

Select Mortality - Aggregated Premium Rates

Erling Falk
Norske Liv Insurance
P.O. Box 1166 Sentrum N-0107 Oslo, Norway

Telephone: + 47 22 48 61 95
Facsimile: + 47 22 48 57 70
E-mail: erling.falk@norskeliv.no

Abstract

This paper deals with pure death risk insurance, based on a natural premium system. Due to the underwriting procedure the underlying death risk for an insured person will differ from the mortality of another insured person of the same age, if their policies have been issued at different points of time - the phenomenon of select mortality among insured lives. Despite this, it will be necessary to base the tariff on an aggregated rating system. Basing the tariff on select mortality tables will result in healthy people continuously lapsing their contracts and demanding the issuing premium rate, according to their actual age.

Setting an aggregated premium rate that is not loss-bringing is discussed, taking into consideration lapsing of contracts and including also a presumption regarding the force of entry as a function of age. The framework for the analysis is a multi-state time continuous Markov model.

Keywords

Death risk insurance, select mortality, force of entry, lapsing, aggregated rating.

Introduction

Ratings based on natural premiums, i.e. risk premiums reflecting expected claims, dominate the Norwegian death risk market. Hence, it follows that each premium, on continuous basis, depends on the insured person's age (and sex).

The companies use a classic filtration method before allowing new individuals into the portfolio. The person applying for insurance has to satisfy the individual company's requirements concerning health. However, there are wide limits for entering, but the less healthy will have the insurance cover priced at higher rates.

After having passed the underwriting procedure in such a way that the applicant is accepted at ordinary rates, the underlying death risk will be influenced by the fact that the applicant could document that he/she was healthy at the time the insurance was written. Hence the company theoretically should have offered premiums based on select mortality tables, in order to use a "fair rating", with premiums according to expected claims. Nevertheless, a rating system based on select mortality tables, combined with natural premiums, would not be practical, so long as healthy people can at any time lapse their policies and demand the ordinary issuing rate for their specific age, by continuously declaring their healthiness to the underwriters. At the same time, those who can no longer pass the underwriting test will remain in the system, paying the rates from the select table, which by then would be insufficient to cover the actual risk represented. Thus the basis for using the select tables explicitly for rating purposes is no longer present.

Hence, the natural premium has to be determined from an aggregated mortality table. Accordingly it will be a challenge for the company to offer a competitive premium rate, with an expected profitability. Furthermore, it is important that the rate is not subjected to frequent changes inasmuch as it is assumed that the market will have an aversion to such price instability.

Having established the principle of an aggregate rating system for this product, the question of pricing is discussed in the paper, where the impact of lapsing is taken into account as well as the impact of the initial selection according to the underwriting procedure.

Assumptions concerning the pattern of entry (the distribution of the age of the applicants) are made. To elucidate the impact that lapsing of contracts has on the claims ratio in the portfolio, the mortality in the death risk portfolio is compared with the mortality in the population.

The rating problem is analysed using a simple time-continuous Markov model with age-dependent forces of transition. The discussions are limited to initial selection, lapsing and pattern of issuing as technical elements. The randomness connected with death and disability is not dealt with.

The model

The Markov model outlined in Figure 1 is adopted from Norberg (1988). The population is grouped into four categories according to further definitions. In addition there is a category comprising the dead. At all times, each individual is assigned to one and only one state.

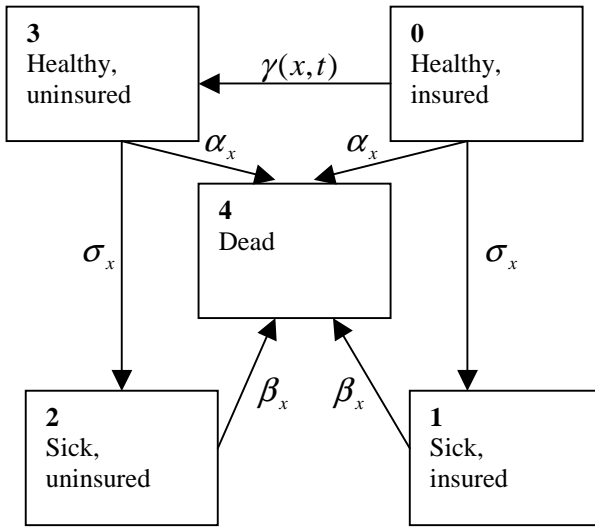


Figure 1. Markov model.

