

The Garch model and their application to the VaR

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Topic 4

Abastract

This paper is a summary of the Arch models publicity since 1982 and their application on the Value at Risk of Portfolio, with an analysis of the impact of the bad news on the "market humors" and as a result on the assets volatility. It try to explain the different tests used on the selection of the best model for the estimation of the volatility. The results suggest that in our market, like in others, each series has its own "personality", so for each series I applied the model, which more fix to predict the volatility. I found how the returns on stocks are affected for the news, detecting euphoric markets (Merval 1990-1991) and markets strongly impacted by the crises.

Keywords: Arch, Garch, Egarch, Tarch, Value at Risk, Portfolio, Volatility, Fractal, Hurst

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Ricardo A. Tagliafichi*

Introduction

It is a lot what has been written and investigated in the application from the non linear models to the Capital Markets, for that is considered the importance of the use of these models with the appropriate pattern to each one of this applications.

The most important variable, is the volatility of assets returns of the different ones that they are negotiated in the market through the different operations that are carried out in the same one, each one of them with different objectives. It is known that the operations of purchase of shares to the effects of forming a portfolio, they are selected using different models as the CAPM, APT, etc. Together with the purchase an asset you begins to generate a possibility to take earnings or loses according to the evolution of the prices of the acquired asset and of this for the economic units present aversion to loses ones, therefore it is generated the estimation of the Value at Risk or VaR.

On the other hand, according to the position of each operator, exist the derivatives operations (futures and options) that are used to cover different risks that it assumes the company, through the commitments of merchandise sales with differed delivery, debt adopted in foreign currencies, or to reduce the position of VaR for the taken portfolio.

In all these situations, the volatility is the invited of honor, to the effects of the estimates of the primes and the value at risk of each asset. In consequence the modeling of the volatility allows to adjust with best precision the results obtained for the decision making.

In the first part of this work it is presented a brief summary of the classic models and the incidence of the fractal models like Hurst coefficient to the effects of knowing if the markets have memory of the last facts. In the second part justifies the landing of the Arch models, with their two variants, the symmetrical models, Arch and Garch and the asymmetric like, Egarch and Tarch, to the effects of estimating the behavior of the volatility. In the third part is made an analysis of the behavior of different assets and their application for calculate VaR.

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I. The Hurst process or the memory of the process

1. The classic hypotheses

When the capital markets are analyzed the premise is that the behavior of the market is perfect. This concept is based on the following premises: to) The market has rates in continuous form, so that the space of time among each rate is very small b) it is defined like return in the following way: $R_t = \ln(P_t) - \ln(P_{t-1})$ that is it formulates that it capitalizes in continuous form the lost ones or the earnings in the period of time $t, t-1$ c) These returns $R_t, R_{t-1}, R_{t-2}, \dots, R_{t-q}$ doesn't have any relationship among if, the fact that one period is positive, doesn't imply that the following one is positive or negative. In other words if the market today was rising, it doesn't imply that tomorrow is negative or positive. d) The returns are distributed identically and applying the hypotheses of the Central Theorem of Limits, the distribution of probabilities follows the normal law.

If the realizations of the returns don't have any relationship among them, then it is said that we are in presence of a Random Walk. That is to say that the price of an asset is the resultant of the price of the previous day but the return that he can take any positive or negative value according to the rate that he takes the asset. If we denote Y_t to the $\ln(P_t)$ and we define as ε_t to the difference between Y_t and Y_{t-1} we can express the following:

$$Y_t = Y_0 + \sum_{i=1}^t \varepsilon_i \quad (1.1)$$

Taking the expected values, is obtained $E(Y_t) = E(Y_{t-s}) = Y_0$, therefore the mean of a Random Walk is a constant. In consequence all the stochastic shocks doesn't have effect of decline in the sequence of $\{Y_t\}$. In knowledge of the first t results of a process $\{\varepsilon_t\}$ the conditional mean of Y_{t+1} it is:

$$E[Y_{t+1}] = E[Y_t + \varepsilon_{t+1}] = Y_t$$

In the same way the conditional mean of Y_{t+s} for each $s > 0$ can be obtained of

$$Y_{t+s} = Y_t + \sum_{i=1}^s \varepsilon_{t+i} = Y_t \quad \therefore E[Y_{t+s}] = Y_t + E\left[\sum_{i=1}^s \varepsilon_{t+i}\right] = Y_t \quad (1.2)$$

In consequence the series Y_t this permanently influenced by the shock of ε_t , then the variance is dependent of the time as you can appreciate:

$$\text{Var}(Y_t) = \text{Var}(\varepsilon_t + \varepsilon_{t-1} + \dots + \varepsilon_1) = t\sigma^2 \quad (1.3)$$

$$\text{Var}(Y_{t-s}) = \text{Var}(\varepsilon_t + \varepsilon_{t-s-1} + \dots + \varepsilon_1) = (t-s)\sigma^2 \quad (1.4)$$

As a result the variance is not constant, $Var(Y_t) \neq Var(Y_{t-s})$ then the Random Walk is not stationary. When $t \rightarrow \infty$ the variance of Y_t also approaches to infinite. This practices it is derived of the observations carried out by Einstein that show that the increment of the distance that a particle travels, in Brownian motion¹, is similar to the square root of the time that uses to travel her.

In consequence the Random Walk walks without direction without showing any tendency to grow or to decrease If the covariance is analyzed (γ_{t-s}) between Y_t e Y_{t-s} we obtain:

$$\begin{aligned} E(Y_t - Y_0)(Y_{t-s} - Y_0) &= E[(\varepsilon_t + \varepsilon_{t-1} + \dots + \varepsilon_1)(\varepsilon_{t-s} + \varepsilon_{t-s-1} + \dots + \varepsilon_1)] \\ &= E[(\varepsilon_{t-s})^2 + (\varepsilon_{t-s-1})^2 + \dots + (\varepsilon_1)^2] \\ &= (t-s)\sigma^2 \end{aligned} \quad (1.5)$$

Starting from the covariance of the values of Y_t , Y_{t+s} , we can calculate the value of the correlation coefficient ρ_s that arises of the relationship of the covariance and the volatilities as:

$$\rho_s = \frac{(t-s)\sigma^2}{\sqrt{(t-s)\sigma} \sqrt{t}\sigma} = \frac{(t-s)}{\sqrt{(t-s)t}} = \sqrt{\frac{(t-s)}{t}} \quad (1.6)$$

This coefficient is important in the detection of the non-stationary series. In the first correlations for small values of s the value of $(t+s)/t$ is approximately similar to the unit. In consequence when the value of s grows, the autocorrelaci3n function, ρ_s , decrease. Therefore it is impossible to use the autocorrelaci3n function to distinguish among a process with unitary root $a_1 = 1$ and a process where a_1 are near to the unit. In the models ARMA this decline is very slow in the function as s grows. This is an indication that the process is not stationary, that is to say, the values of Y_t are not independent of the time.

2. The performance of the market

It is interesting to check that the market doesn't behave according to the hypotheses of the efficient markets, and in consequence the returns don't travel the way of the Random Walk.

If they are analyzed the procedures of the distribution of probabilities, where applying non parametric test and using Kolmogorov Smirnov's distribution, we can affirm that the returns don't follow a normal distribution of probabilities. To say that the central theorem of it limits it is not completed it is very intrepid, but “.. *although I cannot say that all the crows are black, but all the crows that I saw, they are black* “ Also other tests like those of squared Chi or that of Jarque Bera takes to the same previous conclusion. The test of KS is one of the most potent because the distribution that is used for its contrast is independent of the normal distribution of probabilities since it is built starting from equations lineal recursive.

¹ *The Brownian motion is due to Robert Brown who observed the movement of little particles under a gas injection*

Thinking that the returns follow a Random Walk, an analysis carried out to most of the rates of the markets, like stocks, bonds, indexes, currencies and commodities, in all the analyzed series of returns, the autocorrelación functions and partial autocorrelación don't show a conduct of white noise, in contrary they show certain structure that it allows to indicate that there is something systematic that we can predict in the time series of the returns.

3. The periodic structure of the volatility

Together with the considerations about the efficiency of the markets and the one on the way to the Random Walk, applying the distribution of probabilities, appears the periodic structure of the volatility in a period of given time. As it was demonstrated in (1.3) and (1.4), the annual volatility you can calculate starting from the daily volatility as being $\sigma_{annual} = \sigma_{diaria} \sqrt{n}$ where n is the quantity of days or operations days that there is in a year.

However in spite of this not very precise method for annualize the volatility, is very well-known that the volatility climbs to higher speeds that the square root of the quantity of periods used to calculate one period. Turner and Weigel (1990), Schiller (1989) and Peters (1991) on one hand they demonstrated that the annual volatility climbs to a bigger speed that being n the quantity of periods included in him finishes to estimate. On the other hand Young (1971), prove the hypotheses $H_o : \sigma_n = \sigma_1 \sqrt{n}$ against $H_1 : \sigma_n \neq \sigma_1 \sqrt{n}$ demonstrating that you could not accept the null hypothesis. As a result of the exposed by Young I test the hypotheses mentioned with the conclusion that in 22 actions and seven funds of the Argentinean market, the yen against the dollar, the index Merval, Dow Jones, Nikkei and S&P 500 and 8 commodities that quote in the CBOT, you cannot accept the null hypothesis outlined by Young.

