FURTHER ON EXCESS-OF-LOSS REINSURANCE

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Note 1:

On a Combination of the Excess-of-Loss and Largest Claims Reinsurance treaties

Abstract

The largest claims reinsurance treaty is combined with an excess-of-loss cover. An exact premium formula and a premium bound are given for the combined treaty under general conditions. Both are also specialized to more special, ideal model assumptions.

Keywords

Largest claims reinsurance, excess-of-loss, premium theory.

Note 2:

Limit Determination for the Excess-of-Loss Treaty in Case of Simple Retrocession

Abstract

The classical excess-of-loss reinsurance cover is reconsidered. The problem of determining an adequate limit is treated in case of simple retrocession. By applying a former result of Bühlmann on retention-determination, a senseful solution is given.

Keywords

Excess-of-loss, retrocession, limit-determination.

Note 1

On a Combination of the Excess-of-Loss and Largest Claims Reinsurance Treaties

1 Introduction

Reinsurance mathematics is one of the classical fields of mathematical risk theory (see e.g. Bühlmann (1970)). First important contributions go back to the thirties (see e.g. Thesen (1937)), newest ones to the last few years (see e.g. Kremer (1999)).

Certain classical reinsurance treaties became of new interest to several researchers. E.g. the famous stoploss treaty was reinvestigated for many times (see e.g. De Vylder & Goovaerts (1983), Kremer (1990c)) and a comprehensive new theory was developed for reinsurance treaties of the largest claims type (see e.g. Kremer (1984), (1985), (1986), (1988), (1990a), (1990b), (1992), (1994), (1998), (2000)).

Also combinations of treaties were brought to actuarial treatments. So most important the combination of the quota- and excess-of-loss-treaties (see e.g. Centeno (1985)) and the combination of the surplus and excess-of-loss-treaties (see e.g. Benktander and Ohlin (1967)). As far as the author knows, combinations of the largest claims treaty with another treaty were not investigated up to now. Nearlying it is to combine the largest claims treaty with an excess-of-loss-treaty. This combined treaty might be quite attractive for both sides. For the first insurer it works similar like an excess-of-loss-treaty and for the reinsurer it fixes in addition the maximum number of possible excess-claims. In the following a premium theory for that combination is developed. Results of previous papers of the author are adapted to the new situation.

2 Model

Consider a collective of a first insurer and a reinsurer that likes to reinsure that collective. Denote with the random variables $X_1, X_2, \ldots$ on $(\Omega, \mathcal{A}, Q)$ the claims sizes of the collective and with N on $(\Omega, \mathcal{A}, Q)$ the number of claims. The reinsurer likes to take from the claims, exceeding a certain priority P, the k largest ones excess the P. So denote with $Y_1, Y_2, \ldots$ the excess-claims:

$$(X_i - P)$$

with $X_i > P$ and with M the corresponding claims number. Assumed shall be for the sequel that:

(A.1) the $M, (Y_1, Y_2, \ldots)$ are independent

(A.2) the $X_1, X_2, \ldots$ are identically distributed.

With the in nonincreasing size ordered Y_i:

$$Y_{M:1} \geq Y_{M:2} \geq \ldots \geq Y_{M:M} \,,$$

one gets as **claims amount** taken over by the reinsurer:

$$S = \sum_{i=1}^{k} Y_{M:i}$$

(with $Y_{M:i} = 0$ for $i > M$). In the present note it shall be investigated the **net premium** of that **combined treaty**:

$$\nu = E(S)\,.$$

For deriving results on ν it is assumed that the reinsurer knows:

1) the mean claims number $\lambda = E(N)$,

2) for a given $a \in (0, P)$ the probability $p = Q(X_i > a)$ that a claim exceeds the a.

