer: Insurer economic value!**
  - Shareholders (equity) capital is used to satisfy solvency requirements imposed by a regulator;
  - Shareholders of the insurance company are well diversified in a capital market;
  - Conclusion: the main (natural) insurer's objective is to maximize the shareholders' value under solvency constraints imposed by a regulator.

What do insurers optimize?

- Stock insurers resemble financial corporations: they leverage themselves by issuing risky debt, i.e. insurance policies;
- Why issue insurance debt? Insurers have competitive advantage in creating value by borrowing in insurance (not capital) market;
- Insurers are financed by their principals (shareholders);
- **Answer: Insurer economic value!**
    - Shareholders (equity) capital is used to satisfy solvency requirements imposed by a regulator;
    - Shareholders of the insurance company are well diversified in a capital market;
    - Conclusion: the main (natural) insurer's objective is to maximize the shareholders' value under solvency constraints imposed by a regulator.

Incentives to reinsure

- **Overcapitalization does not mean high return on equity:** there is tradeoff between the purchase of reinsurance and the risk capital required to maximize shareholders' value;
- **Frictional costs such as corporate tax and financial distress costs:**
  - reinsurance may create an additional layer of synthetic equity capital to mitigate expected financial distress costs;
  - dividend and capital structure play a key role in mitigating frictional costs

Incentives to reinsure

- **Overcapitalization does not mean high return on equity:** there is tradeoff between the purchase of reinsurance and the risk capital required to maximize shareholders' value;
- **Frictional costs such as corporate tax and financial distress costs:**
  - reinsurance may create an additional layer of synthetic equity capital to mitigate expected financial distress costs;
  - dividend and capital structure play a key role in mitigating frictional costs

– Models and analysis –

something old, something new

Single period models

Models of reinsurance optimization

Model M1 (*maximization of return on equity using two control variables: change-loss reinsurance and risk capital*):

$$\begin{array}{ll} \textbf{maximize} & 1+\rho(u;a,b) = \frac{V(u,a,b)}{u} = \frac{\mathbb{E}[\max\{0,u+P(a,b)-I_{a,b}(X)\}]}{u}, \\ \textbf{subject to} & u \geq u_{\min} \text{ and } (a,b) \in [0,1]\times[0,\infty), \end{array}$$

where

$$I_{a,b}(X) = X - a(X-b)_+ \quad \text{is the retained risk ;}$$

$$u = u_{\min} + v = \mathrm{VaR}_\alpha[X] - P + v \quad \text{is the risk capital}$$

Single period models (cont)

Model M2 (maximization of return on equity through reinsurance):

$$\begin{aligned} &\textbf{maximize} \quad 1+\rho(a,b) = \frac{V(a,b)}{u_{\min}(a,b)} = \frac{\mathbb{E}[\max\{0, u_{\min}(a,b)+P(a,b)-I_{a,b}(X)\}]}{u_{\min}(a,b)}, \\ &\textbf{subject to} \quad (a,b) \in [0,1] \times [0,\infty), \end{aligned}$$

where

$u_{\min}(a,b) = \mathrm{VaR}_\alpha[I_{a,b}(X)] - P(a,b)$ is the minimal value of risk capital

altered by reinsurance

M1 and M2 with corporate tax

The shareholders' expected after-tax terminal value is equal to:

- within the model M1

$$\begin{aligned}\widetilde{V}(u,a,b) &= \mathbb{E}_i\left[\mathbb{E}_{I_{a,b}(X)}\left[\max\left\{(1+i)\left(u+P(a,b)\right)-I_{a,b}(X);\,0\right\}\mid\, i\right]\right]\\ &\quad -\tau\mathbb{E}_i\left[\mathbb{E}_{I_{a,b}(X)}\left[\max\left\{i\left(u+P(a,b)\right)+P(a,b)-I_{a,b}(X);\,0\right\}\mid\, i\right]\right]\end{aligned}$$

- within the model M2

$$\begin{aligned}\widetilde{V}(a,b) &= \mathbb{E}_i\left[\mathbb{E}_{I_{a,b}(X)}\left[\max\left\{(1+i)\left(u(a,b)+P(a,b)\right)-I_{a,b}(X);\,0\right\}\mid\, i\right]\right]\\ &- \tau\mathbb{E}_i\left[\mathbb{E}_{I_{a,b}(X)}\left[\max\left\{i\left(u(a,b)+P(a,b)\right)+P(a,b)-I_{a,b}(X);\,0\right\}\mid\, i\right]\right]\end{aligned}$$

