Topic: Reinsurance

A NOTE ON THE CALCULATION OF COVARIANCE BETWEEN LAYERS IN MULTILAYER EXCESS OF LOSS PROGRAMMES

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ABSTRACT

Sundt's (1999) multivariate Panjer recursion is used to calculate the covariances in results from excess of loss layers protecting the same underlying risk when this is modelled by a compound distribution. The method developed handles the case where the covers of the layers are reduced with aggregate limits and deductibles and the layer premiums are regulated by reinstatement premiums. The results are used to calculate the loading on standard deviation of the layer structure regarded as one single risk.

KEYWORDS

Excess of loss reinsurance; multilayer; standard deviation loading; multivariate Panjer recursion; reinstatements; aggregate deductible; reinstatement premium.

1. INTRODUCTION

In today's reinsurance market an excess of loss cover of a particular risk is almost always split into more than one contract each often denoted a *layer*. In the following multiple excess of loss layers covering the same risk will, when regarded as a whole, be called a(n *excess of loss*) *program*.

The ability to aim the various layers at different segments in the market is the reason most often stated for multiple contract constructs. In reality the reinsurer often sign shares on more than one layer. In some cases one even see a *cross signing* where all the layers are written by the same reinsurer. The cedent sometimes makes cross signing a condition for participating on the program. In these cases the market segment argument does not apply and the split into multiple layers must be explained by the possibilities of differentiated aggregate cover limitations and premium payment terms across the layers.

If the reinsurer quotes multiple layers on the basis of either the standard deviation or the variance premium principle the interlayer covariance must not be disregarded. Otherwise inconsistent results often appear (an example of this is presented in Section 7). Further more as Benktander (1961) states the layer correlations might be of interest in themselves. The interest could for example stem from the fact that it is possible for the reinsurer to accept a larger share of the programme if the layer correlations are low.

Even though the quotation of multilayer excess of loss programmes is of great practical importance the subject has only been investigated by Benktander (1961). In this reference claims dependant premiums are not considered.

Whilst the reinsurer should have knowledge of the loaded premium of the multilayer construct as a whole, the individual layer premiums must not be disregarded. The quotes of the individual layers are still regarded as separate offers and the reinsurer should thus not take a cross signing into account when this is only tentative.

2. THE SETUP

Assume in the following that a risk is covered by n individual excess of loss contracts with limits L_j and retentions R_j with $R_j < R_{j+1}$, $j = 1,\ldots, n - 1$. It is clear that $R_j + L_j \leq R_{j+1}$, $j = 1,\ldots, n - 1$ to avoid double insurance. In most practical cases the cover would be seamless that is $R_j + L_j = R_{j+1}$. Assume further that the cover of layer j is restricted with an aggregate deductible $D_j \geq 0$ and aggregate limit $(1 + r_j)L_j$, $r_j \in \mathbf{N}$. The aggregate limit is here, like in any practical application, given as a number of single claim limits. The factor r_j is called the number of reinstatements referring to the number of extra times the original cover can be exhausted. Sundt (1991) and Rytgaard (1991) give more thorough explanations of these concepts.

As the expected number of claims is decreasing in their size, we can assume that

$$r_j + \frac{D_j}{L_j} \geq r_{j+1} + \frac{D_{j+1}}{L_{j+1}}, \text{ for } j = 1,\ldots,n - 1. \tag{1}$$

The number of total claims needed to consume the total cover and deplete the aggregate deductible is thus assumed to decrease up through the program. The aggregate limit and deductible of the individual layer reflects the ceding companies expectation of the number of claims that at most is expected to hit the cover. If (1) did not apply this would mean that the cedent was expecting more claims to hit the higher layers than the lower. This would obviously not make sense.

Let Y_i, $i = 1, 2,\ldots$ denote the stochastic claims to the portfolio covered by the programme and N the stochastic number hereof. The total claims to a cover of L_j XS R_j with no aggregate deductible or limit is given by

$$\underline{X}_j = \sum_{i=1}^{N} \min\left(L_j, \left(Y_i - R_j\right)^+\right) \tag{2}$$

under the convention that $\sum_{i=1}^{0} x_i = 0$ and where the positive part of x, $(x)^+$, is defined as $\max(x, 0)$. The aggregate claim to contract j is given by

$$\overline{X}_j = \min\left(\left(1 + r_j\right)L_j, \left(\underline{X}_j - D_j\right)^+\right). \tag{3}$$

If an excess of loss cover has reinstatements, extra premium is usually paid if some of these are employed. The extra premium payment is usually a percentage of the original premium with no respect as to when the claim occurs. The cheapest reinstatements are almost always used first. Assume a cover of 10 XS 1 with one reinstatement at 50% and one reinstatement at 100% is hit with a full claim and claim of 6. The reinstatement premium would in this case be one times 50% plus one half of 100% of the original premium since half the cover is consumed by the second claim.