3) the distribution function G of the condtional distribution of the X_i given the event $X_i > a$, i.e.:

$$G(x) = Q(X_i \leq x | X_i > a)\ .$$

It shall hold:

$$p > 0,\ \ G(P) < 1\,.$$

One has under the above conditions the

Lemma

1) the distribution function H of the Y_i is given according

$$H(y) = (\frac{1}{q}) \cdot [G(P+y) - G(P)], \quad \text{for} \quad y \geq 0$$

with the probability

$$q = 1 - G(P)\,.$$

2) the mean value $\Lambda = E(M)$ is given as

$$\Lambda = \lambda \cdot p \cdot q\,.$$

Proof

Denote with F the distribution function of the X_i.

1) One has

$$(*) \qquad G(x) = \frac{F(x) - F(a)}{1 - F(a)}$$

and analogously

$$H(y) = \frac{F(P+y) - F(P)}{1 - F(P)}\,.$$

Obviously

$$H(y) = \left(\frac{F(P+y) - F(P)}{1 - F(a)}\right) \cdot \left(\frac{1 - F(a)}{1 - F(P)}\right)\,.$$

The first factor is according to $(*)$ just $[G(P+y) - G(P)]$ and the second just $[1 - G(P)]^{-1} = 1/q$.

2) One knows that

$$\begin{aligned} \Lambda &= \lambda \cdot (1 - F(P)) \quad = \\ &= \lambda \cdot (1 - F(a)) \cdot \left(\frac{1 - F(P)}{1 - F(a)}\right)\,. \end{aligned}$$

The second factor is just p and because of (2.1) the third just $[1 - G(P)] =: \ q$ □

3 Exact premium

For this section assume in addition

(A.3) $\qquad Y_1, Y_2, Y_3, \ldots$ are independent

and

(A.4) $\qquad G$ is continuous on $[P, \infty)$.

Denote with $M^{(i)}(\cdot)$ the i-th derivative of the probability generating function $M(\cdot)$ of the M:

$$M(t) = \sum_{m=0}^{\infty} Q(M = m) \cdot t^m\,.$$

One has the

Theorem 1

Under the above assumptions holds for the **net premium** of the combined reinsurance treaty

$$\nu \;=\; \sum_{i=1}^{k}\left(\frac{1}{q^i\cdot\Gamma(i)}\right)\cdot\int_0^q G^{-1}(1-t)\cdot t^{i-1}\cdot M^{(i)}(1-t/q)\,dt$$

$$-P\cdot\sum_{i=1}^{k}\frac{1}{\Gamma(i)}\cdot\int_0^1 t^{i-1}\cdot M^{(i)}(1-t)\,dt$$

where $\Gamma(i)=(i-1)!$ and G^{-1} is the pseudo-inverse of the distribution function G

$$G^{-1}(t)=inf\,\{x:G(x)\geq t\}$$

with in addition

$$G^{-1}(1-q)=P\ .$$

Proof:

One applies Theorem 1 in Kremer (1985), giving

(3.1) $$\nu=\sum_{i=1}^{k}\frac{1}{\Gamma(i)}\cdot\int_0^1 H^{-1}(u)\cdot(1-u)^{i-1}M^{(i)}(u)\,du\ .$$

One inserts just

(3.2) $$H^{-1}(u)=G^{-1}(1-(1-u)\cdot q)-P\,,$$

splits up the integral into two integrals and makes certain substitutions then. □

More special one gets the

Corollary 1

Assume in addtion to the conditions of Theorem 1

(A.5) M is Poisson-distributed, i.e.:

$$Q(M=m)=\left(\frac{\Lambda^m}{m!}\right)\cdot\exp\,(-\Lambda),\quad m=0,1,2,3,\dots\ ,$$

(A.6) G is the Pareto-distribution with parameter $\alpha>1$, i.e.

$$G(x)=1-(x/a)^{-\alpha}\ ,\quad \text{for } x\geq a\ .$$

Then one has for the net premium

$$\nu = (\lambda \cdot p)^{1/\alpha} \cdot \sum_{i=1}^{k} \left(\frac{\Gamma_\Lambda(i - 1/\alpha)}{\Gamma(i)} \right) \cdot a - P \cdot \sum_{i=1}^{k} \frac{\Gamma_\Lambda(i)}{\Gamma(i)}$$

with

$$\Lambda = \lambda \cdot p \cdot q$$

and the incomplete Gamma-function

$$\Gamma_\Lambda(z) = \int_0^\Lambda s^{z-1} \cdot \exp(-s)\, ds \ .$$

Proof:

