What are the Essential Features of a Good Economic Scenario Generator?

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Richard Urbach (DFA Capital Management Inc.)

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What is an Economic Scenario Generator

- An ESG generates all the financial economic and macro-economic variables necessary for risk management.
- Produces a forward looking simulated global economy of financial markets including the pricing of derivatives and alternative assets.
- The simulated global economy manifests as a distribution of possible economic futures.
- Simulation engenders unexpected, but plausible outcomes critical to assessing risk.
- An ESG is not a predictive tool.
What is an Economic Scenario Generator

- Model parameterization should induce distributions and dynamics that reflect not just the salient features of the benchmark, but also novel behavior.
- The random aspects of the simulation enable us to capture plausible scenarios that have not yet been observed.
- Those are the unexpected outcomes critical to assessing risk.
Components of the Process: A Good ESG

- Coverage of critical financial variables such as: interest rates, total return, and macro variables.
- Coverage of a broad range of asset classes.
- Dynamic relationships between these variables are captured.
- Models can be estimated and calibrated using benchmark financial data.
- Models can be fully validated.
Measuring the Performance on an ESG

- In order to assess the performance of an ESG, one must understand and measure a vast array of empirical facts.
- One must be able to judge the relationships among the financial economic variables produced by the ESG against what is understood in the historical record.
- Let us look at some of the interesting empirical facts.
Testing portfolio protection strategies like CPPI requires daily returns.

Data vs. the average 80 year simulation

### Daily returns S&P 500 1926 - 2005

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>SVJ</th>
<th>GBM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.02%</td>
<td>0.02%</td>
<td>0.02%</td>
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<tr>
<td>StdDev</td>
<td>1.2%</td>
<td>1.2%</td>
<td>1.2%</td>
</tr>
<tr>
<td>Skew</td>
<td>-0.5</td>
<td>-0.5</td>
<td>0</td>
</tr>
<tr>
<td>XS-Kurtosis</td>
<td>21</td>
<td>18</td>
<td>0</td>
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### Jumps exceeding

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<tr>
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<th>Data</th>
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<th>GBM</th>
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<tr>
<td>-4.50%</td>
<td>90</td>
<td>71</td>
<td>2</td>
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<tr>
<td>-7%</td>
<td>24</td>
<td>28</td>
<td>0</td>
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<tr>
<td>-9%</td>
<td>9</td>
<td>16</td>
<td>0</td>
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<tr>
<td>1987 &gt;-20%</td>
<td>1</td>
<td>0.1</td>
<td>0</td>
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The “Volatility Question” for Equity

The historical record shows a broad range of average volatility levels.

<table>
<thead>
<tr>
<th>Standard Deviation</th>
<th>Annualized Volatility</th>
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<tbody>
<tr>
<td>Jan26 - Dec36</td>
<td>0.09848375</td>
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<table>
<thead>
<tr>
<th>Data Ending April, 2009</th>
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<tbody>
<tr>
<td>Standard Deviation</td>
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<tr>
<td>---------------------</td>
</tr>
<tr>
<td>March, 1871</td>
</tr>
<tr>
<td>January, 1900</td>
</tr>
<tr>
<td>January, 1926</td>
</tr>
<tr>
<td>January, 1946</td>
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<td>January, 1960</td>
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<td>January, 2000</td>
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</table>
The “Volatility Question” for Equity

5-Year & 25-Year Trailing Annualized Volatility (Jan1900 - Apr2009)
The “Volatility Question” for Equity

The following statistics are characteristic of an SVJ model large cap calibration.