Assume an excess of loss cover of L_j XS R_j with r_j reinstatements is hit with aggregate claims of $\overline{X}_j$ from (3). The part of the cover being reinstated from the s'th reinstatements is given by

$$\min\left(L_j, \left(\overline{X}_j - (s-1)L_j\right)^+\right). \tag{4}$$

Let $\overline{\pi}_j$ denote the initial premium paid for the cover and c_j^s the percentage of this premium paid for the s'th reinstatement where $c_j^1 \le c_j^2 \le \ldots \le c_j^{r_j}$. The reinstatement premium paid for the s'th reinstatement is thus given by (4) divided by L_j (the share of the s'th reinstatement being employed) times $c_j^s \overline{\pi}_j$. By summation over the r_j reinstatements we get that the total reinstatement premium paid is given by

$$\overline{\pi}_j^R\left(\overline{X}_j\right) = \overline{\pi}_j \sum_{s=1}^{r_j} c_j^s \frac{\min\left(L_j, \left(\overline{X}_j - (s-1)L_j\right)^+\right)}{L_j}. \tag{5}$$

The reinstatement premium is stochastic as it is depending on the total claims to the layer. The same is the case for the total premium paid for layer j, denoted $\pi_j\left(\overline{X}_j\right)$ as the sum of $\overline{\pi}_j$ and $\overline{\pi}_j^R\left(\overline{X}_j\right)$.

3. STANDARD DEVIATION LOADING WITH CLAIMS DEPENDENT PREMIUMS

In the following the contracts will be priced by the standard deviation principle. Under the standard deviation principle the total premium for the contract *j* should, according to Sundt (1991), be given by

$$E\pi_j\left(\overline{X}_j\right) = E\overline{X}_j + \gamma_j\sqrt{V\left(\overline{X}_j - \overline{\pi}_j^R\left(\overline{X}_j\right)\right)},$$

where the loading factor γ_j is determined by the management usually between 8% and 15%. Since the premium $\pi_j\left(\overline{X}_j\right)$ can be divided into initial non-stochastic premium and a stochastic reinstatement premium we get, with a bit of rearrangement, that the initial premium is given by

$$\overline{\pi}_j = E\left(\overline{X}_j - \overline{\pi}_j^R\left(\overline{X}_j\right)\right) + \gamma_j\sqrt{V\left(\overline{X}_j - \overline{\pi}_j^R\left(\overline{X}_j\right)\right)} = EX_j + \gamma_j\sqrt{VX_j}\ . \qquad (6)$$

In the last equality in (6) the risk was rephrased in terms of the ultimate net loss to the contract (claims minus any reinstatement premium paid) $X_j = \overline{X}_j - \overline{\pi}_j^R\left(\overline{X}_j\right)$. The concept of ultimate net loss was investigated by Rytgaard (1991).

Assume that all layers are loaded with γ. Summing over *j* in (6) gives

$$\overline{\pi} = EX + \gamma\sum_{j=1}^{n}\sqrt{VX_j}\ , \qquad (7)$$

where $X = \sum_{j=1}^{n} X_j$ is the ultimate net loss to all the layers and $\overline{\pi} = \sum_{j=1}^{n}\overline{\pi}_j$ is the total initial premium paid for the whole programme. If the standard deviation principle on the other hand where applied to the layer structure regarded as one single risk this would lead to the initial premium $\hat{\pi}$ given by

$$\hat{\pi} = EX + \hat{\gamma}\sqrt{VX}\ . \qquad (8)$$

It follows from the triangle inequality that $\sum_{j=1}^{n}\sqrt{VX_j} \geq \sqrt{V\left(\sum_{j=1}^{n} X_j\right)}$ with equality if and only if X_j and $X_{j'}$ are perfectly correlated for all j and j'. It is noted that, as perfect correlation is not the case in all practical applications, the quoted premium will be less when loaded on the total programme ultimate net loss provided that the same loading factor is used. As discussed in section 1 the initial quote of a multilayer program should include no discount for interlayer correlation unless a known signing on each layer is ensured.