Insert into (3.1) and (3.2) the special cases

$$\begin{aligned} M^{(i)}(t) &= \Lambda^i \cdot \exp\left(\Lambda \cdot (t-1)\right) \\ G^{-1}(t) &= a \cdot (1-t)^{-1/\alpha} \end{aligned}$$

Make a substitution and use that

$$q = \left(\frac{a}{P}\right)^\alpha \ .$$

□

Remark 1

When Λ is larger, then one is willing to replace Γ_Λ by $\Gamma_\infty = \Gamma$, what gives as **approximate result**

$$\nu \approx (\lambda \cdot p)^{1/\alpha} \cdot \left[\frac{\Gamma(k + 1 - 1/\alpha)}{\Gamma(k)} \right] \cdot \left(\frac{\alpha}{\alpha - 1} \right) \cdot a - k \cdot P \ .$$

The first expression is just the classical net premium formula for the (classical) largest claims reinsurance cover, like derived already in 1964 by the Swiss Ammeter (see Ammeter (1964)). Consequently, the approximate result means that the net premium of the combined treaty is just the net premium of the largest claims cover minus k times the priority of the excess-of-loss cover. This is just what the nonmathematical practitioner would have guessed to come out. But note that in practice the Λ does **not** need to be such larger! In typical situations Λ will be comparatively small. □

4 Premium bound

In this section suppose that one has (A.1), (A.2), and (A.4), but not necessarily (A.3).

Define now for $\Pi \geq 0$:

$$\rho(\Pi) = \left(\frac{1}{q \cdot \mu}\right) \cdot \int\limits_{P+\Pi}^{\infty} \big(x - (P+\Pi)\big)\, G(dx)$$

with

$$\mu = \frac{1}{q} \cdot \int\limits_{P}^{\infty} x\; G(dx)\ .$$

One has the

Theorem 2

The net premium ν of the combined treaty is bounded by

$$\left[\sum_{n=1}^{k} \frac{1}{n} \cdot \left(\sum_{m=n}^{\infty} m \cdot Q(M=m) \cdot \rho(\Pi_{mn})\right)\right] \cdot \mu + \sum_{n=1}^{k} \sum_{m=n}^{\infty} Q(M=m) \cdot \Pi_{nm}$$

where

$$\Pi_{mn} = G^{-1}\left(1 - \left(\frac{m}{n}\right) \cdot q\right) - P.$$

Proof:

Just apply the Theorem in Kremer (1998) with there H, Π inserted for F, P and remember (3.2). □

Again more special one gets

Corollary 2:

Assume in addition to the conditions of Theorem 2 (A.6). The upper bound of Theorem 2 reduces to

$$P \cdot \left[\sum_{n=1}^{k} n^{-1/\alpha}\right] \cdot E\left(M^{1/\alpha}\right) \cdot \left(\frac{\alpha}{\alpha - 1}\right) - k \cdot P - R$$

where the remainder term R is given according

$$R = P \cdot \left[\sum_{n=1}^{k} n^{-1/\alpha} \cdot \sum_{m=1}^{n-1} m^{1/\alpha} \cdot Q(M=m)\right] \cdot \left(\frac{\alpha}{\alpha-1}\right) + P \cdot \sum_{n=1}^{k} Q(M \leq n-1).$$

Proof:

One has simply

$$\begin{aligned}\Pi_{mn} &= P \cdot \left[\left(\frac{n}{m} \right)^{1/\alpha} - 1 \right] \\ \mu &= \left(\frac{\alpha}{\alpha - 1} \right) \cdot P \\ \rho(\Pi) &= \left(\frac{1}{\alpha} \right) \cdot (1 + (\Pi/P))^{1-\alpha}.\end{aligned}$$

Inserting all these formulas into the result of the Theorem 2 implies after some routine calculations the statement of the Corollary 2. □