<table>
<thead>
<tr>
<th></th>
<th>All Simulated Data</th>
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<tbody>
<tr>
<td>Standard Deviation</td>
<td>0.080488</td>
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<tr>
<td>Monthly Standard Deviation</td>
<td>0.046470</td>
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<tr>
<td>Annualized Standard Deviation</td>
<td><strong>0.160976</strong></td>
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<table>
<thead>
<tr>
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<th>Volatility Averages over Paths</th>
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<tr>
<td>Min</td>
<td>0.087192</td>
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<tr>
<td>Max</td>
<td>0.236134</td>
</tr>
<tr>
<td>Average</td>
<td><strong>0.158971</strong></td>
</tr>
</tbody>
</table>
The “Volatility Question” for Equity

Relative Frequency by Path for Annualized Large Cap Return Vol

Annualized Large Cap Return Vol

Relative Frequency
The “Volatility Question” for Equity

Max 5-Year Trailing Annualized Large Cap Return Vol by Path

Relative Frequency

Annualized Large Cap Return Vol

0.115 0.125 0.135 0.145 0.155 0.165 0.175 0.185 0.195 0.205 0.215 0.225 0.235 0.245 0.255 0.265 0.275 0.285 0.295 0.305 0.315 0.325 0.335 0.345 0.355 0.365 0.375 0.385
Pathwise Behavior versus Averages

- Reminder of a common error:
  - Average mean and variance are not a measure of the diversity of events that can emerge on a path (scenario.)
  - Average behavior suppresses individual behavior.
  - A scenario is the development of one possible coherent economy over time.
  - It is pathwise behavior that matters when assessing investment programs and risk management.
Historical S&P 500 Log Level Data for the Period 1926 - 2005

- Loss of 32 years growth within 3 years (87% drop)
- To make it up, took another 25 years
80 Years Simulated S&P 500 (Log Scale)

30 years of growth

disappear in 6 years
Global Equity Modelling

Trailing 24 Month Large Cap Equity Correlations (Feb, 1873 to Feb, 2009)
Global Equity Modelling

Trailing 24 Month Large Cap Equity Correlations (Feb, 1955 to Feb, 2009)

-0.40
-0.20
0.00
0.20
0.40
0.60
0.80
1.00


DE:UK  DE:US  UK:US
Interest Rate Levels

US - Coupon Interest Rates (Sep92 to Nov08)

- 3-Month
- 2-Year
- 10-Year

Dates:
- 30-Sep-1992
- 24-Dec-1993
- 19-Mar-1995
- 1-Jun-1995
- 4-Sep-1997
- 28-Nov-1997
- 21-Feb-2000
- 16-May-2001
- 9-Aug-2002
- 2-Nov-2003
- 25-Jan-2005
- 20-Apr-2005
- 14-Jul-2007
- 6-Oct-2008
Interest Rate Volatilities

US - 24 Month Trailing Standard Deviation (Aug94 to Nov08)
Global Interest-Rate Movements

UK 10y
US 10y
DE 10y
Japan Interest Rate Volatility

Japan - 24 Month Trailing Standard Deviation (Aug94 to Nov08)

<table>
<thead>
<tr>
<th>Date</th>
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<tbody>
<tr>
<td>31-Aug-1994</td>
</tr>
<tr>
<td>24-Nov-1995</td>
</tr>
<tr>
<td>16-Feb-1997</td>
</tr>
<tr>
<td>12-May-1998</td>
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<td>5-Aug-1999</td>
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<td>28-Oct-2000</td>
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<td>21-Jan-2002</td>
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<td>16-Apr-2003</td>
</tr>
<tr>
<td>9-Jul-2004</td>
</tr>
<tr>
<td>2-Oct-2005</td>
</tr>
<tr>
<td>26-Dec-2006</td>
</tr>
<tr>
<td>20-Mar-2008</td>
</tr>
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</table>

Graph showing Japan - 24 Month Trailing Standard Deviation (Aug94 to Nov08) with 3-Month, 2-Year, and 10-Year data points.
Market Relationships

- There is danger in building in “ideas” about market behaviour that are not thoroughly vetted.
- Some have advocated the “conventional wisdom” that bond yields have something to do with bond returns. (Starting points vs. equilibrium levels matter but …)
- Let’s have a look at what history tells us.
Market Relationships: Historical 1926-2009 US
10 year Treasury Yield and 10 year OTR Return

[Graph showing historical data for 10-year yield and 10-year OTR return from 1926 to 2009]
Market Relationships: Historical 1926-2009 US
Same as Previous with a Treasury Bond Index