The states are defined according to the criteria healthy/sick and insured/uninsured. Note that we assume only one level for each of the states "healthy" and "sick" even though there should be an approach to a multi-state model of a far wider range, defining sub-categories of sick individuals ("frailty at level I" - "frailty at level II" - etc.) to give a more adequate description of the health situation. Norberg applies the terms "insurable" and "uninsurable" which might at first sight appear to be more precise, as the terms are linked to the result of a (theoretical) underwriting carried out, based on certain standards. However, most of those who apply for insurance will achieve the state "insured" on strengthened premium terms, and hence, if we compare the category "sick" and the, literally, small sub-category "uninsurable", the former is probably a more suitable term with regard to numerical implementation.

To be "insured" in this case means to have the death risk covered for a given insured sum from the time the insurance is issued and through to a common termination age ω .

This analysis concentrates on those people who find themselves in state **0** (after having been categorised as "healthy" following a health check). These will later change their state in accordance with a time-continuous Markov chain with age-dependent forces of transition. As indicated in the figure we based ourselves on the following forces of transition:

- From the "healthy" state (insured or uninsured) to the "sick" state at the age of x : σ_x
- From the "healthy" state (insured or uninsured) to the "dead" state at the age of x : α_x
- From the "sick" state (insured or uninsured) to the "dead" state at the age of x : β_x

Irrespective of the criteria used for the definition of the "sick" state all people with deadly illnesses will be in the sick state. It is therefore clear that mortality in this state will exceed mortality in the healthy state and we have

$$(1) \quad \beta_x > \alpha_x \quad \forall x > 0.$$

Norberg's model has been extended with the assumption of forces of transition $\gamma(x,t)$ between state **0** and state **3**, where t indicates the period of insurance cover. This force

indicates the lapsing frequency, i.e. the occurrence of withdrawal from the portfolio before reaching the age ω , and without any insurance event incurring. Even though this force of lapsing will reasonably be a function of both the age of the insured person and the length of the period the insurance has run, it is difficult to make model assumptions that can result in an explicit formula expression.

At the early stage of the insurance it is assumed that the rate of lapsing will be strongly influenced by changes in the needs that were the direct cause of the insurance being taken out. One might envisage aggregated lapsing tendency as an increasing function of age, as a result of the following two circumstances: a) a real decreasing need as the period of time from taking out the insurance increases (a general improvement in financial status, including lower debt liabilities and less need of extra inflow of dependants' capital) and b) accelerating premiums when the natural premium principle is applied.

In general it must also be assumed that the lapsing tendency is affected by social, economic and other trends in society.

Nevertheless in the calculations we use a constant γ based on alternatives that reflect the observed level for total lapsing in such portfolios (to be more precise portfolios of risk insurance sold through banks). We also make the following reasonable assumptions:

- The probability of returning to state **0** from states **1** and **3** is negligible.
- The probability of lapsing from state **1** is negligible.

By assuming that the "healthy" have forces of transition toward "dead" and "sick" that are independent of whether they are insured or not and that the same applies for a transition from "sick" state to "dead" state, we can also express the forces of mortality for a randomly selected x -year old in the population, based on the assumptions in the model.

We set the lowest age for this death risk insurance at x_0 , in other words a person must have reached the age of x_0 before being allowed to take out such insurance cover.

A key reference to multistate modelling in life insurance is Hoem (1969, 1988). The following notation is adopted:

Let $S(x)$ be the state of a randomly selected x -year old insured person and denote the relevant transition probabilities in the Markov process $\{S(x); x > x_0\}$ by

$$P_{0j}(u, x) = P\{S(x) = j \mid S(u) = 0\}; u < x; j = 0,1.$$

Solving the Kolmogorov differential equations derived from the Markov model (see Hoem), leads to the following expressions for the transition probabilities:

$$(2) \quad P_{00}(x-t, x) = e^{-\int_{x-t}^x (\sigma_u + \alpha_u + \gamma) du},$$

$$(3) \quad P_{01}(x-t, x) = \int_{x-t}^x e^{-\int_{x-t}^z (\sigma_u + \alpha_u + \gamma) du} \cdot \sigma_z \cdot e^{-\int_z^x \beta_u du} dz, \quad \text{for } 0 < t < x - x_0.$$