In the following it will be assumed that the individual initial layer premiums are fixed at $\overline{\pi}_j$, $j = 1,\ldots, n$ along with the reinstatement premium terms. For the reinsurer trying to get an idea of the quality of the programme on the offered terms as a whole, the loading on the programme's total ultimate net loss is a better indicator than the individual layer loadings. From (8) we get that

$$\hat{\gamma} = \frac{\overline{\pi} - E(X)}{\sqrt{V(X)}} = \frac{\sum_{j=1}^{n}\overline{\pi}_j - \sum_{j=1}^{n} EX_j}{\sqrt{V\left(\sum_{j=1}^{n} X_j\right)}} = \frac{\sum_{j=1}^{n}\overline{\pi}_j - \sum_{j=1}^{n} EX_j}{\sqrt{\sum_{j=1}^{n} VX_j + 2\sum_{j=2}^{n}\sum_{j'=1}^{j-1} COV\left(X_j, X_{j'}\right)}}. \tag{9}$$

We thus need to determine the mean and variance of the ultimate net loss of each contract along with the interlayer covariances in order to determine the loading with respect to the programme's ultimate net result.

In the following section the mean and variance of the ultimate net loss to the layers will be found. The covariances are calculated in section 5. Section 6 will give an example of the application of the developed method.

4. CALCULATION OF MEAN AND STANDARD DEVIATION

Assume that an excess of loss cover of L_j XS R_j with an annual aggregate deductible D_j and aggregate cover limited to r_j reinstatements with reinstatement premium percentages of $c_j^1, c_j^2, \dots, c_j^{r_j}$ pays an initial premium $\overline{\pi}_j$. It can quite easily be shown that the ultimate net loss to the contract can be rewritten as

$$X_j = \sum_{i=1}^{m_j} b_{ji} \min\left(a_{ji}, \left(\underline{X}_j - A_{ji-1}\right)^+\right), \tag{10}$$

where

$$m_j = r_j + 1,\; a_{js} = \begin{cases} D_j, s = 0 \\ L_j, s = 1,\dots, m_j \end{cases},\; A_{ji} = \sum_{s=0}^{i} a_{js} \text{ and } b_{js} = \begin{cases} 1 - c_j^s \dfrac{\overline{\pi}_j}{L_j}, s = 1,\dots, m_j - 1 \\ 1, s = m_j \end{cases}. \tag{11}$$

Rytgaard (1991) shows that the first and second moment of a variable of the form (10) is given by

$$EX_j = \sum_{i=1}^{m_j} b_{ji} \left(\int_{a_{ji-1}}^{a_{ji}} y - A_{ji-1} dF_j(y) + a_{ji} \int_{A_{ji}}^{\infty} dF_j(y) \right) \tag{12}$$

and

$$EX_j^2 = \sum_{i=1}^{m_j} 2b_{ji} \int_{a_{ji-1}}^{a_{ji}} \left(b_{ji}(y - A_{ji-1}) + \sum_{k=1}^{i-1} a_{jk} b_{jk} \right)(1 - F_j(y))dy\,. \tag{13}$$

With m_j, b_{js} $s = 1,\dots, m_j$ and $A_{js,}$ $s = 1,\dots, m_j - 1$ given by (11) and F_j denoting the distribution of $\underline{X}_j$, (12) and (13) are respectively the first and second moment of the ultimate net loss to the contract.

To find the two moments of X_j we thus initially need to calculate the distribution F_j. Let G_j denote the transformed distribution of $\min\left(L_j, \left(Y_i - R_j\right)^+\right)$. F_j is then a compound distribution of G_j given by $F_j = \sum_{n=0}^{\infty} q_n G_j^{*n}$, where G_j^{*n} denote the *n*-fold convolution of G_j and $q_n = P(N = n)$. If N is a poisson, binomial or negative binomial distributed stochastic variable (and only one of these, cf. Sundt et. al. (1981)) and G_j a discrete random variable F_j can be found by Panjer recursion (Panjer (1981)). As the claim size is almost always modelled by a continuous distribution, G_j would be the continuous distribution discretizised to equidistant points $R_j + l\frac{L_j}{\kappa}$, for $l = 1,\ldots, \kappa$. Usually $\kappa = 100$ will be enough to give sufficient precision. Under these circumstances the Panjer recursion takes the form