Results of M1 and M2

- **Result 1.** The model M1 does not induce demand for reinsurance in maximization of return on equity. For every predetermined level of return on equity the excess of risk capital decreases with the amount of reinsurance.
- **Result 2.** The model M1 induces demand for reinsurance in maximization of return on equity in the presence of corporate tax.
- **Result 3.** An optimal tradeoff between the required minimal level of the risk capital and purchase of reinsurance occurs in the model M2.

Krvavych, Y. and Sherris, M. **Enhancing insurer value through reinsurance optimization**, *Insurance: Mathematics and Economics*, Vol. 38, pp. 495-517, 2006.

Results of M1 and M2

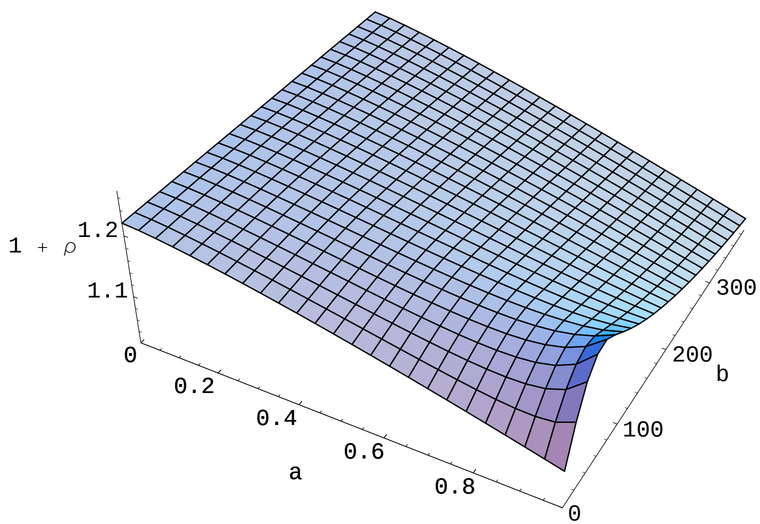
Graphical illustrations of the results:

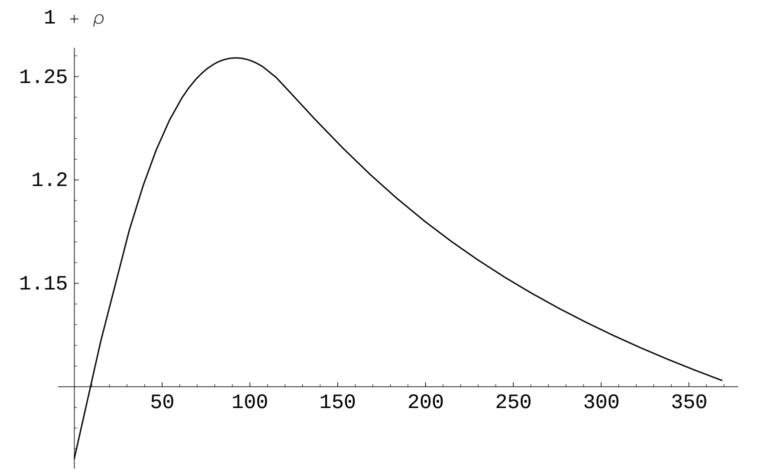
Assumptions:

- Claims are exponentially distributed $F(x) = 1 - e^{-\gamma x}$, $\gamma = 0.01$;
- $\mathrm{VaR}_\alpha[X] = -\frac{\ln(1-\alpha)}{\gamma}$ with $\alpha = 0.975$;
- The mean value premium principle is applied: $P = (1+\theta)\mathbb{E}_F[X] = \mathbb{E}_G[X]$, where $G(x) = F(kx)$ and $k = \frac{1}{1+\theta} \in (0.1)$ is a risk adjustment coefficient.
- Given insurer's risk loading θ (there is no unique θ in incomplete insurance market) reinsurer's loading for change-loss reinsurance contract is $\eta(b,\theta) = \frac{1}{k}\left(\int_{bk}^{\infty}(1-F(x))dx\right) / \left(\int_{b}^{\infty}(1-F(x))dx\right) - 1 > \frac{1}{k} - 1 = \theta;$ $\theta = 0.4$