The selective mortality in the portfolio

The probability that a person who has been allowed to enter the insurance portfolio (state **0**) at the age of $x-t$ still being alive and a member of the portfolio at the age of x , will be

$$(4) \quad {}_t p_{[x-t]} = P_{00}(x-t, x) + P_{01}(x-t, x),$$

and the corresponding select force of mortality

$$(5) \quad \mu_{[x-t]+t} = \frac{\alpha_x \cdot P_{00}(x-t, x) + \beta_x \cdot P_{01}(x-t, x)}{P_{00}(x-t, x) + P_{01}(x-t, x)},$$

which by substituting from (2) and (3) gives

$$(6) \quad \mu_{[x-t]+t} = \alpha_x + \frac{(\beta_x - \alpha_x) \cdot \int_{x-t}^x \sigma_z \cdot e^{\int_{x-t}^z (\sigma_u + \alpha_u - \beta_u + \gamma) du} dz}{1 + \int_{x-t}^x \sigma_z \cdot e^{\int_{x-t}^z (\sigma_u + \alpha_u - \beta_u + \gamma) du} dz}.$$

Which can also be written as follows

$$(7) \quad \mu_{[x-t]+t} = \alpha_x + \frac{(\beta_x - \alpha_x)}{1 + r(x, t; \gamma)},$$

$$\text{where } r(x, t; \gamma) = \frac{1}{\int_{x-t}^x \sigma_z \cdot e^{\int_{x-t}^z (\sigma_u + \alpha_u - \beta_u + \gamma) du} dz}.$$

What one intuitively expects has been given an explicit expression in (7), namely that the select mortality may be described by the force of mortality for a healthy x -year old, α_x , being given an additive component. Since $\beta_x > \alpha_x$ per assumption, and at the same time $r > 0$, the additive component must be positive for all $t > 0$.

Inasmuch as $r(x, t)$ is a decreasing function of t , then $\mu_{[x-t]+t}$ will be an increasing function of t . However, we see also that $r(x, t)$, for given x and t , is a decreasing function of γ . The distance between α_x and $\mu_{[x-t]+t}$ thus increases with intensified lapsing.

Select insurance mortality versus mortality in the population

We revert to the Markov model and envisage degeneration by merging the states **0** and **3** and by merging states **1** and **2**. The insurance element is thus removed and we are left with three states in the model, namely healthy, sick and dead. Since the transitions are defined irrespective of the insurance state the degenerated model can be used to find an expression of

forces of mortality in the population $\bar{\mu}_x$. By taking as our starting point a newly born child who is included in the healthy state at birth, we can set $t=x$ in the equations (4) and (5), skip the forces of lapsing, and thus arrive at the following equation:

$$(8) \quad \bar{\mu}_x = \alpha_x + \frac{(\beta_x - \alpha_x) \cdot \int_0^x \sigma_z \cdot e^{\int_0^z (\sigma_u + \alpha_u - \beta_u) du} dz}{1 + \int_0^x \sigma_z \cdot e^{\int_0^z (\sigma_u + \alpha_u - \beta_u) du} dz},$$

which can be written

$$(9) \quad \bar{\mu}_x = \alpha_x + \frac{(\beta_x - \alpha_x)}{1 + r(x, x; 0)}.$$

In the same way that we have found the select insurance mortality above, the mortality in the population for the age x can be arrived at for people in the "healthy" state being given an additive component. This additive component will depend on age alone.

Mortality in the population will thus develop in line with mortality in a theoretical obligatory insurance scheme in which one becomes a member at birth and from which one cannot withdraw.

If we denote the mortality for an x -year old in an insurance scheme where withdrawal is not allowed (i.e. where $\gamma = 0$), with membership starting at $x-t$, by $\bar{\mu}_{[x-t]+t}$, we will necessarily have $\bar{\mu}_{[0]+x} = \bar{\mu}_x$, i.e. equality for $t = x$, which is t 's theoretical maximum value (membership in the scheme from birth).

In order to compare $\bar{\mu}_{[x-t]+t}$ and $\bar{\mu}_x$ for $t < x$ we consider

$$\bar{\mu}_x - \bar{\mu}_{[x-t]+t} = (\beta_x - \alpha_x) \cdot \frac{r(x, t; 0) - r(x, x; 0)}{(1 + r(x, x; 0))(1 + r(x, t; 0))}.$$

Since r is a decreasing function of t , and $\beta_x > \alpha_x$, we have

$$(10) \quad \bar{\mu}_{[x-t]+t} < \bar{\mu}_x \quad \forall t < x.$$

In the interpretation the difference is because the underwriting procedure that was carried out at the age of $(x-t) > 0$ ensures positive selection, and thus a lower force of mortality for x -year olds who have been insured for a period in an insurance scheme they cannot withdraw from, than the force of mortality for x -year olds in general in the population.

What can we say about the relation between $\bar{\mu}_x$ and $\mu_{[x-t]+t}$, i.e. the forces of mortality for a randomly selected x -year old and an x -year old who has been insured for t years in an ordinary

voluntary insurance scheme that can be withdrawn from at any point in time? Which impact has the phenomenon “lapsing” on the underlying force of mortality in the portfolio?