$$f_j\left(l\frac{L_j}{\kappa}\right) = \frac{1}{1-ag_j(0)}\sum_{l'=1}^{\min(l,\kappa)}\left[a+b\frac{l'}{l}\right]g_j\left(l'\frac{L_j}{\kappa}\right)f_j\left((l-l')\frac{L_j}{\kappa}\right), \text{ for } l=1, 2,.. \qquad (14)$$

starting with

$$f_j(0) = \begin{cases} e^{-b(1-g_j(0))}, a = 0 \\ \left(\frac{1-ag_j(0)}{1-a}\right)^{-\frac{a+b}{a}}, a \neq 0 \end{cases}. \qquad (15)$$

a and b in (14) and (15) are the constants satisfying

$$q_n = \left(a+\frac{b}{n}\right)q_{n-1}, \; n = 1, 2,\ldots \qquad (16)$$

With the distribution F_j thus calculated, the first and second moment of X_j are calculated by (12) and (13) respectively. It is noted that it only is necessary to calculate $f_j(y)$ for $y < D_j + (1 + r_j)L_j$. Finally the loading can be calculated by $\gamma_j = \frac{\bar{\pi}_j - EX_j}{VX_j}$.

5. COVARIANCE CALCULATION

In the following it will be assumed that the first two moments of the ultimate net losses to the layers in the programme have been calculated by the method presented in the previous section. It follows from (9) that the covariances between any two net losses of the contracts are the only thing missing in order to calculate the loading to the programme regarded as one risk. As

$$COV\left(X_j, X_{j'}\right) = EX_j EX_{j'} - EX_j X_{j'} \qquad (17)$$

this is equivalent to calculating $EX_jX_{j'}$ for all $j > j'$, $j, j' \in \{1,\ldots, r_j\}$. Denote the simultaneous density of $(X_j, X_{j'})$, $f^{(j,j')}$. From the representation of the ultimate net loss (10) and (11) we get

$$\begin{aligned}
&EX_jX_{j'} \\
&= E\left[\left(\sum_{i=1}^{m_j} b_{ji}\min\left(a_{ji}, \left(\underline{X}_j - A_{ji-1}\right)^+\right)\right) \times \left(\sum_{i'=1}^{m_j} b_{j'i'}\min\left(a_{j'i'}, \left(\underline{X}_{j'} - A_{j'i'-1}\right)^+\right)\right)\right] \\
&= \sum_{i=1}^{m_j}\sum_{i'=1}^{m_{j'}} b_{ji}b_{j'i'}E\left[\min\left(a_{ji}, \left(\underline{X}_j - A_{ji-1}\right)^+\right) \times \min\left(a_{j'i'}, \left(\underline{X}_{j'} - A_{j'i'-1}\right)^+\right)\right] \\
&= \sum_{i=1}^{m_j}\sum_{i'=1}^{m_{j'}} b_{ji}b_{j'i'}\int_{A_{ji-1}}^{A_{ji}}\int_{A_{j'i'-1}}^{A_{j'i'}} \left(x_1 - A_{ji-1}\right)\left(x_2 - A_{j'i'-1}\right) f^{(j,j')}(x_1, x_2)dx_2dx_1 \qquad (18) \\
&+ \sum_{i=1}^{m_j}\sum_{i'=1}^{m_{j'}} b_{ji}b_{j'i'}\int_{A_{ji}}^{\infty}\int_{A_{j'i'-1}}^{A_{j'i'}} a_{ji}\left(x_2 - A_{j'i'-1}\right) f^{(j,j')}(x_1, x_2)dx_2dx_1 \\
&+ \sum_{i=1}^{m_j}\sum_{i'=1}^{m_{j'}} b_{ji}b_{j'i'}\int_{A_{ji-1}}^{A_{ji}}\int_{A_{j'i'}}^{\infty} \left(x_1 - A_{ji-1}\right)a_{j'i'} f^{(j,j')}(x_1, x_2)dx_2dx_1 \\
&+ \sum_{i=1}^{m_j}\sum_{i'=1}^{m_{j'}} b_{ji}b_{j'i'}\int_{A_{ji}}^{\infty}\int_{A_{j'i'}}^{\infty} a_{ji}a_{j'i'} f^{(j,j')}(x_1, x_2)dx_2dx_1
\end{aligned}$$

with m_j, b_{js} $s = 1,\ldots, m_j$ and $A_{js,}$ $s = 1,\ldots, m_j - 1$ given by (11).