Results of M1 and M2

Graphical illustration of the Result 2:





Graphical illustration of the total return $1 + \rho(u_{\min}, a, b)$ in the model M1 with corporate tax $\tau = 30\%$

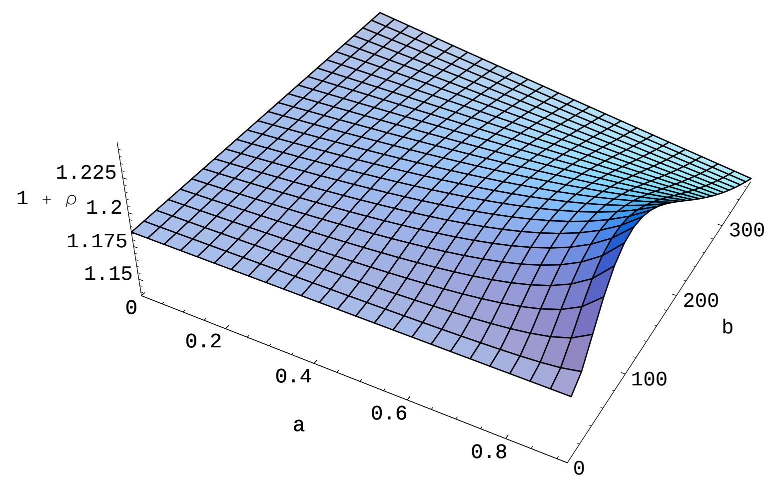
Results of M1 and M2

Illustration of the Result 2:

τ	Optimal reinsurance	Maximal return on equity ρ^*
15%	$b^* = \infty$ or $a^* = 0$	26.83%
20%	$b^* = \infty$ or $a^* = 0$	25.492%
25%	$b^* = 99.31$ and $a^* = 1$	26.47%
30%	$b^* = 93.73$ and $a^* = 1$	26.01%
35%	$b^* = 87.69$ and $a^* = 1$	25.302%
40%	$b^* = 82.07$ and $a^* = 1$	24.58%

Results of M1 and M2

Graphical illustration of the Result 3:



Graphical illustration of the total return on equity as the function $1 + \rho(a, b) = \frac{V(a,b)}{u(a,b)}$ defined on $\{a \in [0, a_1]\} \cap \{b \leq \mathrm{VaR}_\alpha[X]\}$ (domain of feasible reinsurance contracts)

Results of M1 and M2

Remarks:

1) Model M2 induces demand for reinsurance in frictionless environment (without tax) under the assumption that gross premiums are not dependent on capital or reinsurance of the insurer (i.e. the gross premium does not reflect the effect of insolvency on policy payoff);
2) Model M2 should not induce any demand for reinsurance when the gross premium is adjusted with respect to the value of insolvency exchange option, unless frictional costs such as taxes and costs of financial distress are included. The adjusted gross premium P is a solution to the equilibrium system of two equations

$$\begin{aligned} P &= e^{-r}\mathbb{E}_{\mathbb{Q}}\left[L_1 - (L_1 - A_1)\,\mathbf{1}_{\{A_1<L_1\}}\right] \\ E_0 &= e^{-r}\mathbb{E}_{\mathbb{Q}}\left[(A_1 - L_1)\,\mathbf{1}_{\{A_1>L_1\}}\right], \end{aligned}$$

where $A_1 = (1 + r_A)(P + E_0)$ is the terminal value of assets; E_0 - present value of equity.

Yet another single period model: M3

Maximization of shareholders value in the presence of financial distress costs

Consider an insurance company over the period of time $[0, T]$ (the period between two consecutive audits), and three economic states of the insurer:

- "financially distressed ($m_T^S \leq D$) & solvent ($F(S_T) > S^*$)";
- "healthy ($m_T^S > D$) & solvent ($S_T > S^*$)";
- "insolvent",

where S_t - the company's surplus at time $t \in [0, T]$;
$m_T^S = \min_{t \in [0,T]} S_t$; D $(0 < D < S^*)$ - financial distress barrier;
$S_T - F(S_T)$ - financial distress (FD) costs;
S^* - the company's minimal capitalization level (regulatory capital).