0.20
0.15
0.10
0.05
0.00
-0.05
-0.10

10-Year Yield
10-Year TR
Treas TR 1y+
Annual (Cal Yr) US Trsy and Equity: 1871 - 2008

US 10y Treasury TR

US S&P500 TR
Empirical Facts: Spread Overlap

- Source: Fons: Using Default Rates to Model the Term Structure of Credit Risk
- Investment grade bonds exhibit a broad range of yield spreads by class at any point in time.
- Spreads overlap.
Empirical Facts: Default Rates

- Default rates exhibit clustering behaviour.
- Sudden changes in default behaviour are important risk factors in managing corporate bond returns.
- A good ESG must pick this up.
Corporate Bond Returns

Corporate Bond Correlations

Rolling Trailing 10-Year Correlations with Corp Quarterly TR with S&P 500 Quarterly TR

03/31/99 08/12/00 12/25/01 05/09/03 09/20/04 02/02/06 06/17/07 10/29/08

AAA AA A BBB High-Yield
Inflation and expected inflation modeling is required for most insurance and pension applications. Ideally, an ESG provides both.

Some applications require measures of several specific types of inflation beyond basic consumer price inflation. This may require the modeling of several inflation sub-indices.

A link between Treasury yields and inflation must be respected.

Seasonality effects must be accounted for.
Inflation Models

Correlation (10-Year Trailing) - US YoY Inflation vs. 1-Year T-Bill Rate (Dec35 - Feb09)
Inflation Models

Scatter Plot for 1-Year T-Bill Rate vs. CPI NSA YoY Rate

CPI NSA YoY Rate

1-Year T-Bill Rate

-0.15 -0.10 -0.05 0.00 0.05 0.10 0.15 0.20 0.25

-0.15 -0.10 -0.05 0.00 0.05 0.10 0.15 0.20 0.25

-0.05 0.00 0.05 0.10 0.15 0.20 0.25

Annual (Cal Yr) US Trsy TR and CPI: 1876 - 2008
Inflation and the Real Term Structure

- Ideally, one should employ an arbitrage-free inflation model that, in addition to generating inflation, can:
  - Produce the real term structure of interest rates
  - Produce market expectations of inflation
  - Price inflation-linked bonds and derivatives
  - Coupled with an econometric model to produce inflation sub-indices
- The model should be flexible and efficient to estimate.
### AAA Variable Annuity Guidelines (Equity)

#### 1-Year

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<th>AAA Requirement</th>
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<td>0.78</td>
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<td>0.050</td>
<td>0.7954</td>
<td>0.84</td>
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<td>0.100</td>
<td>0.8728</td>
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<td>0.900</td>
<td>1.3454</td>
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<td>0.950</td>
<td>1.4107</td>
<td>1.35</td>
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<tr>
<td>0.975</td>
<td>1.4738</td>
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#### Overlapping Intervals

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<td>0.900</td>
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<td>0.975</td>
<td>1.4699</td>
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#### Non-Overlapping Intervals

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<td>0.100</td>
<td>0.8900</td>
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<tr>
<td>0.950</td>
<td>2.3081</td>
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<td>0.975</td>
<td>2.4562</td>
<td>2.72</td>
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#### 5-Year v1

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#### Overlapping Intervals

<table>
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#### Non-Overlapping Intervals v1

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#### Non-Overlapping Intervals v2
What to Do About 2008?

- How does one take the lessons of 2008 and incorporate them into an ESG?
- A necessary first step is to understand the drivers of the events of 2008.
- What suite of economic variables can be used to explain the credit crunch?
  - Credit Spreads
  - Macroeconomic Variables
  - House Price Appreciation (HPA)
Exhibit 21: Segmentation of Outstanding First-Lien Residential Mortgage Debt