The remaining problem is to find the simultaneous distribution of $(X_j, X_{j'})$. Multiple methods are available in the literature for recursive calculation of multidimensional compound distributions. Some of these (see e.g. Hesselager (1996) and Ambagaspitiya (1998)) model the dependence by correlation of the number processes. These do not apply in this specific case where the simultaneous distribution of the claim size can be found. In this case the method developed by Sundt (1999) is applicable.

Assume that $j' < j$ where $j, j' \in \{1, 2,\ldots,n\}$ and let $N_{j'}$ denote the number of claims that hits the lower layer j'. Assume as before that the individual claims, Y_i, $i = 1,\ldots, N_{j'}$, are mutual stochastic independent and identically distributed with discrete distribution g_0 and independent of $N_{j'}$. Let $(q_n)_{n\geq 0}$ denote the discrete distribution of $N_{j'}$ and assume that this satisfies (16). The set of losses to layer j and j', i.e. $\left(\min\left(L_j, \left(Y_i - D_j\right)^+\right), \min\left(L_{j'}, \left(Y_i - D_{j'}\right)^+\right)\right)$, is thus also mutual stochastic independent and independent of $N_{j'}$ with a two-dimensional density denoted $g^{(j,j')}$. The compound distribution of the total claims to the two layers, in the following denoted $f^{(j,j')}$, can under these circumstances be found by Sundt’s multivariate recursion formula (Sundt (1999), Theorem 1, p. 33).

Assume initially (correspondent to the single dimensional case) that the support of the density $g^{(j,j')}$ is restricted to the grid

$$M = \left\{0, \frac{L_j}{\kappa}, 2\frac{L_j}{\kappa}, \ldots, (\kappa - 1)\frac{L_j}{\kappa}, L_j\right\} \times \left\{0, \frac{L_{j'}}{\kappa}, 2\frac{L_{j'}}{\kappa}, \ldots, (\kappa - 1)\frac{L_{j'}}{\kappa}, L_{j'}\right\}. \tag{19}$$

To avoid being choked in notation the following is introduced:

$$\begin{aligned} f_{(l,l')}^{(j,j')} &= f^{(j,j')}\left(l\frac{L_j}{\kappa}, l'\frac{L_{j'}}{\kappa}\right), \\ g_{(l,l')}^{(j,j')} &= g^{(j,j')}\left(l\frac{L_j}{\kappa}, l'\frac{L_{j'}}{\kappa}\right). \end{aligned} \tag{20}$$

Sundt’s recursion can be computationally very heavy to implement in the general case. Three features of this particular problem makes the method computationally feasible:

First we have that the claim for the upper layer is j is nil if the claim for the lower layer j' is nil. It is thus possible to develop the recursion after $x_{j'}$ and avoid to shift variable as explained in Sundt (1999), p. 34.

Second, there cannot be claims for layer j without layer j' having a full claim. We thus have that

$$\min\left(L_{j'}, \left(Y_i - D_{j'}\right)^+\right) < L_{j'} \Rightarrow \min\left(L_j, \left(Y_i - D_j\right)^+\right) = 0 .$$

With the notation from (20) we have that the support of $g_{(l,l')}^{(j,j')}$ in (19) can be restricted to

$$\overline{M} = \{(0,0), (0,1), (0,2), \ldots, (0, \kappa - 1), (0, \kappa), (1, \kappa), (2, \kappa), \ldots, (\kappa, \kappa)\}. \tag{21}$$

The third feature making this problem even simpler is the fact that there must be at least k full claims to the lower layer j' for there to be claims to k’th reinstatement of the upper layer *j*. We will thus have that $f^{(j,j')}_{(l,l')} = 0$ for $l > \left[\frac{l'}{\kappa}\right]\kappa$ where [*x*] is the integer part of *x*.