At first we have to define a technical basis. In the calculations that follow in this paper, the forces of mortality α_x and β_x are defined in accordance with Gompertz-Makeham. The same applies to the force of disability σ_x .

We make a numerical evaluation and the following assumptions:

- $x_0 = 20$,
- $\alpha_x = 0,0008 + 0,0000109 \cdot 10^{0,046 \cdot x}$,
- $\beta_x = 0,00126 + 0,0000109 \cdot 10^{0,046 \cdot x}$,
- $\sigma_x = 0,0008 + 0,0000021 \cdot 10^{0,0716 \cdot x}$.

The chosen α_x gives a representation of male mortality in the first year as insured (related to insurance sold through banks). σ_x is defined with reference to experienced force of transition in a relevant death risk portfolio, from "healthy" state to a "sick" state where the insured is entitled to disability payment in accordance with the Norwegian life companies standard terms. To determine β_x we suffer from lack of data, but we have been aiming at a determination of β_x which, together with the chosen α_x and σ_x , will lead to a "calculated mortality in the population", cf. (8), fairly close to the true male mortality in the Norwegian population.

A comparison between the "calculated mortality in the population" and the experienced Norwegian mortality in 1999 in accordance with SSB (2000), is shown in figure 2.



Figure 2 Comparing the calculated mortality with the experienced mortality.

To carry out a comparison between $\bar{\mu}_x$ and $\mu_{[x-t]+t}$ we have to make assumptions regarding γ . In the further calculations we give γ the values: 0, $\ln(1,10)$ and $\ln(1,20)$.

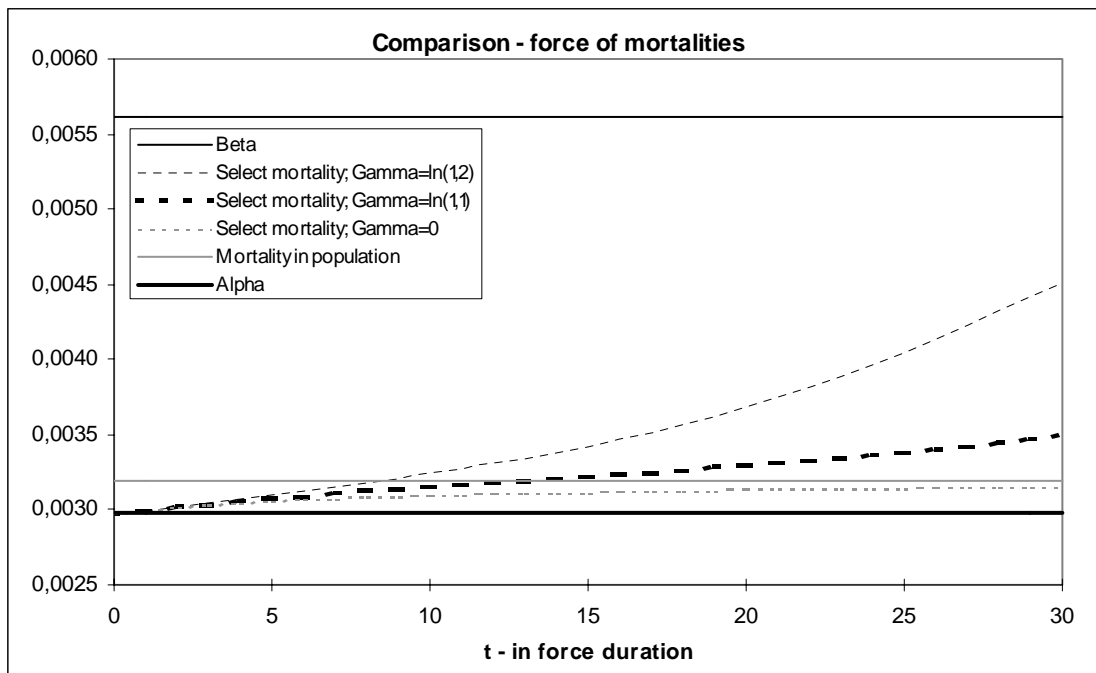


Figure 3 A comparison between Mortality in Population and Insurance Mortality. Male, age 50.