Model M3 (contd)

Model insurer's surplus (net worth) by geometric Brownian motion (M. Powers, 1995):

$$dS_t = \mu S_t dt + \sigma\, S_t dW_t,$$

where $\mu = a\lambda\,(\pi_L(1-\varepsilon_P) - (\varepsilon_L + \varepsilon_P) + r_I) + r_I$,
$\sigma = \sqrt{a^2\lambda^2\sigma_L^2 + (a\lambda+1)^2\sigma_I^2}$;
company's assets consist of insurance loss reserves (L) and surplus (S), which are invested in the capital market.

Model M3 (contd)

The terminal value of company's surplus net of regulatory capital is:

$$\begin{aligned} V_0 &= \mathbb{E}_Q\left[(S_T - S^*)\mathbf{1}_{\{\text{Healthy}\}} \right. \\ &\quad \left. + (F(S_T) - S^*)\mathbf{1}_{\{\text{FinDistress}\,\&\,\text{Solvent}\}}\right] \\ &= \mathbb{E}_Q\left[(S_T - S^*)^+\,\mathbf{1}_{\{m_T^S > D\}} + (F(S_T) - S^*)^+\,\mathbf{1}_{\{m_T^S \le D\}}\right] \\ &= \mathbb{E}_Q\left[(S_T - S^*) - (S_T - F(S_T))\,\mathbf{1}_{\{m_T^S \le D\}\cap\{F(S_T) > S^*\}} \right. \\ &\quad \left. + (S^* - S_T)\left(\mathbf{1}_{\{S_T \le S^*\}} + \mathbf{1}_{\{m_T^S \le D\}\cap\{S_T > S^* \ge F(S_T)\}}\right)\right] \end{aligned}$$

where Q is an equivalent martingale measure (we use the Numeraire Invariance Theorem and set risk-free rate to 0)

Model M3 (contd)

Possible forms of FD costs:
1) deadweight losses are proportional to the terminal value of company's surplus with proportionate coefficient $1 - w,\ w \in (0, 1)$:

$$F_1(S_T) = w\, S_T,$$

(empirical studies show that in practice $1 - w$ is 10%-20% for production firms and 15%-25% for insurance companies);
2) deadweight losses are in the form of lost upside potential of terminal value of company's surplus:

$$F_2(S_T) = (S_T - S^*)^+ - (S_T - (S^* + U))^+,$$

where $U > 0$ is the parameter of FD costs.

Model M3 (contd)

We consider $F(S_T) = F_2(S_T)$ (i.e. model with FD costs that come in the form of lost upside potential of surplus), and maximize the value V_0, w.r.t. company's risk $\sigma(\alpha, \sigma_I)$, as a value of two different barrier options:

$$\begin{aligned} V_0 &= \mathbb{E}_Q\left[(S_T - S^*)^+ \mathbf{1}_{\{m_T^S > D\}} + (F(S_T) - S^*)^+ \mathbf{1}_{\{m_T^S \le D\}}\right] \\ &= \mathbb{E}_Q\Big[(S_T - S^*)^+ \mathbf{1}_{\{m_T^S > D\}} + (S_T - S^*)^+ \mathbf{1}_{\{m_T^S \le D\} \cap \{S_T \le S^* + U\}} \\ &\quad + U\, \mathbf{1}_{\{m_T^S \le D\} \cap \{S_T > S^* + U\}}\Big] \end{aligned}$$

Results of the Model M3

Result 4. There are risk-management incentives in maximization of the value V_0 (i.e. shareholders value, since solvent company pays dividends from the value V_0). At time 0 risk managers optimally choose a level of the company's risk $\widehat{\sigma}(a, \sigma_I)$ to maximize the value V_0:

$$\widehat{\sigma}^2 = \frac{1}{T} \frac{\ln\left(\frac{D^2}{S^*(S^*+U)}\right) \ln\left(\frac{D^2 S^*}{S_0(S^*+U)}\right)}{\ln\left(\frac{S^*+U}{S^*}\right)}$$

Krvavych, Y. and Sherris, M. **Reinsurance optimization in the presence of financial distress cost**, *UNSW Working Paper*, 2006.

Results of the Model M3

Result 5.