An analysis of the speed was also made of having climbed for different terms of volatility obtaining the following results:

Table 1

days \ Asset	1	2	4	8	16	32	64	128	256
Siderca σ_t	0,0373	0,05466	0,07686	0,11120	0,16395	0,23945	0,33762	0,47099	0,72834
Exponent or Speed of increment	*****	0,54910	0,52036	0,52448	0,53340	0,53601	0,52928	0,52228	0,53561
Bono Pro2 dólar σ_t	0,0112	0,01580	0,02333	0,03365	0,04941	0,06631	0,08054	0,10476	0,13636
Exponent or Speed of increment	*****	0,49744	0,52990	0,52932	0,53553	0,51333	0,47450	0,46092	0,45084

days \ Asset	1	2	4	8	16	32	64	128	256
Merval σ_t	0,0294	0,04305	0,06014	0,08740	0,12863	0,18312	0,25426	0,36346	0,53265
Exponent or Speed of increment	*****	0,54742	0,51488	0,52297	0,53161	0,52719	0,51825	0,51785	0,52206

As a result of that exposed there is a different procedures between the bonds, the shares and the Merval index, finding that although the hypotheses of Young are accepted, the increment speeds or of having climbed starting from daily volatilities they are bigger at 0.5 in the shares and the index and in the bond the increment speeds are bigger in smaller periods to 60 days and smaller in more periods to 60 days

4. The Hurst exponent and the coefficient R/S. A measure of the memory of the process -

a) The Hurst exponent

The coefficient or exponent of Hurst is a number that this related with the probability that an event this auto correlated That is to say that there are events that condition the appearance or event of another similar event in some moment. This coefficient is born of the observations that was carry out by engineer Hurst analyzing the currents of the Nile River for the location of Assuan dam. He found that the overflows ones were followed by other overflows ones, for that he builds this coefficient starting from the following equation:

$$R / S = c n^H \quad (4.1)$$

$$Ln(R / S) = Ln(c) + H Ln(n) \quad (4.2)$$

It is important to highlight that to make this analysis the rescaled or R/S, it doesn't require of any gaussian process, for what is independent of the same one, neither it requires of any parametric process, for what is also independent of all underlying distribution.

According to the original theory if $H = 0.50$ imply that the process doesn't have memory, therefore it is a process where the series of data is independent among if that is to say in definitive a process Random Walk.

If $0.50 < H < 1$ imply that the series is persistent, and a series is persistent when this characterized by a long memory of its process. In terms of a dynamic process of chaos, it means that there is a dependence of the initial conditions. This long memory is indifferent of the scale of time, that is to say that the daily returns will be correlated with daily returns and the weekly returns will be correlated with weekly returns. There is not a characteristic scale of time, the

characteristic key of the series it is a process fractal. The conflict between the symmetry of the geometry Euclidian and the asymmetry of the real world can be extended to our concept of time. Traditionally the events can be seen as eventual or deterministic. In the deterministic concept, all the events through the time have been fixed from the moment of the creation. In contrast the nihilistic groups consider to all the events like random that derive of the structure lack of order through the time. In the time fractal, the chance and the determinism, the chaos and the order coexist.

If $0 < H < 0.50$ mean that the series is antipersistence. System antipersistence covers less distance than a system at random. So that a system covers less distance than a system at random, this it should change but frequently that a process at random.

The most common series that plows in the nature it plows the persistent series and they it plows those but common in the market of capitals and the economy.

b) The coefficient R/S

The coefficient R/S is calculated through the changes of prices of the market of capitals that can be daily, weekly etc. These changes of prices can analyze them as $Y_t = Ln\left(\frac{P_t}{P_{t-1}}\right)$ in such way that you can calculate the mean value and the volatility of those changes of prices that we will call returns (For engineer Hurst, the changes of prices were the changes in the régime of the river). For calculate this coefficient R/S, they should be normalized the series or said otherwise to transform the series into a new series centered in the half value. In the series of the market of capitals, the stocking of the returns is therefore zero the series it is as this calculated in original form.

They are made then periods but small inside the series in a such way that you began with periods of 10 days (if they are calculating in daily form) until arriving to periods that are as maximum half of the data analyzed in the series that we will call n in such way that we will calculate $R_n = \max(Y_t \cdots Y_n) - \min(Y_t \cdots Y_n)$ and in consequence the coefficient rescaled will be $R/S_n = R_n/S_n$ where S_n is the volatility of this sub period.

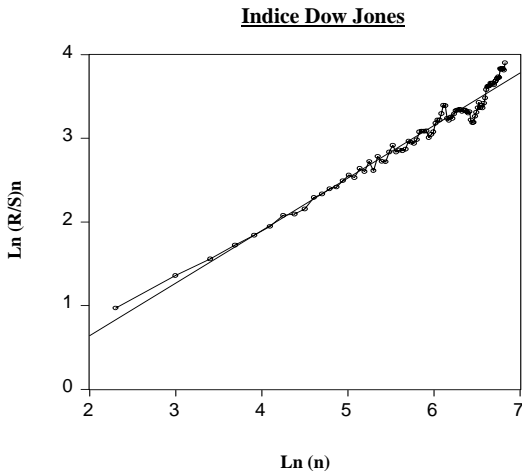
This value this referred to a range rescaled that has half zero and this expressed in terms of the volatility of the sub-period. The value of R/S increases with the increment of the sub period starting from the equation where H is known as the Hurst² exponent value of the power law.

This is the first connection between the exponent of Hurst and the geometry fractal. Let us remember that all the fractales is climbed according to a power law.

² Hurst call this coefficient like K, but Mandelbroot rename the same coefficient like H in honor to Hurst who has discovered the processes

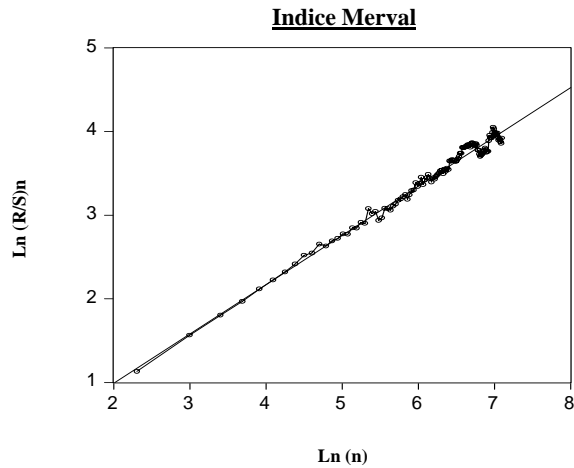
c. Some results of the calculation of the coefficient of Hurst

They are exposed some results of the application of the equations next (4.1) and (4.2) where it is verified that the series are persistent and therefore it is not fulfilled the conditions of the efficient markets. As already you expressed when the value of H is bigger than 0.50, the series is persistent and therefore this assuring that there is a correlation among the returns, or the daily differences of prices. The obtained results and the corresponding graph are the following ones:



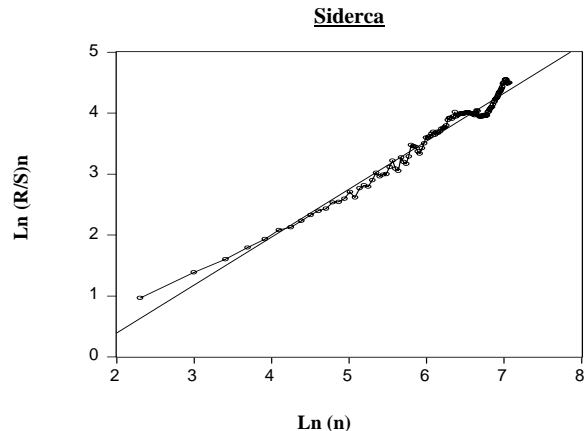
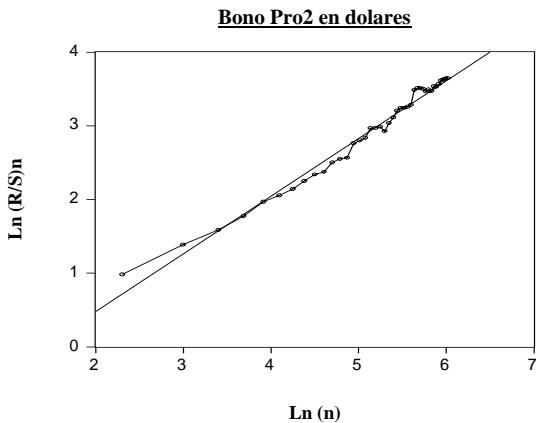
Regression results

