MATHEMATICAL MODELS
AND
THE CREDIT CRUNCH

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Plan

- Philosophy
- Background (brief!)
- Questions:
  - assumptions and limitations
  - model types
  - concealment of complexity and model risk
- What to do in the future
Background to this presentation

Acknowledgements to

• Numerous friends & colleagues

• Osmosis (i.e. accumulation of uncorroborated evidence over many months)

• Several speakers this week
Credit Crunch

- Turner Review (UK regulator):
  - [apparent] misplaced reliance on sophisticated maths
  - complexity ⇒ difficult for top management and boards to assess and exercise judgement over risks being taken
  - complexity of market not matched by improvements in modelling
  - VaR partly to blame
Financial mathematicians must take some blame

Different individuals: some or all of

- Allowing models to be used inappropriately
- Not carrying out due diligence
- Not warning senior management about risks
- Allowing bonus culture to over-rule common sense

→ operational risks
Question 1: assumptions and limitations

Did users of models understand assumptions and limitations of models?

- Hypothesis:

  *nothing wrong with the underlying maths*

  BUT require full specification + testing

- some models are better than others

- models must be fully scrutinised and tested

- underlying assumptions and limitations must be communicated upwards
Assumptions

• Is a specific assumption: (A) true, (B) approximately correct, (C) laughably wrong?

• What will happen if the assumption is incorrect?

• What can be done to mitigate incorrect assumptions?

• e.g. Black-Scholes model + delta hedging
  – Gamma hedging: rebalancing at discrete times, jumps in prices
  – Vega hedging: volatility changes from time to time
Limitations

- Model designed for a specific contract then applied to other contracts
- What about less complex contracts?
- What about more complex contracts?
- Model $\Rightarrow$ price $+$ risk management strategy
- Model might fail if market gets too big
Question 2: pricing versus risk-management models

Did users understand the difference between

- pricing models
- risk-management models
- risk-measurement models?
Pricing models

- Also known as *market models*
- e.g. Black-Scholes model
- Model a subset of all risks
- No-arbitrage assumption + dynamic hedging
- Risk-neutral pricing measure
- Simple enough to allow quick calculation of prices
- Calibration of parameters using today’s market prices
Pricing models

Pros:

- Model is *consistent* with what we observe *today* in the market
- Avoids mispricing of very similar contracts

Cons:

- Model might not be consistent with historical dynamics and data
- Approach to calibration might not be consistent with model assumptions
- e.g. recalibration of $\sigma$ in B-S model
Pricing models

Dangers:

- avoids mispricing of very similar contracts BUT
- extension of pricing to new, less similar contracts creates a market based on the assumed truth of the model
- e.g. (???) Gaussian copula model + credit market
- Reality: embryo market:
  
  pricing models A and B both consistent with limited data
  
  BUT A and B ⇒ different prices in expanded market
Risk MANAGEMENT models

- Also known as *real-world models*
- Wider range of risks
- *Calibrated to historical data*
- Regular recalibration
- Rigorous statistical testing; *model + parameter risk*
- Economic reasonableness
- Rational economic dynamics

⇒ okay for risk control and optimisation
Risk management models

Pros:

• Consistent with the past
• Realistic
• Proper assessment of risk

Cons:

• Difficult to calibrate in real time
• Difficult to price derivatives
• Theoretical prices not exactly equal to market prices
Risk MEASUREMENT models

- Real-world models
- Incorporate market irrationality; inefficiency
  - information asymmetry
  - negative risk premiums
  - pro/counter cyclical dynamics
  - behavioural finance
    - e.g. overconfidence; understatement of risks
- DO NOT attempt to optimise! (⇒ excessive leverage)
- Okay for: robustness of strategy ⇒ ??? risk mitigation
Question 3: complexity and model risk

Do quants and/or traders have the incentive to

- conceal the extent of contract and model complexity from investors?
- downplay model risk?