Remark 2

In the result of Corollary 2 one can use the bound

$$E(M^{1/\alpha}) \leq (E(M))^{1/\alpha} = (\lambda \cdot p)^{1/\alpha} \cdot \left(\frac{a}{P} \right)$$

what follows directly from Jensen's inequality. In case that $Q(M < k)$ is sufficiently small one neglects the remainder term $R(\approx 0)$. But note that in typical practical situations the $Q(M < k)$ will usually be **not** „sufficiently" small. It will be significantly different from zero! □

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Note 2
Limit-Determination for the Excess-of-Loss Treaty in Case of Simple Retrocession

1 Introduction

Reinsurance is well one of the most embracing subfields of the Applied Risk Theory (see Kremer (1999b)). Especially the so-called nonproportional reinsurance was treated with intensively. It is well-known that the most important nonproportional reinsurance treaty is the so-called excess-of-loss cover. Its premium was analyzed extensively by mathematicians (see e.g. Helbig (1953), Gisler et al. (1986), Kremer (1993), and Kremer (1999a)). In connection with the excess-of-loss there appears a very interesting problem, that is how one can determine adequately the priority of that treaty. Also on that topic diverse mathematical treaties appeared (see e.g. Bühlmann (1970), Waters (1979), Schmitter (1984), Zecchin (1987), Kremer (1988)). But nothing one can find in the literature on a related problem, that is the determination of the limit for the reinsurer in retrocession. With that topic it is dealt with in details in the following note and there is given a proposal for solution.

2 Problem

Consider a reinsurer that likes to give the excess-of-loss treaties with priorities P_i, $i = 1, \ldots, h$ into retrocession. For this it is necessary to find adequate limits $L_i(> P_i)$, $i = 1, \ldots, k$ with which the after retrocession at the reinsurer remaining claims amount for an orignal claim Y of the $i-$th treaty is defined as

$$H(Y) = \min\big(L_i - P_i,\ \max\left(Y - P_i, 0\right)\big).$$

Suppose that for determining adequate limits L_i the reinsurer has

(1) the mean claims number λ_i of original claims under the i-th excess-of-loss,

(2) for a given threshold $a_i \in (0, P_i)$ the probability p_i, that the original claim of the i-th excess-of-loss exceeds the a_i,

(3) the distribution function G_i of the conditional distribution of the original claim of the i-th excess-of-loss, conditionally given, that the original claim exceeds the a_i.

Behind this one has obviously the risk-theoretical model assumptions:

i) the original claims number M_i and the original claims sizes Y_{ij}, $j = 1, 2, 3, \ldots$ of the i-th excess-of-loss are stochastically independent.

ii) the original claims sizes Y_{ij}, $j = 1, 2, 3, \ldots$ of the i-th excess-of-loss are identically distributed, say with distribution function F_i on $[0, \infty)$.

Furthermore suppose that

iii) the variables of different excess-of-loss covers are stochastically independent.

It is nearlying to take as model for the claims numbers M_i the classical Poisson-distribution, meaning that

$$\text{(2.3)} \qquad Prob\,(M_i = k) = \left(\frac{\lambda_i^k}{k!}\right) \cdot \exp\,(-\lambda_i), \;\; k = 1, 2, \ldots \;.$$

According to earlier statements of the author it appears to be very suitable to take as model for G_i the so-called **generalized Pareto-distribution function** (see Kremer (1986), (1997), (1998)):

$$\text{(2.4)} \qquad G_i(y) = 1 - \left(1 + (y - a_i) \cdot \left(\frac{g_i}{s_i}\right)\right)^{-1/g_i}$$

for $y \geq a_i$ and parameters $g_i \in (0, 1)$, $s_i > 0$.

Suppose the reinsurer got for his i-th excess-of-loss the total risk premium Π_i.

For giving away the claims amount

$$R_i = \sum_{j=1}^{M_i} \max\,(Y_{ij} - L_i, 0)$$

he himself has to pay a risk premium c_i^L to the retrocessionar. This shall be calculated with the expectation principle, what means, that one has with a given loading factor $\beta_i > 0$

$$c_i^L = (1 + \beta_i) \cdot E(R_i)$$

The **problem for the reinsurance actuary** consists now in giving in the given detailed model a rule for the choice of the limits $L_1, \ldots, L_k$, that is adequate or already in some sense optimal from the reinsurer's point of view.