Mortgage Debt $7,808BB (100%)
- Agency MBS $3,765BB (48.6%)
  - Fannie Mae
    - ARM $1,969BB (25.2%)
      - Interest Only $1,000BB (1.3%)
      - Non-IO $1,969BB (25.2%)
    - Fixed Rate $1,708BB (21.7%)
      - Interest Only $329BB (0.5%)
      - Non-IO $1,379BB (21.3%)
  - Freddie Mac
    - ARM $1,475BB (18.9%)
      - Interest Only $858BB (1.1%)
      - Non-IO $1,017BB (1.3%)
    - Fixed Rate $1,259BB (16.1%)
      - IO $10BB (0.1%)
      - Non-IO $1,249BB (16%)
- Non-Agency MBS $2,064BB (26.4%)
  - Prime Jumbo
    - Fixed Rate $517BB (6.6%)
      - Interest Only $225BB (2.9%)
      - Non-IO $200BB (2.6%)
    - ARM $293BB (3.7%)
      - Interest Only $190BB (2.4%)
      - Non-IO $200BB (2.6%)
      - Option ARM $21BB (0.3%)
  - Alt-A $722BB (9.3%)
    - Fixed Rate $261BB (3.3%)
      - Interest Only $67BB (0.9%)
      - Non-IO $194BB (2.5%)
    - ARM $461BB (5.9%)
      - Interest Only $196BB (2.5%)
      - Non-IO $46BB (0.6%)
      - Option ARM $219BB (2.8%)
- Non-Securitized $1,961BB (25%)
  - Subprime $624BB (10.6%)
    - FRM (Non-IO) $207BB (2.6%)
    - ARM $617BB (7.9%)
      - Interest Only $171BB (2.2%)
      - Non-IO $448BB (5.7%)

Assumptions: Securitization rate 75%; Loan Performance Reporting rate: Prime and Alt-A: 75%, Subprime: 65%
Source: Credit Suisse U.S. Mortgage Strategy
What to Do About 2008?

Riverside, CA

Affordable Price

Median Price

0  50,000  100,000  150,000  200,000  250,000  300,000  350,000  400,000  450,000
0  50,000  100,000  150,000  200,000  250,000  300,000  350,000  400,000  450,000
Treasury to Insured Muni Par-Coupon Spread (June 1994 to February 2009)

Date of Observation


Spread (Treasury - Muni)

5 Year Spread

10 Year Spread

20 Year Spread
What to Do About 2008?

UK 6m LIBOR Spread (Apr91 - Jun09)
What to Do About 2008?

- Unsettled. Just as uncertainty exists about the drivers of the Great Depression, so too will be the case with respect to the Great Recession.
- One is faced with the need to capture some features of systemic risk without destroying historical relationships within the ESG.
- Market collapses and terrible equity returns are associated with a variety of economic conditions within the historical record.
What to Do About 2008?

- If new systemic variables are introduced into the ESG, extreme care must be taken to avoid restricting the nature of the tail events the ESG can produce.

- A good ESG begins with a careful design of the interactions between asset classes. For example, a good ESG will capture the correlations between equity returns and corporate bond returns.
If appropriate systemic risk factors are in place and the model was structured correctly to begin with then changes to the systemic risk structure can be made while maintaining model consistency.
What to Do About 2008?

- There is considerable research ongoing that holds promise in capturing the credit aspects of recent events.
- The work of Errais, Giesecke & Goldberg - Affine Point Processes and Portfolio Credit Risk is one example.
- The following picture gives the flavour of the model.
What to Do About 2008?

\[
\lambda_t = u(t) + \int_0^t h(t-s) \, dL_s
\]

\[
h(v) = \delta e^{-\kappa v}, \quad v \geq 0,
\]
Conclusions

- A good ESG must have a wide range of specific attributes which are essential in obtaining reliable economic scenarios.
- Must provide scenarios that are consistent with the features of real market data. That is, the simulated artificial economy must look and feel just like the real economy both qualitatively and quantitatively.
- Must be computationally efficient and numerically stable.
Conclusions

- Must have a comprehensive estimation and validation support system.
- Must produce extreme but plausible scenarios that encapsulate historical behaviour.
- It is highly desirable, if not essential, to have the same suite of core models running for ALM and other real world applications that are running for embedded value and other pricing applications in the risk-neutral world.