Coefficient H	0.628
Standard error of H	0.011
Probability that H is null	0.000
R ²	0.974
Constant	-0.617



Regression results

Coefficient H	0.589
Standard error of H	0.006
Probability that H is null	0.000
R ²	0.987
Constant	-0.184



Regression results

Coefficient H	0.782
Standard error of H	0.016
Probability of H its null	0.000
R ²	0.983
Constant	-1.083

Regression results

Coefficiente H	0.787
Dispersión de H	0.011
Probability of H its null	0.000
R ²	0.977
Constant	-1.181

Starting from the exposed results the following conclusions of the use of the exponent of Hurst can be made:

- 1) The series presents coefficients H over 0.50, that indicates the presence of persistence in the series
- 2) The Hurst exponent obtained is an estimator whose parameter is not null starting from the dispersion of the same one
- 3) The values of R² are high that allows to consider a good regression
- 4) Starting from the knowledge that one of the properties of the coefficient R/S is the confirmation of the presence of cycles what is proven through the use of the FFT and its significant tests³. The presence of the cycles can detect it when the line of the logarithm of R/S_n separates the regression straight line; in that point the presence of a cycle appears. Also analyzing the coefficient V. Statistic. This ratio goes growing as the quantity of days increases until he/she takes place an it breaks and he/she stops to grow reason for the one which you this in presence of a cycle.
Remember you that the width of the cycle cannot be bigger than the value of the range rescaled -
- 5) The presence of small cycles (the curve this far from the regression straight line to the beginning) they give base to the study wavelets, and the presence of persistence, to a modeling of the returns through the conditioned volatility.
- 6) It is tempting to use to the Hurst exponent like the coefficient to estimate the variance in annual terms. Somebody has proposed that since the speed of growth of the volatility to I finish it is bigger at 0.50, then we can estimate the annual volatilities applying being H the exponent of Hurst and n the quantity of days in one year.

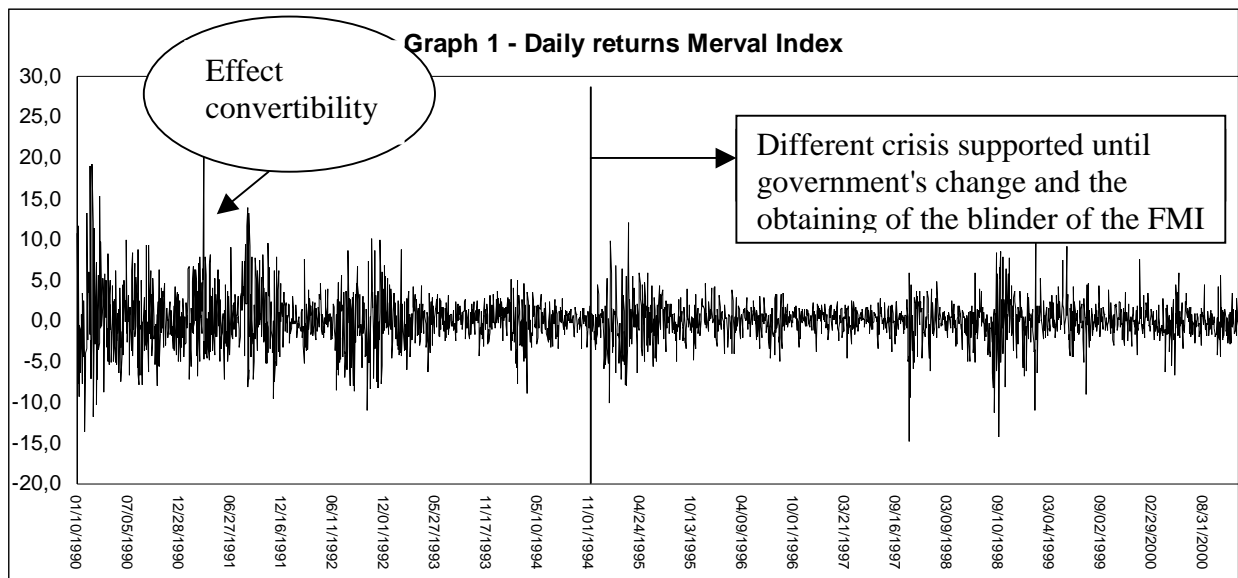
³ R. A. Tagliafichi in 1995 with H. Steinbrun in *The cycles applied to the Capital Markets presented at Congress of Actuaries* and R. A. Tagliafichi in *Statistical Models applied to the Capital Markets, workshop in Palermo University*, has detected the presence of significant cycles with amplitude less than 10 days.

II. The Garch model

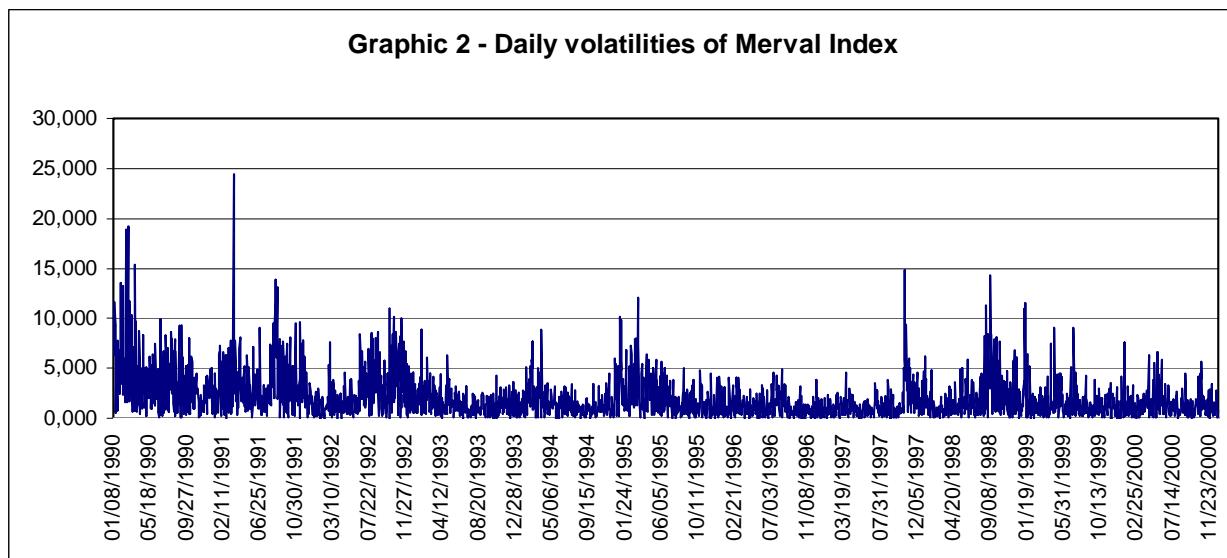
1. The behavior of the series

To the effects of continuing with our work have been considered representative series of the market of capitals of the Argentina especially. Of the series analyzed previously we can carry out the following analysis as for their fundamental characteristics:

Before the detail of the measures we will analyze the movement of the Merval Index (Index of the stock leaders' Buenos Aires, Argentina) in the behavior it has been observed from 1990 until ends of the year 2000, suffering a decade of news, those that were good news with the coming of the convertibility and the bad news corresponding to the crises that have been supported.



A true history of an economic process that has stayed for one decade and that it has supported all type of attacks from the internal crises until the contagion effects of the Mexican, Asian, Russian and Brazilian crises, and the internal problems of the declarations of candidates before the elections, former presidents, and the goings and a new government's comings until the recovery of trust for effect of the financial blinder granted by the IMF for this year 2001, avoiding the default risk.



Accompanying this whole crisis one can observe the movement of the daily volatility of the one referred index that seemed not to have rest after 1995 date of the crisis of Mexico.

To the effects of a better analysis the following square is presented with the results observed in some series of prices:

Table 2

Stock	MERVAL		Siderca		BONO PRO2 EN U\$S	BONO GLOBAL 2027 EN \$
	01/90 11/94	12/95 12/00	01/90 11/94	12/95 12/00	01/95 12/00	12/89 12/00
Observations	1222	1500	1222	1500	1371	504
Mean of Returns	0.109231	-0.015424	0.199808	0.075177	0.051171	-0.022307
Volatility	3.565741	2.321757	4.314257	3.107834	1.294979	1.169380
Skewness	0.739128	-0.383044	0.823084	-0.321122	-0.146268	-0.559697
Kurtosis	7.052907	8.020537	7.204004	7.216238	33.93084	21.97117
Maximum	24.405	12.072	26.020	17.979	14.458	9.38907
Minimum	-13.528	-14.765	-18.232	-21.309	-11.779	-9.94635
Q.STAT for K = 10	21.659	52.999	39.526	51.157	46.384	23.071
Auto correlat Not null	3	5	4	5	3	2
Autocorrelat. Partial not Null	3	4	3	5	3	2

Of the previous results we can appreciate a behavior difference before the Mexican crisis and another behavior stops after the same one. In first part leaves and enjoys the success of the arrival of the plan of convertibility, coarse to notice the value of the asymmetry, positive in the first part of the decade and negative in the second part.