3 Retention determination

A solution to the just given actuarial problem can be derived by applying an older result of Bühlmann (1970) on determining the retention of an excess-of-loss cover. Bühlmann's model and solution shall be sketched in a short now.

Considered shall be a first-insurer, who likes to cite for each of k branches an excess-of-loss treaty with priorities P_i, $i = 1, \ldots, k$.

Let N_i denote the claims number of the i-th branch and X_{ij} the corresponding j-th claims size. Assumed shall be again i), ii), iii) (with N_i instead of M_i, X_{ij} instead of Y_{ij} and „branch“ instead of „excess-of-loss-cover“). Furthermore assume in analogy to (4):

$$(3.5) \qquad Prob\,(N_i = k) = \left(\frac{\eta_i^k}{k!}\right) \cdot \exp\,(-\eta_i), \qquad i = 1, 2, \ldots$$

with $\eta_i = E(N_i)$ and

$$F_i(0) = 0 \quad .$$

Let B_i be the total risk premium income of the first insurer for the i-th branch.

The risk premium c_i^R of the first-insurer for the excess-of-loss treaty of the i-th branch again let be defined by the expectation-principle, what means that one has with a given loading factor $\alpha_i > 0$

$$c_i^R = (1 + \alpha_i) \cdot E(S_i)$$

with

$$S_i = \sum_{j=1}^{N_i} \max\,(X_{ij} - P_i, 0) \ .$$

In this model Bühlmann handled first the so-called **relative retention problem**, what gave as structure for a senseful choice of the priority P_i

$$P_i = K \cdot \alpha_i \ .$$

with a constant $K > 0$ (concerning details see Bühlmann (1970)).

Afterwards he treated with the so-called **absolute retention problem**. More concretely he determined approximately an adequate K with help of ruintheoretical considerations (concerning details see Bühlmann (1970)). The formula for K is given according

$$(3.6) \qquad K = \frac{u}{|ln(\varepsilon)|} + \sqrt{\frac{u^2}{(ln(\varepsilon))^2} + \frac{2 \cdot u \cdot V}{|ln(\varepsilon)|}}$$

where $\varepsilon \in (0, 1)$ is a desired (small) ruinprobability for the whole business (of all k branches) under the excess-of-loss covers and $u > 0$ is a corresponding (bigger) initial reserve. Furthermore

$$(3.7) \qquad V = \left[\frac{\sum\limits_{i=1}^{k} B_i - (1 + \alpha_i) \cdot \eta_i \cdot \mu_i}{\sum\limits_{i=1}^{k} \eta_i \cdot \alpha_i^2}\right]$$

with the mean claims size of an original claim of the i-th branch

$$\mu_i = E(X_{ij}) = \int\limits_{[0,\infty)} x \ F_i(dx) \ .$$

4 Problem solution

Considered shall be the context of part 2, on which simply the model of part 3 will be adapted by identifying the reinsurer of part 2 with the first-insurer of part 3.

Then the claims sizes X_{ij} are just all

$$Z_{ij} = Y_{ij} - P_i$$

with $Y_{ij} > P_i$ and the claims number N_i results as the number of all these $Y_{ij} > P_i$. With this hold obviously $(i), (ii)$ and (iii) (for N_i instead of M_i and X_{ij} instead of Y_{ij}).

