How good are Portfolio Insurance Strategies?

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Introduction and Motivation

- Increasing demand for insurance contracts which also serve as savings towards retirement
- Trade off between security of the retirement savings and participation in the market
- Solution provided to the insured:
  - Payoff of insurance linked to *underlying investment strategy*
  - *guaranteed interest rate*
- Product design: basically *structured insurance products* and CPPI based products
Motivation

Implications for risk management

- Risk management crucially depends on the underlying investment strategy

Perspective of insured

- Does the insured profit from products with capital guarantee?

⇒ When and why are CPPI (OBPI) strategies better than OBPI (CPPI) strategies?

⇒ Mitigate between expected utility maximization and the comparison of stylized strategies
Outline of the talk

- Review of the (well known) optimization problems yielding
  - constant mix,
  - CPPI
  - and OBPI strategies

- Comparison of the optimal strategies and resulting payoffs
- Discussion of some advantages (disadvantages) of the portfolio insurance methods
- Illustration of utility losses caused by the introduction of strictly positive terminal guarantees for a CRRA investor
  - effects of market frictions such as discrete–time trading, transaction costs and borrowing constraints
Model Setup

- Assumptions

\[
\begin{align*}
  d B_t &= B_t r \, dt, \quad B_0 = b \\
  d S_t &= S_t (\mu \, dt + \sigma \, dW_t), \quad S_0 = s
\end{align*}
\]

- \( W = (W_t)_{0 \leq t \leq T} \) standard Brownian Motion
- \( \mu, \sigma \) and \( r \) constant \((\mu > r \geq 0, \sigma > 0)\)

- Value Process \( V = (V_t)_{0 \leq t \leq T} \) of investment strategy \( \pi \)

\[
  dV_t(\pi) = V_t \left( \pi_t \frac{dS_t}{S_t} + (1 - \pi_t) \frac{dB_t}{B_t} \right), \quad V_0 = A
\]
### Benchmark Optimization Problems

#### Optimization problems

<table>
<thead>
<tr>
<th>Problem</th>
<th>Utility Function</th>
<th>Additional Constraint</th>
<th>Optimal Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>$u_A(V_T) = \frac{V_T^{1-\gamma}}{1-\gamma}$</td>
<td>none</td>
<td>CM</td>
</tr>
<tr>
<td></td>
<td>unconstrained CRRA problem</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(B)</td>
<td>$u_B(V_T) = \frac{(V_T - G_T)^{1-\gamma}}{1-\gamma}$</td>
<td>none</td>
<td>CPPI</td>
</tr>
<tr>
<td></td>
<td>subsistence level $G_T$ (HARA)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(C)</td>
<td>$u_A(V_T) = \frac{V_T^{1-\gamma}}{1-\gamma}$</td>
<td>$V_T \geq G_T$</td>
<td>OBPI</td>
</tr>
<tr>
<td></td>
<td>constrained CRRA problem</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Optimal Payoffs

\begin{align*}
V_{T,A}^* &= \phi (V_0, m^*) S_T^{m^*} \\
V_{T,B}^* &= G_T + \alpha_B V_{T,A}^* \\
V_{T,C}^* &= \alpha_C V_{T,A}^* + \left[ G_T - \alpha_C V_{T,A}^* \right]^+ \\

m^* &= \frac{\mu - r}{\gamma \sigma^2} \quad (\text{Merton investment quote}) \\
\text{Fractions } \alpha_B \text{ and } \alpha_C \text{ are} \\
\alpha_B &= \frac{V_0 - e^{-rT} G_T}{V_0} < \alpha_C = \frac{\tilde{V}_0}{V_0} < 1 \\
\text{Relation is also true w.r.t. more general model setups!}
\end{align*}
Motivation
Optimization Problems
Utility loss caused by guarantees
Discrete Trading and Transaction Costs

Model Setup
Benchmark Optimization Problems
Comparison of optimal payoffs
Illustration Optimal Payoffs

Link between payoffs

- $V^*_T, A$ corresponds to $\phi(V_0, m^*)$ power claims with power $m^*$ where the number $\phi(V_0, m^*)$ depends on
  - the initial investment
  - and the optimal investment weight $m^*$

- **Subsistence level** in (B) and terminal constraint in (C) imply
  - reduction of the number of power claims (to afford the risk–free investment which is necessary to honor the guarantee)
Link between strategies

- **CPPI strategy** is a
  - buy and hold strategy of a constant mix strategy
  - with an additional investment into $G_T$ zero bonds

- **Solution of (C) (OBPI) is a**
  - buy and hold strategy of a constant mix strategy
  - with an additional investment into a put with strike $G_T$

→ Put is cheaper than $G_T$ zero bonds such that one can buy and hold more CM strategies in the case of the option based approach
## Parameter constellation

### Basic parameter constellation

<table>
<thead>
<tr>
<th>model parameter</th>
<th>strategy parameter</th>
<th>terminal guarantee</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_0 = 1$</td>
<td>$V_0 = 1$</td>
<td>$G_T = 1$</td>
</tr>
<tr>
<td>$\sigma = 0.15$</td>
<td>$T = 10$</td>
<td></td>
</tr>
<tr>
<td>$r = 0.03$</td>
<td>$\gamma = 1.2$</td>
<td></td>
</tr>
<tr>
<td>$\mu = 0.085$</td>
<td>$m = m^* = 2.037$</td>
<td></td>
</tr>
</tbody>
</table>

**Table:** Basic parameter constellation.
Optimal payoffs $V^{*}_{T,A}$ (solid line), $V^{*}_{T,B}$ (dotted line) and $V^{*}_{T,C}$ (dashed line)
Remarks

- Intersection points with unconstrained solution
- Probability to end up with (only) the guarantee

  - OBPI payoff is equal to guarantee if the put expires out of the money
  - In contrast to the CPPI method, this implies a positive point mass for the event that the terminal value is equal to the guarantee
    → This can cause a high exposure to gap risk, i.e. the risk that the guarantee is violated, if market frictions are introduced.