Other singular data are the autocorrelación functions for each one of the series. The autocorrelación is the relationship that there are between the value ρ_1 and ρ_n with the participation of the ρ_k values intermissions, and it demonstrates that it is not a white noise, since the estimators takes values outside of the areas of acceptance to say that the parameter ρ_k is null. The same thing happens with the function of partial autocorrelación that is the relationship that there are between the values of ρ_{11} and ρ_{nn} , without the participation of the ρ_{kk} values intermissions. In the previous square they are shown the quantity of non-null values in the first 10 lags.

The value of Q-statistic calculated for the lag 10 in each one of the series this accepting the idea that there is a correlation among the data, and this correlation this contradicting the classic hypothesis that indicates that the markets are efficient. This statistical one developed by Ljung-Box is good to determine if there is autocorrelación among the values of those. This statistical one is calculated in the following way:

$$Q = n(n+2) \sum_{i=1}^k \frac{\rho_i^2}{n-i} \rightarrow \chi_k^2$$

Fixing as null hypothesis that the coefficients are not correlated, and determining 5% of probability for the area of rejection, for a value of with 10 degrees of freedom, the opposing value is 18.31, in consequence all value of $Q > 18.31$ allow to reject the null hypothesis and to accept the alternative that the coefficients are correlated. These results don't make but that to confirm the presence by heart of the process just as it could demonstrate himself with the values obtained with the exponent of Hurst.

Another of the analyses to carry out is to demonstrate if the returns have a normal distribution, that is to say that the returns are distributed identically. To such effects they have undergone the series test of kindness of the adjustment applying Kolmogorov Smirnoff's distributions, and the test of Chi - square that been is obtained that they are shown in the table 3

This table was made starting from test of kindness of the applied adjustment Kolmogorov Smirnoff's distributions where it is to prove that the maxim differs between the values of a theoretical distribution and the values of the real distribution, (D_n), it fulfills the following equation, $P(D_n < \varepsilon_{n,0.95}) = 0.95$ being $\varepsilon_{n,0.95} = 1.3581/n^{0.5}$. The same thing has been carried out applying Chi -square's well-known test, being a value of the test coming from the following equation being also a difference among the theoretical and real values found in each interval.

Table 3

Asset	Number of Observations	Kolmogorov Smirnov test		Chi – square test	
		D_n	$\epsilon_{n,0.95}$	χ^2	Probability of refuse zone
Merval Index	2722	0.0844	0.026534	387.91	0.00
Siderca	2722	0.0658	0.026534	769.86	0.00
Bono Pro2 en dollars	1371	0.2179	0.036678	2666.97	0.00
Bono Global 27 en Pesos	504	0.2266	0.060376	2849.35	0.00

Of that exposed in the previous square you cannot accept there is hypothesis of normality applying the central theorem of it limits, questioning that the variables are distributed identically.

What happens if we calculate the value of the mean conditioned through a regression of the series on a constant. In such way the regression model is $R_t = c + \epsilon_t$, where the residuals are $\epsilon_t = R_t - c$. Using this procedure the difference among the value that the variable takes in the moment t and the constant obtained by the regression is denoted as ϵ_t

If the residuals are coverts as square residuals it will be been in presence of the variances of the series under study and in consequence some conclusions could be reached with regard to the variances and to the volatility.

Making a regression on the constant of the series that we have under study has produced the following results as for the correlation functions and partial autocorrelación of the variances of the regression.

Table 4

Asset	Q – Statistic For k = 10	Number of lags not null	
		Auto-correlation	Partial Autocorrelation
Merval Index 01/90 al 11/94	359.48	10	8
Merval Index 12/94 al 12/00	479.52	10	8
Siderca 01/90 al 11/94	477.92	10	7
Siderca 12/94 al 12/00	392.65	10	8
Bono Pro2 en dollars 26/01/95 al 30/12/00	52.953	6	7
Bono Global 17 01/12/89 al 30/12/00	152.33	5	7

Of the results exposed in the table 3 it arises clearly that the variances are correlated among if, therefore the homocedasticity principle is not completed. To accept this hypothesis is to accept that there is a structure among the variances that we can model, and therefore one can affirm that the volatility is not constant during the whole period, in consequence to these series you the flame conditioned heterocedasticity since there are periods in which the variance is relatively high (to see graphic 2).

In the traditional econometrics models the variance of the errors is assumed as constant, and like we see in some cases this supposition it is inappropriate. A owner of a bond or stock market should be interested in the prediction of the return rate and the variance during the period in that he will be possessor of the asset. In consequence the knowledge of the non-conditional variance is not important mainly when he wants to buy in the moment t and to sell it in t+1. If we remit ourselves to the market of having derived and we center ourselves in the operations of options of American type, where the volatility is the adjustment variable to determine the prime's price. -

2. The model Garch, or as modeling the volatility

Engle presented the ARCH model in 1982 modeling the inflation rate in the United Kingdom showing like the stocking and the variance can be calculated simultaneously.

To know the methodology applied by Engle considers you a model always of conditional prediction of the type:

$$E_t y_{t+1} = a_0 + a_1 y_t$$

If we use the conditional mean to budget y_{t+1} , y_{t+1} depending on the last values of y_t that is to say it is a process of Markov, then the variance of the error of the conditional prediction is.

$$E_t \{(y_{t+1} - a_0 - a_1 y_t)^2\} = E_t \varepsilon_{t+1}^2 = \sigma^2.$$

If the non conditional variance, the non conditional prediction is used it is always the mean value of the whole y_t series that is similar to:

$$\begin{aligned} E_t y_{t+1} &= a_0 + a_1 y_t \\ E_t y_{t+1} - a_1 y_t &= a_0; \quad \text{si } y_t = y_{t+1} \\ y_t(1 - a_1) &= a_0 \\ y_{t+1} &= a_0 / (1 - a_1) \end{aligned}$$

In consequence the variance of the non conditional prediction is:

$$E_t \left[\left(y_{t+1} - \frac{a_0}{1 - a_1} \right)^2 \right] = E_t \left[\frac{\varepsilon_{t+1}^2}{1 - a_1^2} \right] = \frac{\sigma^2}{1 - a_1^2}$$

Then if the non-conditional prediction has a bigger variance than the conditional variance, therefore the conditional prediction is preferable since the last realizations of the series are known.

In the same way the variance of the ε_t is not constant and we can calculate some tendency or prediction of its movements, through the analysis of the autocorrelation functions and partial autocorrelation, as shows the table 4, that demonstrate us the absence of white noise.

Before beginning with the development of the models it is important to determine the notation that will use:

ω is the constant of the model

$\alpha_i, \beta_i, \gamma_i$ are the parameters of the model

σ_t is the predicted volatility for period t or the conditional volatility

ε_t is the real volatility ($R_t - \bar{R}$)

R_t is the return of an asset at moment t

The following one is a synthesis of the analyzed models that ARCH begins on the base of the pattern presented by Engle in 1982 for the analysis of the residuals of a prediction model and that

applied to our well-known case $\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2$ as ARCH(q) and that then it was generalized

by Bollerslev in 1986 like model GARCH (q,p) in the way $\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2$ where

the volatility that in the pattern ARCH was conditioned now to the real volatility of last periods in the generalized pattern this not conditioned alone to the real volatility of last periods but rather also this conditioned to the conditional volatility for the previous periods.

The identification of the pattern GARCH (q,p) it follows the limits of the identification of a model ARMA (p,q) The important thing is to find that the square residuals of a prediction model behave in form homocedasticity starting from the application of a model certain ARCH.

The temptation that is presented is to calculate the model for separate, that is to say the best model of prediction traditional ARMA or a linear regression and to add a modeling of the square residual.

After the selection of to model of prediction so much ARMA or a linear regression, the new estimation of the parameters is considered starting from the model Garch that removes the heterocedasticity, transforming to the residuals into homocedasticity and recalculate the values of the parameters of the pattern ARMA or a regression by means of to process of likelihood, making that these parameters plows those of smaller variance or like hood.