- For intermediate FD costs (i.e. $\exists U' : \forall U > U'$) the company's optimal risk decreases with an increase in the FD costs;
- In this case risk managers can decrease the optimal value of the company's (integrated investment-underwriting) risk

$$\widehat{\sigma} = \widehat{\sigma}(\alpha, \sigma_I) = \sqrt{a^2\lambda^2\sigma_L^2 + (a\lambda + 1)^2\sigma_I^2}$$

by both the quota share a of proportional reinsurance and the investment risk σ_I.

Multi-period period optimization model

Insurer preference ordering under solvency constraints

- Insurer's solvency constraints are primarily defined by:
  - ε - the absolute ruin probability (primary risk measure, e.g. 1-in-750 yrs),
  - the probability of regulatory distress (secondary risk measure, e.g. 1-in-10 yrs chance of surplus falling below MCR);

Multi-period period optimization model

Insurer preference ordering under solvency constraints

- Insurer's solvency constraints are primarily defined by:
  - ε - the absolute ruin probability (primary risk measure, e.g. 1-in-750 yrs),
  - the probability of regulatory distress (secondary risk measure, e.g. 1-in-10 yrs chance of surplus falling below MCR);
- Using the primary risk measure one can estimate the premium to be $P = \mathbb{E}[L] + \frac{\rho}{2}\mathrm{Var}[L] + o(\rho)$, where L is a total underwriting loss, and ρ denote the adjustment coefficient;

Multi-period period optimization model

Insurer preference ordering under solvency constraints

- Insurer's solvency constraints are primarily defined by:
  - ε - the absolute ruin probability (primary risk measure, e.g. 1-in-750 yrs),
  - the probability of regulatory distress (secondary risk measure, e.g. 1-in-10 yrs chance of surplus falling below MCR);
- Using the primary risk measure one can estimate the premium to be $P = \mathbb{E}[L] + \frac{\rho}{2}\mathrm{Var}[L] + o(\rho)$, where L is a total underwriting loss, and ρ denote the adjustment coefficient;
- On the other hand, using zero utility premium principle one can show that $P \approx \mathbb{E}[L] + \frac{r(x)}{2}\mathrm{Var}[L]$, where $r(x)$ is the Arrow-Pratt measure of absolute risk aversion, and $S_0 = x$;

Multi-period period optimization model

Insurer preference ordering under solvency constraints

- Insurer's solvency constraints are primarily defined by:
  - ε - the absolute ruin probability (primary risk measure, e.g. 1-in-750 yrs),
  - the probability of regulatory distress (secondary risk measure, e.g. 1-in-10 yrs chance of surplus falling below MCR);
- Using the primary risk measure one can estimate the premium to be $P = \mathbb{E}[L] + \frac{\rho}{2}\,\mathrm{Var}[L] + o(\rho)$, where L is a total underwriting loss, and ρ denote the adjustment coefficient;
- On the other hand, using zero utility premium principle one can show that $P \approx \mathbb{E}[L] + \frac{r(x)}{2}\,\mathrm{Var}[L]$, where $r(x)$ is the Arrow-Pratt measure of absolute risk aversion, and $S_0 = x$;
- This implies that $r(x) = \frac{|\ln(\varepsilon)|}{x}$, and thus the insurer's utility function U is isoelastic under solvency constraints and equals to $U = \frac{x^{1-m}}{1-m}$ with $m = |\ln(\varepsilon)|$

Multi-period period optimization model

- **We model insurer's surplus (net worth)** by geometric Brownian motion (M. Powers, 1995):

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where $\mu = a\lambda\left(\pi_L(1-\varepsilon_P) - (\varepsilon_L + \varepsilon_P) + r_I\right) + r_I$, $\sigma = \sqrt{a^2\lambda^2\sigma_L^2 + (a\lambda+1)^2\sigma_I^2}$; a denote the retention level of quota share proportional reinsurance; λ is the leverage ratio;

- Taking into account the dividend payments at the rate $d_t = \delta\, S_t$, we obtain the *reflected Itô diffusion process* of the company's surplus

$$dS_t = \mu S_t dt + \sigma S_t dW_t - dD_t = (\mu - \delta) S_t dt + \sigma S_t dW_t,$$

where $D_t = \int_0^t d_t dt = \delta \int_0^t S_t dt.$

Multi-period period optimization model

Shareholder utility maximization

- We define the shareholder value V as the expected present value of utility of future dividend payments up to insolvency time τ:

$$V(x) = V(x; a, \lambda, \delta) = \mathbb{E}\left[\int_0^{\tau} \mathrm{e}^{-\gamma s} U(d_s)\, ds + \mathrm{e}^{-\gamma\tau}\, U(B)\right],$$

with the boundary condition $V(x^*) = U(B)$, where $x^* < S_0 = x$ denote the minimal capitalization level, B is the insolvency cost, and γ is the force of interest;