By considering mortality for insured people aged 50 years we find from the model that with a lapsing frequency at the 20% level people who have been insured for approximately 8 years will have a mortality rate in line with the mortality rate in the population. With a lapsing frequency of 10% we can see that for people who have been insured for less than 14 years mortality will be below the mortality rate of the population. But since $\mu_{[x-t]+t}$ is a rising function of t 50-year olds who have been insured for a longer period will, according to the model, have a mortality rate that is gradually considerably higher than the mortality rate in the population and this even higher than the mortality rate related to the "healthy state".

This is because the model assumes that the voluntary lapsing among the insured people only takes place among those who are "healthy". Thus, the mortality rate of people who have been insured over a longer period of time will exceed the mortality rate in the population for the same age group. The customary conception that an insured life (which at some point in time has been through an underwriting procedure) has a lower mortality rate than that of the population in general is not absolute.

However, with $\gamma = 0$ the insurance mortality rate will consistently be lower than the mortality rate in the population, cf. (10).

The matter of premium rates

How then should the tariffs be set for these 50-year olds, or the insured people in this type of scheme in general?

Define

$$\mu_u(x) = \mu_{[x_0]+x-x_0},$$

i.e. that we have

$$(11) \quad \mu_u(x) = \alpha_x + \frac{(\beta_x - \alpha_x)}{1 + r(x, x - x_0; \gamma)}.$$

This indicates the "ultimate value" for the insured x -year olds mortality i.e. μ -value for the x -year olds that have been insured for as long as it has been possible to be insured, namely for the period $x - x_0$.

We can also look at $\mu_u^{10}(x) = \alpha_x + \frac{(\beta_x - \alpha_x)}{1 + r(x, \min(x - x_0; 10))}$, i.e. that we look at the ultimate

values at that point in time where the longest possible insurance period is 10 years. This would be the case, for example, if the company or the product has not existed for more than 10 years.

The forces of mortality $\bar{\mu}_x, \alpha_x, \beta_x, \mu_u(x)$ and $\mu_u^{10}(x)$ are shown in table 1 for $x < 70$.

Age	Beta	Select mortality	Select mortality, with max 10 years	Mortality in the population	Alpha
20	0,001441	0,000891	0,000891	0,000900	0,000891
25	0,001568	0,000957	0,000957	0,000967	0,000954
30	0,001783	0,001073	0,001073	0,001080	0,001061
35	0,002148	0,001273	0,001260	0,001272	0,001244
40	0,002768	0,001628	0,001583	0,001603	0,001554
45	0,003821	0,002275	0,002146	0,002176	0,002081
50	0,005610	0,003495	0,003152	0,003194	0,002975
55	0,008647	0,005879	0,005038	0,005076	0,004493
60	0,013805	0,010600	0,008801	0,008775	0,007072
65	0,022564	0,019661	0,016670	0,016483	0,011452
70	0,037439	0,035786	0,032592	0,032379	0,018889

Table 1 Forces of mortality $\bar{\mu}_x, \alpha_x, \beta_x, \mu_u(x)$ and $\mu_u^{10}(x)$, with $\gamma = \ln(1,1)$.

Figure 4 is a graphic representation of these forces of mortality.

We see that from a start of ultimate values that are characterised by $\mu_u(x_0) = \alpha_{x_0}$, the ultimate values and the mortality rate in the population approach β -mortality of the older age groups.

As a curiosity we observe that μ_u^{10} virtually coincides with mortality in the population in figure 4.



Figure 4 Forces of mortality with $\gamma = \ln(1,1)$.

If we alter the γ -value to $\gamma = \ln 1,2$ we see that the impact is considerably stronger, and μ_u^{10} is also significantly higher than $\bar{\mu}$ for the upper part of the area under consideration.