↔ Creating complexity in order to profit from the ignorance of others
A scenario – the illusion of understanding

• Marketing team: good idea for a new product

• Investors will only buy when
  – they think they understand the risk profile
  – they believe the product will help hedge risk or make money (alpha)

• Quants enlisted to help “educate” investors
A scenario – the illusion of understanding

- Quants enlisted to help “educate” investors

- One model, one calibration:
  Result: “enlightenment” and SALES

- Many models + parameter uncertainty
  Result: confusion and NO sales and NO bonuses
  (even when the product is good for risk reduction)
Variant – The Marketing team

- incentive to conceal complexity and model & parameter risk from senior managers and directors

Regulators: need to enforce Prudent Person Principle

- Would you sell this product to your grandmother?
- Do you understand the risks fully?
- Does your customer understand the risks fully?
What do we need to be doing in the future?

- Improved stochastic modelling
- Better availability and use of historical data
- A stronger voice for quants
- Alternatives to short-horizon quantile risk measures
- Stronger dialogue between academics, regulators + banks
- ...

...
Future Model Types

• Solvency II ⇒
  Need combined Pricing + risk management models

• Why?
  S-II ⇒ need market-consistent values in 1 year

BUT: is S-II too focused on short-term balance-sheet volatility?
Combined pricing + RM models

Requirements:

- Realistic, multi-factor
- Process parameters \((\mu, \sigma, \rho, \ldots)\) calibrated using historical data
- State variables \((S(t), r(t), \sigma(t), \ldots)\) calibrated using market prices
- Dynamics of state variables consistent with model assumptions (c.f. pricing models)
Regime shifts

Unsecured versus collateralised short-term loans

1-month LIBOR minus 1-month REPO

Spread (%)
Improved risk-management/measurement models

- Liquidity; buying/selling spreads; asymmetric info.
- Extreme regime shifts
  - Liquidity, volatility, (perceived) information asymmetry, ...?
- Other latent variables
- Large-scale, destabilising feedback, hysteresis
- Fat tails, stochastic volatility
- Market irrationality, behavioural finance etc.
FTSE-100, 13-17 October 2008; log returns

+7.94%, +3.17%, −7.43%, −5.50%, +5.09%

- A: i.i.d. normal model
- B: non-central-t distribution + stochastic volatility

\(p\)-values (should be i.i.d. \(\sim U[0, 1]\)):

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Improved modelling: Augmented by

- thorough analysis of model and parameter risk

  \[\Rightarrow\text{discourages excessive leverage!}\]

- scenario analysis, stress tests and black swans
A stronger voice for quants

- Quants; risk management team; external experts
- Walker Review (UK):
  Senior management and non-executive directors
  (NEDs*) ⇐ external advice

(*) NED: Scots slang: Non-Educated Delinquent
A stronger voice for quants

• a greater number of NEDs should have a strong knowledge of QRM & ERM:
  
  *enough to be able to ask the right questions*

• Regulator $\Rightarrow$ “fit and proper” test

• Risk Committee NEDs should have access to middle and junior staff
The role of Value-at-Risk

- In theory: VaR $\Rightarrow$ quantile
- In practice ???
  
  “VaR” $\Rightarrow$ quantile + i.i.d. multivariate normality
- *VaR is not a coherent risk measure*
  
  - “non-coherence” was not a cause of the crisis
  - Expected-shortfall $+$ stochastic volatility $+$ fat tails $\Rightarrow$ bigger crisis ??
  - BUT optimise VaR $\Rightarrow$ small probability, high-severity risks
Improving on traditional Value-at-Risk

- Use better models!
- How to avoid pro-cyclicality?
  - ????
  - Take the long term view
    e.g. run-off of life insurance liabilities
    (⇒ greater emphasis on cashflow matching)
- Does Solvency II go far enough?
Time to stop rambling ...
Securitisation

Reasons:

- Convert illiquid into liquid assets
- Capitalise future illiquid cashflows
- Trading new risks $\Rightarrow E_{subj}[utility] \nearrow$ for all

Issues:

- Complex repackaging of existing liquid traded risks $\Rightarrow$ DANGER
  ???
- Securitisation of OWN risks $\Rightarrow$ moral hazard
- Insurable interest
- Risk-reduction or gambling?
Better availability of data

- Longer runs of data
- Buying and selling prices (liquidity)
- Available for free (with a time lag) for non-commercial research