The log likelihood function for k parameters is: $\ell(\vartheta_1, \vartheta_2, \dots, \vartheta_k)$ or is equal to $\prod_{i=1}^n f(x_i, \vartheta_1, \vartheta_2, \dots, \vartheta_k)$. The likelihood is maximum when is resolved the system of k equations as a result of derive the function to each parameter and consider a maximum when de equation is equal to zero

An estimator is efficient asymptotically if it is consistent, with distribution of probabilities normal asymptotically and it has a matrix of covariance asymptotically that is not bigger than the matrix of covariance asymptotically of other consistent estimators and with distribution of probabilities asymptotically normal

Applying this concept, if we estimate the parameters of a Garch (1,1), the log likelihood function for this type of models is as below it is detailed, assuming that the errors have a normal distribution of probabilities.

$$\ell_t = \log \sqrt{2\pi} + \log \sqrt{\sigma_t^2} - \frac{(y_{t-1} - \hat{y}_{t-1}\pi)^2}{2\sigma_t^2}$$

where

$$\sigma_t^2 = \omega + \alpha(y_{t-1} - \hat{y}_{t-1}\pi)^2 + \beta \sigma_{t-1}^2$$

In consequence the value of the log like hood coefficient is calculated starting from the dear coefficients of the equation. When the value of this function converges, these they are the good values and its calculation:

$$\ell = -\frac{n}{2} \left[1 + \log(2\pi) + \log \left(\frac{\hat{\varepsilon} \cdot \hat{\varepsilon}}{n} \right) \right]$$

$$\text{donde } \hat{\varepsilon} \cdot \hat{\varepsilon} = \sum_{i=1}^n (\hat{y}_i - y_i)^2$$

Does this formula define the log likelihood function for the moment t and does it understand each other that it maximizes the sum of the N - 1 observations, starting from the initial data calculated for the parameters by means of a lineal regression for α and β

Let us consider the series of returns of the index Merval in the period January of 1990 until December of 2000 inclusive. Since it had been demonstrated that the variances were correlated (Table 2) we proceeded to estimate the variances with of the pattern GARCH. As you it can appreciate comparing the graph 3 the modeling of the conditional volatility it follows the movements of the real volatility while the non-conditional volatility (constant in the time) it doesn't capture the heterocedasticity.

The results of the modeling one of the volatility applying a model Garch is those that continue:

Table 5

	01/90 al 12/00		01/90 al 11/94		12/94 al 12/00		11/98 al 12/00	
Number of obs.	2722		1222		1500		503	
ω	0.125	(0.0019)	0.088	(0.030)	0.203	(0.035)	0.503	(0.138)
α	0.141	(0.0090)	0.137	(0.018)	0.152	(0.012)	0.122	(0.020)
β	0.847	(0.0090)	0.862	(0.016)	0.814	(0.015)	0.760	(0.040)
Probability of white noise for 8 lags	0.516		0.774		0.799		0.757	
Log likelihood	-6214.22		-3037.95		-3171.32		-1058.91	
Log likelihood average	-2.28		-2.49		-2.11		-2.11	
AIC⁴	4.56		4.98		4.23		4.22	
SC⁵	4.58		4.99		4.24		4.25	

These coefficients are the log like hood obtained by means of a numeric optimization getting their convergence after several iterations with a value of the function. Of the exposed results an equation that allows modeling the volatility through three estimators arises that given the value of its variance, they allow confirming that their respective parameters are not null.

If it is analyzed the autocorrelación function and partial autocorrelación of the residuals you can appreciate that the residuals behave as a white noise, given the probabilities of white noise for the first 8 lags according to the coefficient Q - Statistic.

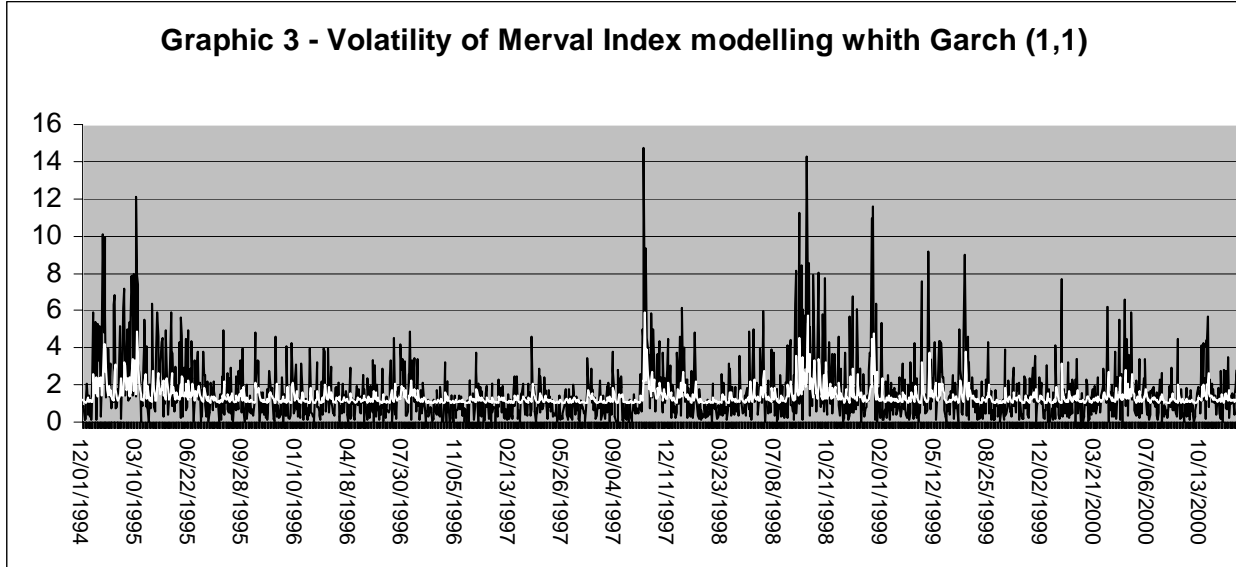
Everything seems to indicate until here that we have arrived to the most precise values for this estimate, and the obtained results help to predict the variance or the volatility for the period $t+1$. Analyzing the coefficients of the equation of the estimate of the variance α and β can we calculate the non-conditional variance in the one that $\sigma_t^2 = \sigma_{t-1}^2 = \sigma^2$. Solving for σ is:

$$\sigma = \frac{\omega}{1 - \alpha - \beta}$$

So that the pattern is stationary the sum of the parameters α and β should it be smaller to the unit. This sum it denominates persistence of the model. The beautiful of this specification is that to parsimonious model provides, with few parameters, and that we would adjust well according to the graphic 3 where the white line is the dear volatility for that period.

⁴ Akaike info Criteria = $-2\ell/n + 2k/n$ where ℓ is the value of the log likelihood, n is the number of observations and k is the number of parameter to be estimate. This coefficient evaluates de parsimony of the prediction. The equation not has excess of parameters. The selection criteria is to take the minimum value of this coefficient

⁵ Schwarz Criteria = $-2\ell/n + k \log n/n$



If the volatilities of financial series of the capital market are analyzed we can observe that the persistence of the model Garch (1,1) used it comes closer to the unit, but is what is noticed that the values of α and β are different to each other.

Two alternatives are presented for the representation of the equation of the variance of the model:

- 1) The equation of the conditioned variance estimated by a Garch (1,1) we can write it starting from considering that in the following way:

$$\begin{aligned}\varepsilon_t^2 &= v_t + \sigma_t^2 \quad \therefore \quad \sigma_t^2 = \varepsilon_t^2 - v_t \\ \varepsilon_t^2 - v_t &= \omega + \alpha\varepsilon_{t-1}^2 + \beta(\varepsilon_{t-1}^2 - v_{t-1}) \\ \varepsilon_t^2 &= \omega + (\alpha + \beta)\varepsilon_{t-1}^2 + v_t - \beta v_{t-1}\end{aligned}$$

Then the square error of a heterocedasticity process seems an ARMA (1,1). The autoregressive root that governs the persistence of the shocks of volatility is the sum of $(\alpha + \beta)$

- 2) It is possible in recursive form to substitute the variance of the previous periods in the right part of the equation and to express the variance like the sum of a pondered average from all the residuals to the last square.

$$\begin{aligned}\sigma_t^2 &= \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \\ \sigma_t^2 &= \omega + \alpha\varepsilon_{t-1}^2 + \beta(\omega + \alpha\varepsilon_{t-2}^2 + \beta\sigma_{t-2}^2) \\ &\vdots \\ \sigma_t^2 &= \frac{\omega(1-\beta^k)}{1-\beta} + \alpha\sum_{j=1}^k \beta^{j-1}\varepsilon_{t-j}^2 + \beta^k\sigma_{t-1}^2\end{aligned}$$

You can appreciate that in the Garch (1,1) the variance decays in exponential form pondering the last residuals, giving him but importance to the residuals nearer and less importance to the residuals more distant. If $k \rightarrow \infty$ then $(1 - \beta^k) = 1$ because $\beta^k = 0$ also affecting to the last one adding of the right. In consequence the variance for the period t in recursive form is in the following way:

$$\sigma_t^2 = \frac{\omega}{1 - \beta} + \alpha \sum_{j=1}^k \beta^{j-1} \varepsilon_{t-j}^2$$

Being able to express a fundamental concept, it doesn't interest the whole history of the financial, alone series they interest us the recent facts. This concept is easier to be explained to it to an operator of the stock market that to an econometrician or an actuary, since "the operators have an ARCH put in its head"

The order (1,1) in a model Garch (1,1) it means that in the prediction of the future variances (period $t+1$) or the conditioned variance this referred to the errors of the estimate of the variance of last period and the dear variance for last period. A model ARCH is a special case of the specification GARCH where the last values of the estimate of the variance don't have incidence in the calculation of the prediction of the conditional variance for the period $t+1$.