Concerning (6) one gets

Lemma 1
Under the assumptions (4) on the original claims numbers the N_i is Poisson-distributed with parameter

$$\eta_i = \lambda_i \cdot Prob\,(Y_{ij} > P_i)\ .$$

Proof:
Denote with g_i the generating function of M_i and with φ the characteristic function of

$$W_{ij} = 1_{\{X_{ij} > P_i\}}$$

(1_M : indicator function of the set M). Because of the representation

$$N_i = \sum_{j=1}^{M_i} W_{ij} \tag{4.8}$$

one has for the characteristic function ψ_i of N_i

$$\psi_i(t) = g_i(\varphi(t))\,. \tag{4.9}$$

One knows that

$$g_i(z) = \exp\left(\lambda_i(z-1)\right)$$

and

$$\varphi(t) = Prob\,(Y_{ij} \leq P_i) + Prob\,(Y_{ij} > P_i) \cdot \exp\,(i \cdot t)\,.$$

Insertion into (10) gives

$$\psi_i(t) = \exp\left(\eta_i \cdot (\exp\,(i \cdot t) - 1)\right)\ ,$$

with the η_i according to the formula of the Lemma. The rhs of the last formula is the characteristic function of the Poisson-distribution (6). □

Remark
Also in case that (4) does not hold for the original claims, one can assume according

to a theorem of the theory of point processes that the N_i are approximatively Poisson-distributed. The parameter of that Poisson-distribution results simply from (9) as

$$E(N_i) \cdot E(W_{ij}) = \lambda_i \cdot Prob\,(Y_{ij} > P_i)\,.$$

□

Since $Prob\,(Y_{ij} > P_i)$ is not known to the reinsurer, one needs a different representation of the η_i.

Lemma 2
One has

$$\eta_i = \lambda_i \cdot p_i \cdot q_i \ . \tag{4.10}$$

with

$$q_i = 1 - G_i(P_i)\ .$$

Proof:
It is

$$\begin{aligned} Prob\,(Y_{ij} > P_i) &= Prob\,(Y_{ij} > P_i, Y_{ij} > a_i) \\ &= Prob\,(Y_{ij} > a_i) \cdot Prob\,(Y_{ij} > P_i | Y_{ij} > a_i) \\ &= p_i \cdot q_i\ . \end{aligned} \tag{4.11}$$

□

Finally one has

Lemma 3
For the distribution function F_i of X_{ij} one has the representation

$$F_i(x) = [G_i(P_i + x) - G_i(P_i)]/q_i\ .$$

Proof:
One has

$$F_i(x) = Prob\,(Z_{ij} \leq x | Y_{ij} > P_i) = \frac{Prob\,(Y_{ij} \leq P_i + x, Y_{ij} > P_i)}{P(Y_{ij} > P_i)}$$

and

$$\begin{aligned} Prob\,(Y_{ij} \leq P_i + x, Y_{ij} > P_i) &= Prob\,(Y_{ij} \leq P_i + x, Y_{ij} > P_i, Y_{ij} > a_i) \\ &= Prob\,(Y_{ij} \leq P_i + x, Y_{ij} > P_i | Y_{ij} > a_i) \cdot \\ &\quad \cdot Prob\,(Y_{ij} > a_i) \end{aligned}$$

Now use (12) and $p_i = Prob\,(Y_{ij} > a_i)$ and recall the definition of G_i. □

This implies

$$\mu_i = \int_{[0,\infty)} x \; F_i(dx)$$

(4.12)
$$= \left(\frac{1}{q_i}\right) \cdot \int_{[P_i,\infty)} (y - P_i) \, G_i(dy)$$

and more special under the model (5):

(4.13)
$$\mu_i = \left(\frac{s_i}{1 - g_i}\right) \cdot \left(1 + (P_i - a_i) \cdot \left(\frac{g_i}{s_i}\right)\right) \ .$$

Now all is together.

According to part 3 one has as senseful formula for the limits

$$L_i = P_i + K \cdot \beta_i \ ,$$

where K is determined by (7), (8) with Π_i, β_i instead of B_i, α_i and the η_i, μ_i according to (11), (13) or (14).

5 Final remark

For application of the results of part 4 the reinsurer only needs to ask the first-insurer for estimates of the λ_i and p_i. From his own collection of claims data (all claims sizes of the first-insurer that exceed the a_i) he has to calculate estimates for the parameters g_i, s_i of (2.2). An efficient method for calculating these parameter estimators was published by the author several years ago (see Kremer (1994)). Finally the reinsurer has to ask the retrocessionar for the β_i.

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Correction: in the formula for S_i must be on the rhs the summationindex j instead of i!

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