With the introduced notation and considering the remarks made above Sundt's recursion formula can be written as

$$f^{(j,j')}_{(l,l')} = \begin{cases} \sum_{n=0}^{\infty} q_n \left(g^{(j,j')}_{(0,0)}\right)^n, & l' = 0, l = 0 \\ \dfrac{I(l' \geq \kappa)}{1 - a g^{(j,j')}_{(0,0)}} \left\{ \sum_{i=1}^{\min(l,\kappa)} \left(a + b\dfrac{\kappa}{l'}\right) g^{(j,j')}_{(i,\kappa)} f^{(j,j')}_{(l-i,l'-\kappa)} + \sum_{i'=1}^{\min(l',\kappa)} \left(a + b\dfrac{i'}{l'}\right) g^{(j,j')}_{(0,i')} f^{(j,j')}_{(l,l'-i')}, \right\}, & \\ l' \geq 1, \left[\dfrac{l'}{\kappa}\right]\kappa \geq l \geq 0 & \\ 0, & elsewhere \end{cases} \tag{22}$$

where $I(l' \geq \kappa)$ is the indicator for $l' \geq \kappa$.

The claim size distribution is quite easily discretizised to the set $\overline{M}$ in (21). The area [0, $R_{j'}$) corresponds to the point (0, 0). $\left\{R_{j'} + \frac{L_{j'}}{\kappa}, R_{j'} + 2\frac{L_{j'}}{\kappa}, ..., R_{j'} + (\kappa - 1)\frac{L_{j'}}{\kappa}\right\}$ corresponds to {(0, 1), (0, 2),…, (0, κ - 1)}. In the interval [$R_{j'} + L_{j'}$, R_j] there is a full claim for layer *j'* but no claim for layer *j*, that is the point (0, κ). The following points $\left\{R_j + \frac{L_j}{\kappa}, R_j + 2\frac{L_j}{\kappa}, ..., R_j + (\kappa - 1)\frac{L_j}{\kappa}, R_j + L_j\right\}$ corresponds to {(1, κ), (2, κ),…, (κ, κ)}.

6. EXAMPLE

Assume that the following two-layer programme needs to be evaluated: Layer 1 is 10 XS 10 with an aggregate deductible of 20 and 3 reinstatements each at 100% extra premium. Layer 2 is 20 XS 20 with 2 reinstatements each at 100% extra premium. We thus have the following parameters: $L_1 = 10$, $L_2 = 20$, $R_1 = 10$, $R_2 = 20$, $D_1 = 20$, $D_2 = 0$, $r_1 = 3$, $c_1^1 = c_1^2 = c_1^3 = 1$, $r_2 = 2$, $c_2^1 = c_2^2 = 1$.

The underlying risk is modelled by a Pareto-Poisson-model: The number of claims to layer 1 is Poisson distributed with mean 2 and the individual claims size is Pareto distributed with an alpha-parameter of 1.5. The distribution function of the variables Y_i is thus given by

$$G_0(y) = 1 - \left(\frac{10}{y}\right)^{1.5}, y > 10.$$

The two layers are initially quoted by the method from section 4. $\kappa = 100$ is used with a mean conserving discretization technique. The initial premiums giving a loading factor of 15% are calculated recursively to $\overline{\pi}_1 = 1.68$ and $\overline{\pi}_2 = 6.72$. The corresponding means and variances are $EX_1 = 1.16$, $EX_2 = 5.54$, $VX_1 = 11.81$ and $VX_2 = 62.31$. These premiums are in the following assumed to be the ones offered to the reinsurer.

So how do the two layers perform as a program? To answer this question the method from the previous section is used. The density $f^{(2,1)}(x_1, x_2)$ is calculated by (22) on the area $0 \leq x_1 \leq 60$, $0 \leq x_2 \leq 40$. The distribution function

$$F^{(2,1)}(x, y) = \int_0^x \int_0^y f^{(2,1)}(x_1, x_2) dx_2 dx_1$$

is represented graphically in figure 1. It is seen that the distribution function is increasing for $x_2 = 0$. This is due to the aggregate deductible on layer 1. This makes it possible for layer 2 to be hit without there being claims for layer 1.

A covariance of 15.50 is calculated by (17) with the mean value from (18). This gives a total loading of 17.95 on a cross signing with equal shares on both layers.

In the last calculation κ has also been set to 100. Studies show that a lower κ of 10 or 20 in most cases will be sufficient. $\kappa = 10$ give the same precision as 100,000 simulations. While the simulation takes about 30 seconds for a three layer programme (programmed in APL), the calculations by the method presented here should be almost instantaneous. A total run time of 30 seconds is not feasible if various layer limits, premium or reinstatement terms need to be tested.