This specification taken in the financial context is that of a financial agent or trader that it predicts the period of variance adding a pondered average of many last observations (the constant) but the error of the estimate of the previous period (the value of the ARCH) and the variance observed in the previous period (the value of the GARCH). If the prospective value of the asset comes preceded of big variations, the trader will increase the estimate of the variance to cover in next period. This is consistent with the appreciations that follow other big changes to big changes.

The autoregressive root that governs the persistence of the shocks of the volatility is the sum of $\alpha + \beta$. In many applications this sum gives near values to the unit that it makes that these shocks disappears in smooth form.

In consequence, the persistence ($\alpha + \beta$) allows to predict the volatility for the future periods and in function of the same one we can already advance that if the persistence is a near value to the unit the same one it will maintain the value of the shocks taken place in the series (ε_{t-k}), do read like a shock of a strong alteration in the market, that is to say it will stay the effect of the shock. If the persistence is a value far from the unit, the effect of the shock it will be diluted in very little time.

As the expressed, with a model Garch, it is assumed that the variance of the returns can be a predictable process. The conditional variance not depends alone of you finish them variations but rather it is also based in it finishes it calculated conditional variance. In a long term prediction horizon, using a Garch (1,1), the variance can be calculated for one period T in the following way, taking as n to the quantity of periods that there are between T and $t+1$

$$\sigma_{i,T}^2 = \sigma_i^2 + \sigma_{i+1}^2 + \sigma_{i+2}^2 + \dots + \sigma_T^2$$

$$E_{t-1}(\sigma_{i,T}^2) = E_{t-1}(\sigma_i^2) + E_{t-1}(\sigma_{i+1}^2) + E_{t-1}(\sigma_{i+2}^2) + \dots + E_{t-1}(\sigma_T^2)$$

If it is calculated with a model Garch (1,1) the prediction for 2 periods using the conditional variance calculated for the first period, considering one has:

$$E_{t-1}(\sigma_{i+1}^2) = E_{t-1}(\bar{\omega} + \alpha \varepsilon_i^2 + \beta \sigma_i^2) = \bar{\omega} + (\alpha + \beta) \sigma_i^2$$

$$E_{t-1}(\sigma_{i+2}^2) = E_{t-1}(\bar{\omega} + \alpha \varepsilon_{i+1}^2 + \beta \sigma_{i+1}^2) = \bar{\omega} + (\alpha + \beta) [\bar{\omega} + (\alpha + \beta) \sigma_i^2] = \bar{\omega} + \bar{\omega}(\alpha + \beta) + (\alpha + \beta)^2 \sigma_i^2$$

Substituting n periods for the future one has:

$$E_{t-1}(\sigma_T^2) = \bar{\omega} \frac{1 - (\alpha + \beta)^n}{1 - (\alpha + \beta)} + (\alpha + \beta)^n \sigma_i^2$$

In consequence the total sum of the variance for the n periods are

$$E_{t-1}(\sigma_{i,T}^2) = \frac{\bar{\omega}}{1 - (\alpha + \beta)} \left[(n-1) - (\alpha + \beta) \frac{1 - (\alpha + \beta)^{n-1}}{1 - (\alpha + \beta)} \right] + \frac{1 - (\alpha + \beta)^n}{1 - (\alpha + \beta)} \sigma_i^2$$

In the series of returns applied to the series dollar/DM, yen/dollar, dollar/ BP, DM/dollar, price of the petroleum and the 30 year bond treasure, these they were modeled with a Garch (1,1) for the period 1990-1999 being values of persistence between 0.99 and 0.97. In the series of actions while the Dow Jones showed a persistence of 0.99 applying a Garch (1,1), the index Merval showed a high persistence for the whole period, but as nearer periods are analyzed to the beginning of the millennium, that is to say when we enter in full crisis succession, leave as the persistence it goes diminishing, that is to say a shock takes place and in little time the same one is absorbed.

Accepting the general concept developed for the model Garch, you can appreciate that the persistence doesn't make differences in the shock type that takes place in the series. For model's type, it gives the same thing that the shock is positive or negative, since ε_i is taken like square value. The certain thing is that starting from the appearance of the crises in the markets, from 1995 for our days, the negative shocks for the effects of the appearance of a bad news, they are redeemed in form but slow that a positive shock. The announcements of the FED personified in the Mr. Alan Greenspan on the drops of the rates or the maintenance of the same ones makes that the euphoria, represented in big ascends it is diluted in one or two days to return to the normality, while if the announcement of a default appears this effect it lasted but days, that is to say to one day of big low they will follow it several of days of low until the effect of the shock is wake.

Starting from 1986 they have appeared but of 200 articles that models of prediction of the volatility develop in the finance area directly centered in the calculation of σ_i^2 and that they try to solve these deficiencies.

The characteristic of these models that they appear as an alluvium of information responds to the desire of being able to predict the effect of the catastrophes or the impact of the bad news. Next some model Arch is detailed developed in these articles.

Different ARCH models

ARCH non-linear (Engle y Bollerslev 1986)

$$\sigma_t^2 = \omega + \alpha |\varepsilon_{t-1}|^\gamma + \beta \sigma_{t-1}^2$$

ARCH multiplicative (Mihoj 1987, Geweke 1986, Pantula 1986)

$$\log(\sigma_t^2) = \omega + \sum_{i=1}^p \alpha_i \log(\varepsilon_{t-i}^2)$$

GJR (Tarch) (Glosten, Jaganathan y Runkle 1989; Zakoian 1990)

$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \alpha \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2$$

donde $d_{t-1} = 1$ si $\varepsilon_{t-1} < 0$ y

$d_{t-1} = 0$ en cualquier otro caso

EGARCH (Nelson 1989)

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \alpha \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}}$$

Autoregressive Standard Deviation (Schwert 1990)

$$\sigma_t^2 = \left[\omega + \sum_{i=1}^p \alpha_i |\varepsilon_{t-i}| \right]^2$$

GARCH Asymmetric (Engle 1990)

$$\sigma_t^2 = \omega + \alpha (\varepsilon_{t-1} + \gamma)^2 + \beta \sigma_{t-1}^2$$

GARCH Non linear Asymmetric

$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \alpha (\varepsilon_{t-1} + \gamma \sigma_{t-1})^2$$

VGARCH

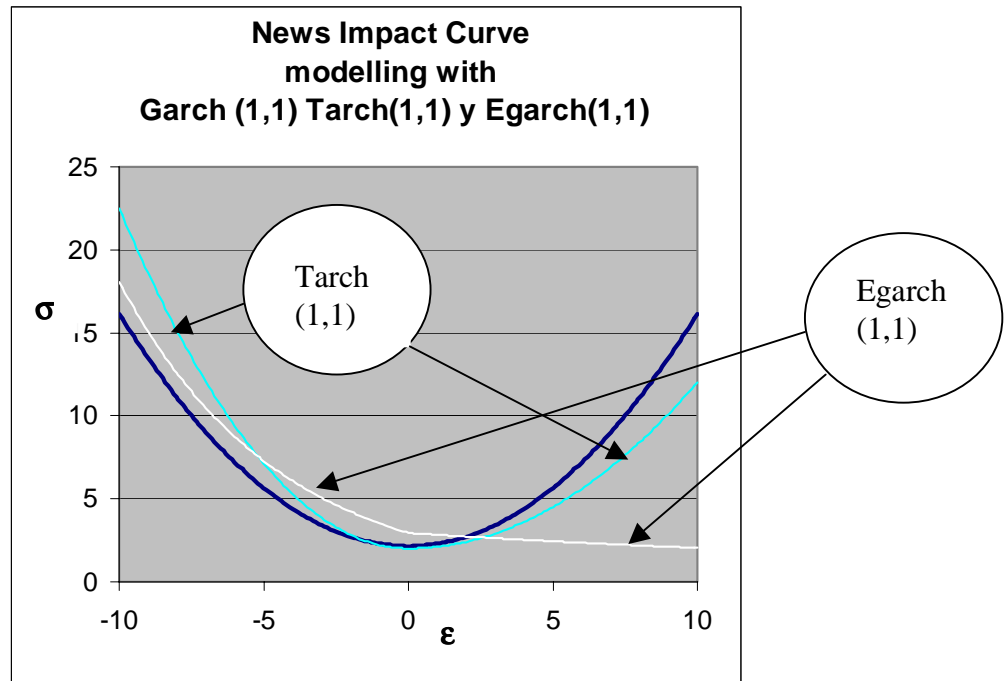
$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \alpha (\varepsilon_{t-1} / \sigma_{t-1} + \gamma)^2$$

3. The asymmetric Garch. Models Egarch y Tarch

The asymmetric models come to fill the hole that the model Garch leaves as for the incidence of the bad news in the aim of the operators, and in consequence in the daily results of the operative one.