- Loss from introducing the guarantee into the unconstrained setup
Loss rate

- Loss rate $l_{T,i}(\pi)$ of the strategy $\pi$ and the utility function $i$ ($i \in \{A, B, C\}$)

\[
l_{T,i}(\pi) := \frac{\ln \left( \frac{CE_{T,i}^*}{CE_{T,i}(\pi)} \right)}{T}
\]

where
- $CE_{T,i}^*$ denotes the certainty equivalent of the optimal strategy $\pi^*_i = (\pi_{t,i})_{0 \leq t \leq T}$
- $CE_{T,i}(\pi)$ the of the suboptimal strategy $\pi = (\pi_t)_{0 \leq t \leq T}$
Loss rates w.r.t. $u = u_A$ for CPPI (solid lines), OBPI (dashed) and CM (dotted) strategies with varying parameter $m$. 
## Motivation

**Optimization Problems**

Utility loss caused by guarantees
Discrete Trading and Transaction Costs

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**Utility loss caused by guarantees**

- Loss rate

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**Illustration**

**Minimal loss rates** ($u_A$–optimal strategy parameter $m$)

<table>
<thead>
<tr>
<th>strategy</th>
<th>$\gamma \setminus T$</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPPI</td>
<td>1.2</td>
<td>0.040 (11.32)</td>
<td>0.035 (7.83)</td>
<td>0.026 (4.91)</td>
<td>0.018 (3.57)</td>
<td>0.010 (2.73)</td>
</tr>
<tr>
<td>OBPI</td>
<td>1.2</td>
<td>0.037 (2.04)</td>
<td>0.031 (2.04)</td>
<td>0.022 (2.04)</td>
<td>0.014 (2.04)</td>
<td>0.007 (2.04)</td>
</tr>
<tr>
<td>CPPI</td>
<td>1.5</td>
<td>0.031 (10.60)</td>
<td>0.026 (7.25)</td>
<td>0.019 (4.45)</td>
<td>0.013 (3.16)</td>
<td>0.007 (2.36)</td>
</tr>
<tr>
<td>OBPI</td>
<td>1.5</td>
<td>0.028 (1.63)</td>
<td>0.023 (1.63)</td>
<td>0.015 (1.63)</td>
<td>0.009 (1.63)</td>
<td>0.005 (1.63)</td>
</tr>
<tr>
<td>CPPI</td>
<td>1.8</td>
<td>0.024 (10.03)</td>
<td>0.020 (6.80)</td>
<td>0.014 (4.10)</td>
<td>0.009 (2.86)</td>
<td>0.005 (2.08)</td>
</tr>
<tr>
<td>OBPI</td>
<td>1.8</td>
<td>0.021 (1.34)</td>
<td>0.017 (1.34)</td>
<td>0.011 (1.34)</td>
<td>0.007 (1.34)</td>
<td>0.004 (1.34)</td>
</tr>
</tbody>
</table>

**Table:** Minimal loss rates ($u_A$–optimal strategy parameter $m$) for varying $T$ and $\gamma$ where the other parameters are given as in Table 1.

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How good are Portfolio Insurance Strategies? 15/20
• Concept of portfolio insurance is impeded by market frictions
• Asset exposure is reduced when the asset price decreases
• A sudden drop in the asset price where the investor is not able to adjust his portfolio adequately, causes a gap risk, i.e. the risk that the terminal guarantee is not achieved.
• Example: trading restrictions in the sense of discrete–time trading and transaction costs

Utility Loss
Gap risk measured by the shortfall probability
Loss rates: continuous–time CPPI (solid line), monthly CPPI without transaction costs (dashed lines) and monthly CPPI with $\theta = 0.01$ (dotted line)
Loss rates: continuous–time CPPI (solid line), monthly CPPI without transaction costs (dashed lines) and monthly CPPI with $\theta = 0.01$ (dotted line)
Distribution of discrete OBPI (CPPI) with transaction costs
Distribution of discrete OBPI (CPPI) with transaction costs
Conclusion

- Main difference between OBPI and CPPI can be explained by their link to constant mix strategies.
- It is also important to take into account the difference between kinked and smooth payoff–profiles.
- **Advantage of OBPI:**
  - Backing up the guarantee is cheaper than for CPPI (closer to unconstrained optimal).
- **Drawback of OBPI:**
  - Implementation is much more difficult than the one of CPPI.
  - Resulting strategies are sensitive against model risk and various sources of market incompleteness.
Thank you for your attention!