Multi-period period optimization model

Shareholder utility maximization

- We define the shareholder value V as the expected present value of utility of future dividend payments up to insolvency time τ:

$$V(x) = V(x; a, \lambda, \delta) = \mathbb{E}\left[\int_0^\tau e^{-\gamma s} U(d_s)\, ds + e^{-\gamma\tau}\, U(B)\right],$$

  with the boundary condition $V(x^*) = U(B)$, where $x^* < S_0 = x$ denote the minimal capitalization level, B is the insolvency cost, and γ is the force of interest;

- The corresponding Hamilton-Jacobi-Bellman (HJB) equation which the value function V must satisfy is

$$\frac{1}{2}\sigma^2 x^2 V''(x) + (\mu - \delta)\, x\, V'(x) - \gamma\, V(x) + U(\delta x) = 0, \quad x \geq x^*$$

Multi-period period optimization model

Optimal solution

- The solution to the HJB equation is:

$$V(x;a,\lambda,\delta) = \left[\frac{B^{1-m}}{1-m} - g(a,\lambda,\delta)\,(x^*)^{1-m}\right]\left(\frac{x}{x^*}\right)^{-\theta} + g(a,\lambda,\delta)\,x^{1-m},$$

where $g(a,\lambda,\delta) = -\frac{\delta^{1-m}}{1-m}\left(\frac{1}{2}\sigma^2\,(1-m)^2 + \left(\mu-\delta-\frac{1}{2}\sigma^2\right)(1-m) - \gamma\right)^{-1}$, and
$\theta = \frac{-\left(\mu-\delta-\frac{1}{2}\sigma^2\right)+\sqrt{\left(\mu-\delta-\frac{1}{2}\sigma^2\right)^2+2\gamma\,\sigma^2}}{\sigma^2};$

Multi-period period optimization model

Optimal solution

- The solution to the HJB equation is:

$$V(x;a,\lambda,\delta) = \left[\frac{B^{1-m}}{1-m} - g(a,\lambda,\delta)\,(x^*)^{1-m}\right]\left(\frac{x}{x^*}\right)^{-\theta} + g(a,\lambda,\delta)\,x^{1-m},$$

where $g(a,\lambda,\delta) = -\frac{\delta^{1-m}}{1-m}\left(\frac{1}{2}\sigma^2(1-m)^2 + \left(\mu-\delta-\frac{1}{2}\sigma^2\right)(1-m) - \gamma\right)^{-1}$, and $\theta = \frac{-\left(\mu-\delta-\frac{1}{2}\sigma^2\right)+\sqrt{\left(\mu-\delta-\frac{1}{2}\sigma^2\right)^2+2\gamma\,\sigma^2}}{\sigma^2}$;

- One can maximize the value function V to find optimal uncontrolled variables: *retention level of quota share proportional reinsurance, leverage ratio* and *dividend rate*

$$(a^*,\lambda^*,\delta^*) = \arg \max_{a\in(0,1);\,\lambda>0;\,\delta\in(0,1)} V(x;a,\,\lambda,\,\delta).$$

Conclusion

Conclusion

- In the model M1 (conservative model) an insurer is well-capitalized (overcapitalized?). It does not allow the insurer to reduce the minimum level of capitalization through purchasing reinsurance. In this model there is no demand for reinsurance in a frictionless environment. However, there is demand for reinsurance in this model with reasonably high value of frictional costs, such as corporate tax;
- The model M2 imposes the demand for reinsurance in the frictionless environment (without tax), which is due to the assumption that the gross premium is not adjusted with respect to the value of insolvency put. For the same level of frictional costs the demand for reinsurance in the model M2 is higher than the one in the model M1;
- There are incentives to control the company's risk in the models M3 and M4 of maximization of shareholders value in the presence of FD/insolvency costs;
- The decision to reinsure can be treated as both a risk-management and a capital-structure tool for creating shareholders' value.