If we consider a model Garch, Egarch or Tarch, and take a constant the predicted volatility for the previous period we can observe which will be the behavior of the predicted volatility changing the values of ε_t and we obtain the news impact curve.

Graphic 4



The Egarch (Nelson 1989) adds as additional variable to the dispersion standardized in absolute values, that is to say that annuls the negative values giving them an additional weight. Nelson presents this model as an alternative solution to the limitations that he has the Garch model, like it can be that only takes into account the magnitude of the different errors, without keeping in mind if these they are positive or negative, since the differences study them as the square of the same ones, for that that the Garch model predicts the future variance in function of the last data without caring the sign that this it has. -

The model Tarch or GJR (Glosten Jaganathan and Runkle 1990) is a model that includes as additional variable to the variable d_t that it takes values similar to 1 when ε_t takes negative values and 0 in any other case. When there are errors negative that is to say bad news and fallen in the returns of the market the variable additional d_t takes value and otherwise it is annulled. Besides these three authors, Zakoian for the same date ago do a similar presentation. -

For an Egarch model the equation for the variance prediction is:

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \alpha \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}}$$

This differs with the Garch models in two aspects:

- 1) The Egarch allows to the bad news and the good news different impact in the volatility.
- 2) The Egarch allows to the big news to have a bigger impact in the volatility that what allows it the Garch model.

In the Tarch model the prediction equation is:

$$\begin{aligned} \sigma_t^2 &= \bar{\omega} + \beta \sigma_{t-1}^2 + \alpha \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 \\ \text{donde } d_{t-1} &= 1 \text{ if } \varepsilon_{t-1} < 0 \text{ and} \\ d_{t-1} &= 0 \text{ in other case} \end{aligned}$$

It is simple to see in what differs of the Garch model with the incidence of the negative returns in the equation, giving values at d_{t-1} only when the returns are negative.

In order to determine the existence of asymmetry in the prediction model of the variance, two forms can be used: 1) by using the test of the cross correlations or 2) by using the analysis of the results of the application of the models, like they are the coefficient log likelihood and the coefficients of SC and AIC. This finishes process he/she helps to decide the use of the pattern of the prediction of the variance, estimating a parsimonious and efficient model.

One in the ways of identifying the presence of asymmetry in the models is to carry out the analysis of estimating the cross correlations among the standardized residuals from the estimation and the square standardized residuals for last periods to the effects of determining if these have a white noise or if some structure exists among the same ones. If the model is symmetrical, the crossed correlations after developing a model Garch should behave as a white noise. In the case of having crossed correlations that don't behave as white noise, this means that there is presence of asymmetry, and then it is necessary to select some other asymmetric model. Inside the class of the asymmetric models to be used, the incidences of the model Tarch and Egarch are specially studied.

The cross correlations are calculated in the following way:

$$r_{xy}(l) = \frac{C_{xy}(l)}{\sqrt{C_{xx}(0)}\sqrt{C_{yy}(0)}} \quad \text{Where:}$$

$$C_{xy}(l) \begin{cases} \sum_{t=1}^{n-l} (x_t - \bar{x})(y_{t+l} - \bar{y}) / n & \text{para } l = 0, 1, 2, \dots \\ \sum_{t=1}^{n+l} (y_t - \bar{y})(x_{t-l} - \bar{x}) / n & \text{para } l = 0, -1, -2, \dots \end{cases}$$

The variance of this crossed correlation coefficient is similar to that of the autocorrelación coefficient and partial autocorrelación, in consequence $\sigma(r_{xy}) = 1/\sqrt{n}$

Applying this concept to the crossed correlations among the square of the standardized residuals of the estimate carried out with the standardized residuals of previous periods the coefficients is obtained of $r_{\varepsilon_t \varepsilon_{t-k}}$ for $k = 0, -1, -2, \dots$

For this analysis of cross correlation the standardized residuals are calculated ε_t / σ_t as that are defined as the conventional residual of the equation of the half value, divided by the conditional standard deviation. If the model is correctly specified the residuals will be independent and identically distributed with mean = 0 and dispersion = 1, therefore it is of hoping that this function of cross correlation represents a white noise.

3. 1 Some results applied to the Argentine series

Table 6	Merval Index								
	Periodo 01/90 al 11/94			Periodo 12/94 al 12/00			Periodo 11/99 al 12/00		
Qty. Of cross correlations Not null ⁶	0			5			4		
Model	G(1,1) ⁷	E(1,1) ⁸	T(1,1) ⁹	G(1,1)	E(1,1)	T(1,1)	G(1,1)	E(1,1)	T(1,1)
Average Log Likelihood	-2.49	-2.49	-2.49	-2.11	-2.10	-2.09	-2.106	-2.089	-2.088
AIC	4.98	4.98	4.98	4.233	4.207	4.203	4.229	4.199	4.196
SC	4.99	5.00	5.00	4.247	4.225	4.221	4.262	4.242	4.238
	Bono Global 17 en pesos			Siderca			Bono Pro2 en dólares		
	Periodo 11/99 al 12/00			Periodo 12/94 al 12/00			Periodo 02/95 al 12/00		
Qty. Of C cross ¹⁰	4			5			4		
Model	G(1,1)	E(1,1)	T(1,1)	G(1,1)	E(1,1)	T(1,1)	G(1,1)	E(1,1)	T(1,1)
Average Log Likelihood	-1.36	-1.33	-1.33	-2.44	-2.10	-2.09	-0.246	-0.237	-0.244
AIC	2.73	2.69	2.68	4.880	4.86	4.86	4.58	4.44	4.55
SC	2.76	2.73	2.72	4.897	4.88	4.88	4.66	4.54	4.65

⁶ Is the quantity of times that the value of the cross correlations take values that estimate parameters not null in the first 10 lags.

⁷ Is a Garch (1,1) model

⁸ Is a Egarch (1,1) model

⁹ Is a Tarch (1,1) model

¹⁰ Idem 6

The results obtained in the table 6, with the analysis of the returns of the Merval Index we could appreciate that in the period previous to the big crises the asymmetric models don't produce some effect and evidently the fact that the cross correlations are a white noise, allows to assert that the model Garch (1,1) is the exact model. Consequently if an asymmetric model, the coefficients of the log likelihood, AIC and SC, is considered or they stay equally or they increase discarding in consequence the use of the same ones. On the other hand in all the other analyzed models, the cross correlations present values of non-null parameters, what induces to use the asymmetric models that produce improvements in the analyzed coefficients.

III. The calculate of VaR with Garch model

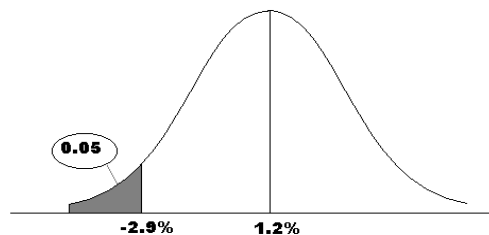
What VaR calculates in the administration of portfolio? Previous to answer we will make it a brief introduction to this estimation. **VaR predicts which is the amount that it can get lost in next period t with a certain probability**

For example consider the following trivial case. Starting from considering that the returns of a stock have a normal distribution with mean = 1.2% and volatility = 2.5%. We could settle down by means of an interval of confidence, which will be the lost maxim that we can have with 5% of probability. -

In consequence if we have an investment of \$1.000.000 in this stock with the characteristics mentioned in the previous paragraph under the same market conditions are 5% of probability of losing, is 2,9% that is to say \$29.000 (2.9% for \$1.000.000. -) The value of 2,9% arises of applying the statistical knowledge. For a probability of 5%, for both sides of the distribution, the value of abscissa of a standardized normal distribution is similar to 1.64. If we multiply 1.64 for 2.5% it will be obtained half of the journey of the interval of trust that is similar to 4.1. Applying the estimate concept for intervals is obtained that the half value of 1.2% less the journey of 4.1 throws a negative value of 2.9% that serious the lost maxim that was supported with a probability of 5%,

This is one of the measures of VaR. If the probability decreases, the lost one will be bigger, since to more certainty less precision in the estimate. If we look at it in the graphic 4 one can have a basic idea of the reasoning before expressed.

Graphic 4



The estimation of the Value at Risk of a portfolio may be done by two forms. It is not the topic of this work to ponder the kindness of the calculation of the VaR and the necessity of the use of the in the companies near to our profession like they are in the banks, financial, insurance companies of pensions, and insurance companies of life and patrimonial, but is the goal to show as the calculation of VaR is influenced by the modeling of the variance.

Of that exposed in the parts I and II, it arises that the returns don't follow a normal distribution and that the variance is not stable through the time since there is heteroscedasticity presence in the regressions that are carried out.

One of the first models that are applied in the capital market to calculate the exhibition to risk of a portfolio is the model of Riskmetrics¹¹ that is developed by J. P. Morgan based on the definition of their manual Technical document 4th edition 1996, where it is specified that the Riskmetrics model of the financial returns presents a modification to the random walk.

