Market Risk Prediction under Long Memory: When VaR is Higher than Expected

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19th International AFIR Colloquium 2009
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Motivation

Several periods of financial market stress:
- the market crash in October 1987,
- a number of accounting scandals at the beginning of the new millennium and
- the recent banking crises

have increased the regulatory and industry demand for effective (market) risk management approaches.

Despite the BIS demands no concrete method, one concept become popular: **Value-at-Risk (VaR)**.
Motivation

GARCH models generate satisfactory volatility forecasts for the very next period.

**Long-term** VaR measures usually require volatility predictions for longer periods:
- several weeks or even
- several months.

Despite their high practical relevance most focus has been placed on **one-day ahead** forecasts.
Motivation

Contribution of the article

- New insights into
  - risk prediction under long memory and
  - issues concerning backtesting for long-term risk measures.

- New scaling based GARCH-LM model for multi-period risk prediction.
In finance scaling is very important, since Basel rules of capital adequacy require banks to calculate VaR numbers for a minimum holding period of at least 10 days.

Square-root-of-time rule:

\[ \text{VaR}(1) \sqrt{\tau} = \text{VaR}(\tau). \]
**Scaling**

**Premises of Square-Root-of-Time Rule**
- independent and
- identically distributed (i.i.d.) returns process

**Problem**
Financial time series are not independent, because e.g. absolute or squared returns are highly correlated.

**Consequences**
- In the presence of long memory, it is not reasonable to scale by a fixed self-affinity parameter \((H = 0.5)\).
- The degree of risk misspecification depends both on the risk horizon and the magnitude of long range dependence.
GARCH-type VaR models are based on the assumption that empirical returns belong to a location-scale family of probability distributions of the form

\[ R_t = \mu_t + \epsilon_t = \mu_t + Z_t \sigma_t. \]

The location \( \mu_t \) and the scale \( \sigma_t \) are \( \mathcal{F}_{t-1} \)-measurable parameters and \( Z_t \sim i.i.d. F(0, 1) \).
The one-day ahead GARCH VaR is obtained by

$$\text{VaR}^\alpha_{t,t+1} = \mu_{t+1} + \sigma_{t+1} F_{\alpha}^{-1},$$

where $\sigma_{t+1}$ is the conditional standard deviation of $R_t$ calculated by GARCH(1,1):

$$\sigma^2_{t+1} = \omega + \alpha \epsilon^2_t + \beta \sigma^2_t,$$

with $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta < 1$. When $\tau \to \infty$, the process $\sigma^2_t$ is finite if and only if $\alpha + \beta < 1$, otherwise the process is non-stationary as $\sigma^2_t \to \infty$. 
The multi-day ahead GARCH variance prediction is obtained by

\[ \sigma^2_{t+\tau} = \mathbb{E}(\epsilon^2_{t+\tau} | F_t) = \sigma^2 + (\sigma^2_{t+1} - \sigma^2)(\alpha + \beta)^{\tau-1} \]

\( \sigma^2 \) denotes the unconditional variance of \( \epsilon_t \).

**Drawbacks**

- If the forecasting horizon \( \tau \) rises and \( \alpha + \beta < 1 \) then \( \sigma^2_t \rightarrow \sigma^2 \).
- All relative weights on past squared returns decline at the same exponential rate \( (\alpha + \beta) \).
The **multi-day ahead** VaR prediction in the novel setting is given by

\[ \text{VaR}_{t,t+\tau}^\alpha = \mu_{t+\tau} + \phi(t + \tau) F^{-1}_{\alpha}. \]

In contrast to GARCH-based VaR forecasts, we substitute \( \sigma_{t+\tau} \) by a scaling based variable \( \phi(t + \tau) \):

\[ \phi(t + \tau) = \tau^H \rho_{|R_t|}(\tau)^{H-\rho_{|R_t|}(\tau)} \sigma_{t+1}. \]

- \( H \) corresponds to the Hurst exponent or self-affinity parameter.
- \( \rho_{|R_t|}(\tau) \) is the autocorrelation coefficient of \( |R_t| \) for the time-lag \( \tau \). Assumption: \( \rho_{|R_t|}(\tau) \neq 0 \).
Special Backtesting Issues

1. Which returns should be used?
   - **Overlapping returns**
     - **Con** Autocorrelation
     - **Pro** Backtesting criteria like Basel traffic light could be achieved easier as in case of non-overlapping returns.
   - **Non-overlapping returns**
     - **Pro** No autocorrelation

2. **Multi-day** VaR figures exhibit an additional backtesting problem.
   - Due to higher risk horizon $\tau$, the spread of $R_{t,t+\tau}$ increases $\Rightarrow$ the distance between $VaR_{t,t+\tau}$ and $R_{t,t+\tau}$ becomes more important in comparison to one-day ahead VaR.
The data contains 8,609 daily closing levels $P_t$ from January 1, 1975 to December 31, 2007 of four international stock market indices:

- DAX
- Dow Jones
- Nasdaq Composite
- S&P 500

We use non-overlapping continuously compounded percentage returns $R_{t,t+\tau}$ for different sampling frequencies $\tau \in \{5, 10, 20, 60\}$ days.
In order to investigate the dependence structure of empirical returns, we calculate estimates of $H$ for all indices, Brownian motion and test the null

- $H_0$: "$H = 0.5$" (no dependence) against
- $H_1$: "$H \neq 0.5$" (dependence).
**Long Range Dependence**

| Index         | $R_t$    | t-value | $R_t^2$   | t-value | $|R_t|$   | t-value |
|---------------|----------|---------|-----------|---------|----------|---------|
| DAX           | 0.520    | 1.91*   | 0.769     | 26.12   | 0.823    | 24.01   |
| DOW JONES     | 0.476    | -1.94*  | 0.614     | 8.32    | 0.769    | 19.99   |
| NASDAQ        | 0.535    | 2.83    | 0.811     | 17.54   | 0.846    | 16.34   |
| S&P 500       | 0.478    | -1.95*  | 0.632     | 9.87    | 0.780    | 18.72   |
| BM(8608)      | 0.490    | -0.76*  | 0.487     | -0.44*  | 0.495    | -0.23*  |

**Table:** Empirical estimates of the Hurst exponent $H$ for daily index data from January 1, 1975 to December 31, 2007. A theoretical estimate for simulated ordinary Brownian motion with 8,608 increments is provided for 10,000 replications. * denotes accepting the null at the 95% confidence level.
**Empirical Analysis**

**Long Range Dependence**

**Long Range Dependence**

**Figure:** ACFs of absolute index returns. Sample period: January 1, 1975 to December 31, 2007.
### VaR Forecasting Performance Results

**DAX**

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<th>Horizon</th>
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<th>LM</th>
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**Table:** 60-day ahead VaR forecasts for the GARCH and GARCH-LM model with skewed $t$-distribution from January 1, 1991 to December 31, 2007. * denotes rejecting the null at the 99% confidence level.
Empirical Analysis

VAR Forecasting Performance Results

Skewed student-$t$ distribution

(a) 5 days
(b) 10 days
(c) 20 days
(d) 60 days