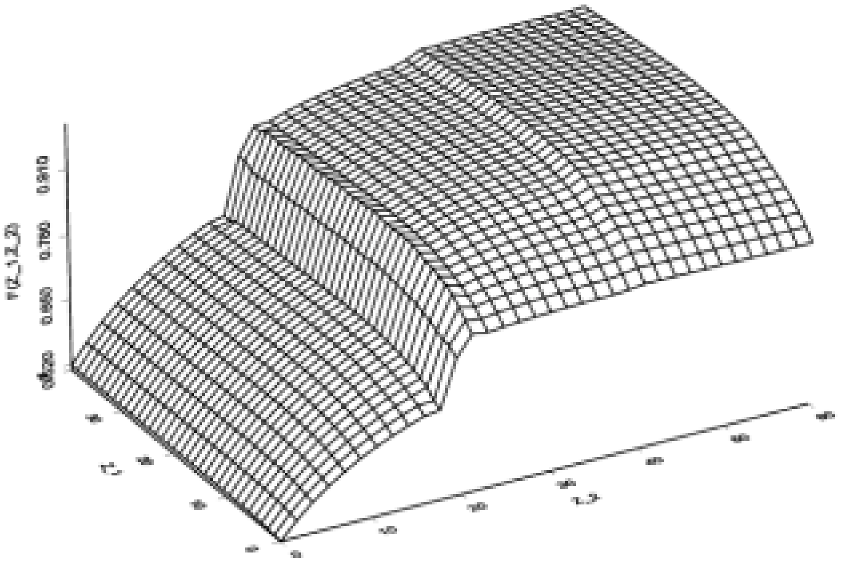

Figure 1: Distribution function for ultimate net loss to example layers 1 and 2 (marked respectively “z_1” and “z_2”.

7. CONCLUSION

Consider the following simpler version of the example in the previous section: Layer 1 is 10 XS 10 with 2 free reinstatements, layer 2 is 20 XS 20 also with two free reinstatements. It is clear that compared to a single layer of 30 XS 10 with two free reinstatements the latter should be the most expensive. Both the single layer solution and the two-layer combination have the same aggregate limit of 90 but the combination of more than one layer restricts the cover on each part of the combined layer. When each layer is quoted separately it often happens that the opposite is the case: The two-layer combination is the most expensive (just set the mean number of claims to 1 and alpha to 1.5 and the prices with a loading of 15% are 6.85 for 10 XS 10, 5.38 for 20 XS 20, and 12.10 for 30 XS 10). These are the kind of irregularities that result if multilayer covers are not loaded on a total basis. The method presented in this article gives a fast and easily implemented way of quoting multilayer covers.

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REFERENCES

Ambagaspitiya (1998); Ambagaspitiya, R. S. *On the distribution of a sum of correlated aggregate claims.* Insurance: Mathematics and Economics, no. 23, 1998, pp. 15-19.

Benktander (1961); Benktander, G. *On the correlation in results from different layers in excess of loss reinsurance.* Preceedings of the XVII'th International Congress of Actuaries 1961, pp. 203-209.

Hesselager (1996); Hesselager, O. *Recursions for Certain Bivariate Counting Distributions and their Compound Distributions.* Astin Bulletin, vol. 26, no. 1, 1996, pp. 35-52.

Panjer (1981); Panjer, H. H. *Recursive Evaluation of a Family of Compound Distributions.* Astin Bulletin, vol. 12, 1981, pp. 22-26.

Rytgaard (1991); Rytgaard, M. *Variations on Typical Excess of Loss Covers.* Preceedings of the XIII'th ASTIN Colloquium, 1991, p. 299.

Sundt et. al. (1981); Sundt, B., Jewell, W. S. *Further results on recursive evaluation of compound distributions.* Astin Bulletin, vol. 12, no. 1, 1981, pp. 27-39.

Sundt (1991); Sundt, B. *On excess of loss reinsurance with reinstatements.* Mitteilungen der Schweiz. Vereinigung der Versicherungsmathematiker, Heft 1, 1991, pp. 51-65.

Sundt (1999); Sundt, B. *On Multivariate Panjer Rekursion.* Astin Bulletin, vol. 29, no. 1, 1999, pp. 29-45.