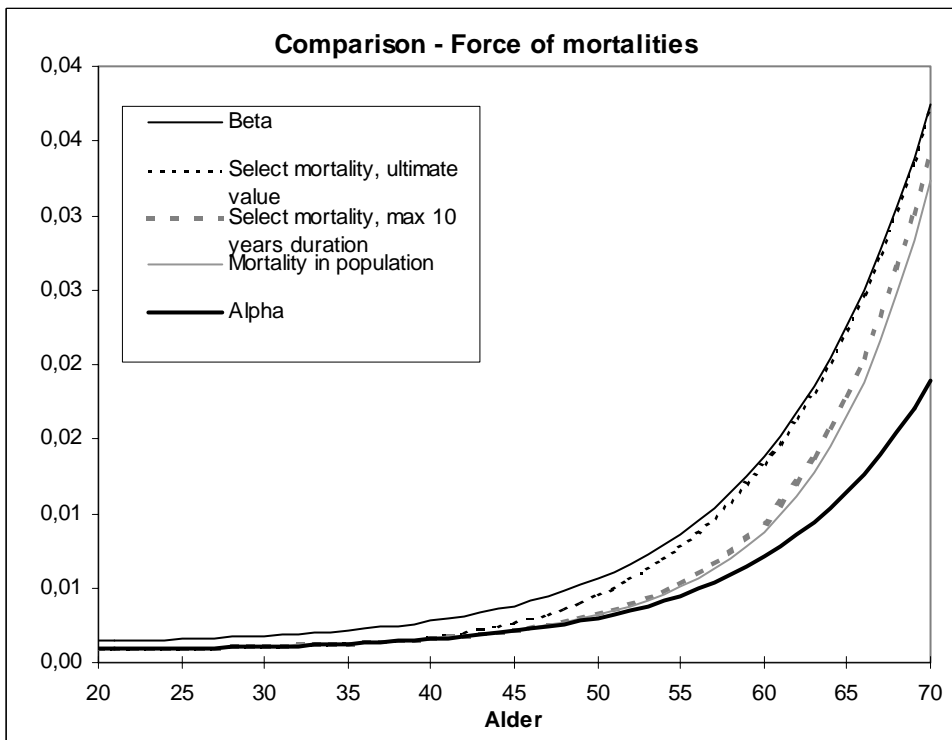


Figure 5 Forces of mortality with $\gamma = \ln(1,1)$.

The importance of new entries

Setting tariffs based on ultimate values for mortality will, as a result of the assumption of a maximum theoretical insurance period imply considerable security margins in relation to the actual risk inherent in the portfolio. For each age group there will be people who have been insured for a shorter period and who thus have a lower mortality than the ultimate value, inasmuch as $\mu_{[x-t]+t}$ increases with t .

Based on a randomly selected person who sooner or later takes out risk insurance cover we can define the cumulative probability function for his entry age X as

$$P(X \leq x) = F(x)$$

If we denote the corresponding probability density $\frac{dF(x)}{dx} = f(x)$, we have defined the following probability:

P (an x -year old uninsured person takes out insurance before reaching the age of $(x + dx)$ | The person will sooner or later take out insurance cover) = $f(x) \cdot dx$.

"The force of entry" thus reflects the tendency to take out insurance cover at the age of x , for those people who actually take out insurance cover.

By applying an assumption for a given force of entry we assume age-related neutrality with regard to the sum insured, or more precisely that the scheme is based on an equal sum insured for all.

At the same time, this "Force of entry" acts as a conditional force of transition between state **3** and state **0** in the Markov model, as a function of age, given that transition takes place.

Based on Norwegian experience regarding risk portfolios we assume that the function

$$(12) \quad f(x, \lambda) = (x - x_0) \cdot \lambda^2 \cdot e^{-\lambda(x-x_0)}$$

can provide a correct picture of the age-distributed force of entry.

With λ set as 0,08 we get a picture that corresponds well with the pattern of entry that has been observed for (bank related) risk insurance in Norway during the last couple of years.

Thus we have $E[X] = \frac{2}{\lambda} + x_0 = 45$, with a maximum for $x = \frac{1}{\lambda} + x_0 = 32,5$.