- a) The volatility of the returns of the stock has heteroscedasticity (they vary in the time) and they are correlated.
- b) The covariance of the returns auto-correlated and they have dynamic models
- c) The fact of assuming that the returns are normal distributed is comfortable for the following thing:
 - 1) Alone the mean and the volatility are required to describe the complete modeling of the distribution
 - 2) The sum of the multivariate returns is usually normal distributed. This fact facilitates the description of the returns of the portfolio, those that are pondered by the sum of the stocks that compose it

Starting from those enunciated precedent Riskmetrics assumes that the returns are generated by the following model:

$$R_{i,t} = \sigma_{i,t} \varepsilon_{i,t} \quad \varepsilon_{i,t} \approx N(0,1)$$

The variance of each return and the correlation entity the returns are functions of the time. The property that the returns have a distribution of probabilities of the normal type it assumes that the returns follow a conditioned normal distribution – conditioned in the time -. As we can appreciate in different presented works, the mean value of the conditioned returns tends to be zero. -

Starting from these concepts, Riskmetrics assumes the conditioned normality of the returns, accepting the presence of heteroscedasticity in the volatility, there being autocorrelación among the volatilities for that you can model the volatility to the effects of calculating VaR

The volatility of the stocks is calculated with the use of a method called EWMA, Exponential Weighted Moving Average. This method consists on the exponential application of a softened to the last volatilities. If we want to express it otherwise one can say that the volatilities of the previous days have a different weight as they move away from the day t. This weight goes decaying in exponential form. That is to say that the incidence of yesterday's volatility is more

¹¹ Riskmetrics is a trademark of J.P.Morgan

important than that of the day before yesterday and so forth. The following one is the it formulates for the I calculate of the volatility for one period but ahead:

$$\sigma_{1,t+1|t}^2 = (1 - \lambda) \sum_{i=0}^{\infty} \lambda^i \sigma_{1,t-i}^2$$

Where λ is a factor of decline, that should be in smaller consequence that the unit.

$$\sigma_{1,t+1|t}^2 = (1 - \lambda) (\sigma_{1,t}^2 + \lambda \sigma_{1,t-1}^2 + \lambda^2 \sigma_{1,t-2}^2 + \dots)$$

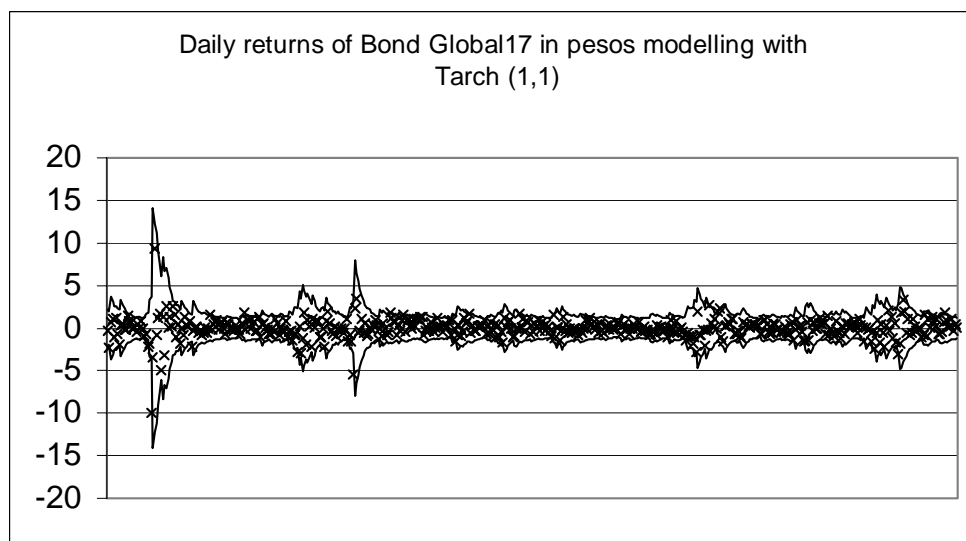
This factor is always the same for all the series. If the effect of the persistence, this method is remembered is it used with success when the sum of $\alpha + \beta$ does it comes closer to the unit. If it is compared to the value of $\lambda = 0.94$ is the same thing that to use a model Garch (1,1) where $\omega = 0$, $\alpha = 1 - \lambda$ and $\beta = \lambda$. It admits that all the series have the same behavior, are they symmetrical and don't they differ among them. Something like that could be thought for the period up to 1994, it has been demonstrated that after 1995 the things have changed and that each series has its "personality" that is to say its model and appropriate coefficients to the behavior of the same one.

If we take a series like that of the bond global 17 in pesos to model the variance is used a model Tarch (1,1) with the following equation:

$$\sigma_t^2 = 0.096 + 0.0825\varepsilon_{t-1}^2 + 0.414d_{t-1}\varepsilon_{t-1}^2 + 0.645\sigma_{t-1}^2$$

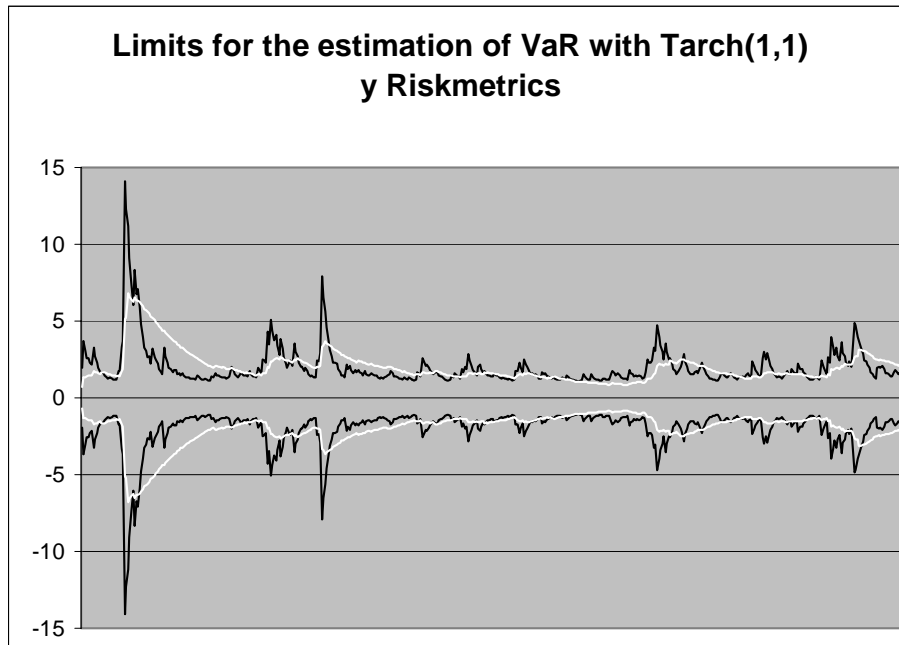
This model, like it has been shown in the table 6, it is asymmetric and the coefficients of the equation guarantee that their parameters are not null. If you graphic the volatility taking ± 2 deviations the Graphic 5 is obtained that it shows that the limits contain the 95% of the data, and the limits they go following the movement of the volatilities.

Graphic 5



In the graphic 6 the modeling of the volatility is shown compared with the use of Riskmetrics, and two appreciations can be made: 1) there is more quantity of data outside of the covering areas, they are without covering 8,5% of the real data and 2) in some periods makes a very wide interval what induces to create reservations in excess.

Graphic 6



In summary, when one needs the reserves this it doesn't exist and when one doesn't need the same one this is constituted in excess.

Conclusions

The chaotic behavior of the time series is generated through the returns of the rates of the financial assets, allows establishing that the process has persistence or memory of the last events and with different measure in each asset under study. The Hurst coefficient contributes solidity to the maintenance that the markets are not efficient and that there is a structure in the returns that it allows establishing certain correlation among the returns. Also open to the presence of cycles, and cycles of few periods, what offers an incentive to the wavelets study.

Demonstrated the behavior that the returns are not distributed identically and that the variance is correlated, we arrive to the formulation of models for forecast the variance conditioned for the next periods taking in account the previous periods using the Garch models. An analysis of the series allows establishing a behavior change before the crises of the middle of the 90 until our days. The asymmetric models are considered to the effects of improving the prediction of the volatility applying the different test and verification of coefficients of the models. The incidence of the bad news, a bad news is front page in the daily ones and then it goes disappearing little by little, a good news appears only one day and it doesn't repeat. Airplane that arrives is not news, airplane that falls if it is news. Nobody thinks to make big notice if a financial entity honors their

obligations, at most it was notified in a small notice that was fulfilled the obligation, but it is front page if the same entity enters in default. The asymmetric models put emphasis to the bad news in the estimate weighted the coefficients

Finally, the incidence of the calculate of the volatility for tomorrow's day to. the effects of making the reservations of the risk that it is wanted to assume, it demonstrates that each series should be modeled in different form and one cannot have a single coefficient for all the assets that you/they enter in the financial market. In consequence the calculate of VaR it should be adjusted by model Garch according to the behavior of each series

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