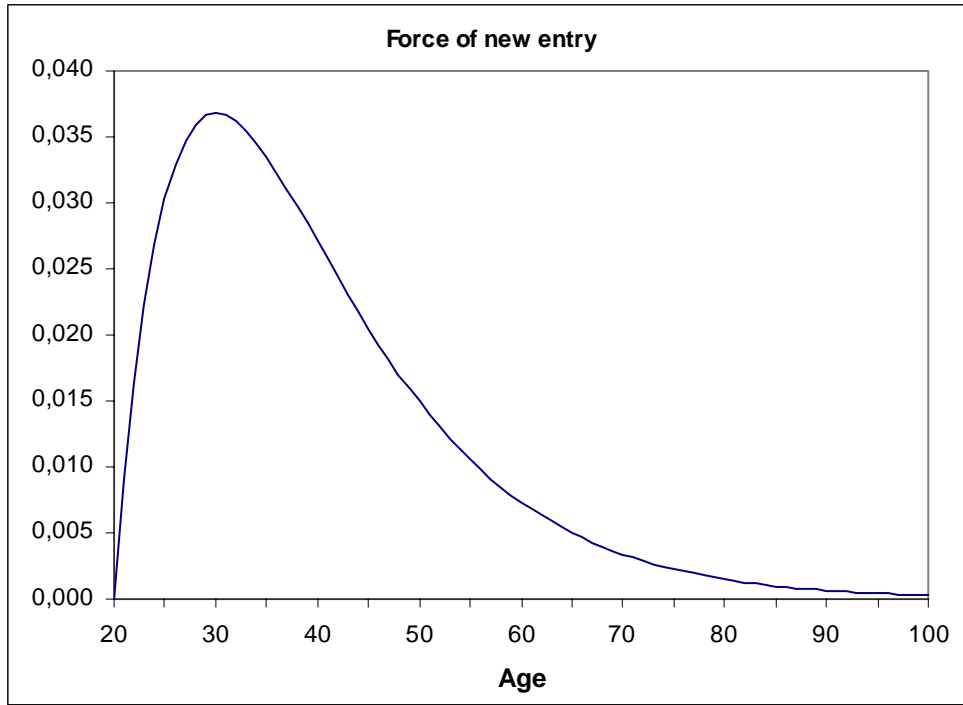


Figure 6 Forces of new entry with $x_0 = 20$ and $\lambda = 0,08$.

Based on the assumed pattern of entry and a given λ , which applied in a tariff relationship is denoted λ_1 , we find the following equivalence in order to obtain a fair, aggregated premium rate:

$$(13) \quad \mu_p(x) \cdot \int_{x_0}^x (\xi - x_0) \cdot \lambda_1^2 \cdot e^{-\lambda_1(\xi - x_0)} \cdot {}_{x-\xi}p_{[\xi]} d\xi = \int_{x_0}^x \mu_{[t]+x-\xi} \cdot (\xi - x_0) \cdot \lambda_1^2 \cdot e^{-\lambda_1(\xi - x_0)} \cdot {}_{x-\xi}p_{[\xi]} d\xi$$

where the left-hand side indicates the premium flow for the group x -year olds, assuming an aggregated continuous premium rate $\mu_p(x)$. The right-hand side gives the claims flow for the group, based on the select risk $\mu_{[t]+x-\xi}$, where t is the age at the time of entry. Noting the integral on the left-hand side as $g_p(x, \lambda_1)$ and the integral on the right-hand side as $g_c(x, \lambda)$, we have

$$(14) \quad \mu_p(x, \lambda_1) = \frac{g_c(x, \lambda)}{g_p(x, \lambda_1)}$$

A comparison between the derived aggregated premium rate μ_p and the calculated mortality in the population is carried out in table 2, based on the values $x_0=20$, $\lambda=0,08$ and $\gamma=\ln(1,2)$

We see that, if the assumption for an pattern of entry is correct, we can use a tariff that at least is under the mortality rate in the population, except for a small area around 60 years of age. With this tariff, expected balance will be achieved between incoming premium and outgoing claims for all age groups.

Age	Aggregated premium rate	Mortality in the population
25	0,000955	0,000967
30	0,001065	0,001080
35	0,001253	0,001272
40	0,001577	0,001603
45	0,002144	0,002176
50	0,003164	0,003194
55	0,005071	0,005076
60	0,008789	0,008775
65	0,016178	0,016483
70	0,030462	0,032379

Table 2 Premium rates and mortality in the population for $x_0 = 20$, $\lambda = 0,08$ and $\gamma = \ln(1,2)$.

If we take into account an expected influx to the portfolio, recruited from all age groups in accordance with the assumed force of entry, the total claims risk for the individual age group will be affected in a positive manner compared to the risk that follows from the isolated ultimate value calculations.

If we set up a "10-year"-tariff taking in consideration the pattern of entry, but otherwise in accordance with the assumptions for ultimate values $\mu_u^{10}(x)$ above, x_0 is replaced with $\max(x_0; (x - 10))$ in the integral limit and in (13).

From figure 7 we see that if tariffs are fixed on the basis of a ten-year period, the tariff can be set even lower and in the upper part of the tariff area will be considerably below mortality in the population. If we revert to figure 5 we see that a corresponding ten-year tariff based purely on ultimate values would be significantly higher than mortality in the population for greater parts of the area under consideration.

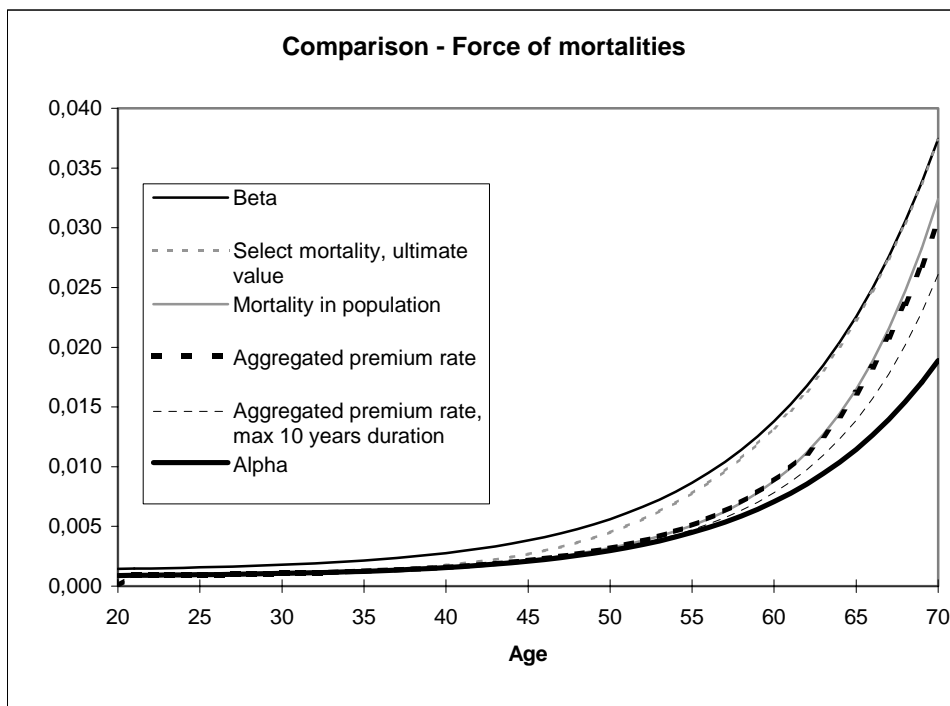


Figure 7 Forces of mortalities, with $\gamma = \ln(1,2)$.

By taking into consideration the assumed pattern of entry when setting tariffs, the company assumes the risk inherent in a less favourable such pattern.

A possible shortfall in new entries into the portfolio can be modelled in a number of ways.

In a situation where new entering has time-continuously and monotonically been lower for all age groups since the tariff was set, the expected entry age for the insured at a given age will be lower. The skew distribution, illustrated by (12) is drawn to the left and this situation can be described by adjusting λ upwards. Correspondingly a regular increase in force of entry can be described by adjusting λ downwards.

For a set continuous premium rate $\mu_p(x, \lambda_1)$, cf. (14), we can introduce a loss function for the age x :

$$l(x, \lambda) = (g_c(x, \lambda) - \mu_p(x, \lambda_1) \cdot g_p(x, \lambda)) / F(x).$$

Obviously we have $l(x, \lambda_1) = 0$. Moreover we find that $\frac{dl(x, \lambda)}{d\lambda} > 0 \quad \forall \lambda > 0$.

For a numerical assessment of the risk level we look at the claims ratio, distributed by age, for alternative λ . We table $CR(x, \lambda) = \frac{g_c(x, \lambda)}{\mu_p(x, \lambda_1) \cdot g_p(x, \lambda)}$, i.e. the claims ratio, for $\lambda = 0.15$ and 0.2 respectively.

This gives us the following values:

Alder	$\lambda = 0,15$	$\lambda = 0,2$
25	1,00007603	1,00015011
30	1,00039592	1,00079454
35	1,00121743	1,00247228
40	1,00306609	1,00626196
45	1,00697026	1,01418849
50	1,01463386	1,02933636
55	1,02765848	1,05380958
60	1,04402169	1,08171718
65	1,05406994	1,09403784
70	1,04748213	1,07603177

Table 3 Claims ratio, distributed by age with $\lambda = 0,15$ and $\lambda = 0,2$.

We find that even with such a substantial change in the tariff-assumed λ from 0.08 to 0.2, the claims ratio will not exceed 110. Analysis of the outcome of $CR(x, \lambda)$ should give a basis for setting a safety loading in the premium rates to meet the risk that a general decline in the force of entry represents.

It seems clear that in the case of aggregate premium rates for select mortality it is necessary to take into account the expected continual renewal of the portfolio. If not, the company will risk offering insurance at rates which the market finds uninteresting and in that case there is little help to be had from favourable expected claims ratios.

Concluding remarks

Based on a simple Markov model we have considered the problems relating to setting premium rates for voluntary death risk insurance based on a natural premium system. In addition to the death and disability risk we have included the risk of lapsing, and the age-distribution of entering into the portfolio.

A formula for determining an aggregated continuous premium rate is derived, and premium rate levels have been clarified and considered for different selected parameters for the relevant forces of transition.

Further work on this subject should probably focus on a possible functional correlation between lapsing frequency and age and the duration in the insurance portfolio respectively.

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