Cautionary remarks about conclusions from the observation of record-life expectancy

IAA Life Colloquium 2009

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Cautionary remarks about conclusions from the observation of record-life expectancy

- Introduction
  - Simulation results
  - Record-life expectancy as forecast-tool
  - Conclusions
Introduction & some definitions

Definitions used in the talk

- Life expectancy (LE) is the expected number of further years of life.
- We study the time-development of specific LEs of nations or countries.
- The record LE in a year is the maximal specific LE of all countries in scope.
- LEs are seen as stochastic processes or time-series.
- We study the slopes of the regression line of specific and record LE processes.

Note: The LE definition of Oeppen&Vaupel is period LE at birth.

What to expect from this presentation

■ Questions raised and (partially) answered
  - What can record LE tell us about LEs specific for a country?
  - What are drivers of record LE beyond specific LE improvements?
  - What issues need to be addressed before record LE can be used to forecast specific LEs?

■ Tenor:
  - Record LE is a biased estimator of specific LEs
  - Inference on specific LEs needs to control all parameters driving the bias
  - Some of those parameters are only marginally relevant to LE development
  - To use record LE in forecasting you need a detailed model of the dependency-structure of global LE improvements
The basic issue – in a nutshell

- **Country A**: current LE is 58 years
  - LE has equal chance of moving up or down 3 month next year
  - Expected improvement is zero or no systematic trend

- **Country B**: same probabilities as A and independent

- **Record LE of A and B?**
  - Probability of moving up: 3/4
  - Probability of moving down: 1/4

- **Record LE is “biased” i.e. it overstates the expected LEs of “A” and “B”**
Drivers of bias

- The simple example is already sufficient to demonstrate some key effects and drivers
  - If three countries participate the probability of “up” is already 7/8
- Number of countries: Increases bias
  - Can be demonstrated in the 2-stage tree: If outcome of B does depend on outcome of A
- Simple correlation: Decreases bias
  - Needs multi-step consideration but is still straightforward
- Initial difference of LEs: Larger initial difference lowers the bias but it is still there
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The simulation laboratory

- Our goal is to demonstrate potential issues with record LE. So we strive for a simple model which is rich enough to show the effects.

- LE simulations are based on mortality rates from a Lee-Carter model\(^1\)
  - Force of mortality: \( \mu(x,t) = \alpha_x + \beta_x \cdot \kappa_t \)
  - \( \alpha_x \) and \( \beta_x \) from England&Wales population data, males, 1978-2007
  - Stochastic time/series \( \kappa_t \), modelled as random walk with drift:
    \[ \kappa_t - \kappa_{t-1} = d + \varepsilon_t \quad \text{with} \quad \varepsilon_t \text{ iid, } \varepsilon_t \sim N(0, \sigma) \]

- Effects can be studied by systematic variation of parameters governing the stochastic properties
  - Drift, volatility, number of time series/countries, correlation-structure

- Reasonable range of variation is fixed by comparison to historic mortality data

- For practical reasons: LEs start with age 20 and are curtailed at 90.

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1) see Lee-Carter (1992) or Cairns et.al. (2007)
Simulated LEs - first example

- Generate stochastic mortality rates
- Choose a number of "countries" (i.e. Time series)
- Calculate for each time \( t \) specific LEs and their record LE

- 5 time series or "countries"
- 50 periods simulated
- Drift is zero
- Random walks are uncorrelated
- Record LE outlined in black
- Different time series make up the record LE line
The record LE regression line

Regress record LE over time to find the slope of record LE

- Time series same as before
- But regression lines are included
- Sampled is one concrete outcome
- Different samples will have different regression lines
Improvements from nowhere

- Repeat sufficiently often to find the **distribution** of slopes of record LEs
- With sufficient countries record LE will show positive slope
- So the record LE shows improvement although all specific LEs have no drift

<table>
<thead>
<tr>
<th>number of countries</th>
<th>Mean slope</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<tr>
<td>5</td>
<td>2.9%</td>
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<tr>
<td>10</td>
<td>3.8%</td>
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<tr>
<td>25</td>
<td>4.9%</td>
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<tr>
<td>50</td>
<td>5.5%</td>
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</table>

Nsim=5000
No rho, no drift, vola standard
start = +.3%
Main_sim.m
Potential amount of bias in record LE

More realistic scenarios varying underlying drift and volatility
There is indeed a possibility of material bias!

- Comparison with data from the human mortality database
  1) see www.mortality.org

- Assumptions on drift and volatility chosen to be reasonable in comparison to 1950-2003 data
  - Historic drift between -0.5 and -1.5 (E&W: -1.1)
  - Standard deviation varies between 90% to 105% of E&W values
  - Initial dispersion of starting positions +/- 3% of average LE 2003: 57 years

<table>
<thead>
<tr>
<th>kappa drift</th>
<th>kappa volatility (of E&amp;W)</th>
<th>specific LE slope</th>
<th>record LE slope</th>
<th>over-statement due to bias</th>
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<tr>
<td>0</td>
<td>90%</td>
<td>0%</td>
<td>5%</td>
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<td>105%</td>
<td>0%</td>
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<tr>
<td>-1.5</td>
<td>105%</td>
<td>20%</td>
<td>20%</td>
<td>4%</td>
</tr>
</tbody>
</table>

nsim = 5000, nperiod = 50
rho = 3%, eps_init = +/- 3%  
bias_estimate.m
How much bias in reality?

- The analysis presented is obviously not conclusive. So we do not know the true current or historic bias in record LE
- Some obvious missing points are:
  - Young and old ages are excluded
  - Only England & Wales data was used for Lee-Carter parameters
  - Static analysis i.e. parameters are fixed in advance
  - Extremely simple dependency: Multivariate normal random walk
- What is the role of the Lee-Carter model?
  - We do not claim that Lee-Carter is a particular good model for this
  - The claim is indeed: It doesn’t matter which model you use
  - Lee-Carter is just a convenient way to generate stochastic mortality rates
- Remember the nutshell example: Volatility => Bias of record LE
Further examples and conclusion

- To decide on bias ALL parameters influencing the joint distribution need to be measured or at least their materiality estimated.

- Example: Dynamic changes
  - Number of countries in scope: probably growing over time?
  - Changes in drift: catch-up to leading countries will lead to clustering, i.e. increased bias due to less differences between leaders
  - More complicated dependency, auto-regression of drift and error, changing volatility ....

- As long as there is no evidence to the contrary it is safer to assume an unknown but potentially material bias in the slope of record LE.
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Record LE as forecasting-tool

- Record LE could be used to forecast specific LEs
- General idea
  - 1) Express any specific LE as a function of the record LE
  - 2) Forecast the record LE
  - 1) and 2) will give you immediately a forecast of your specific LE
- Example: specific LE = record LE - gap (Andreev, Vaupel 2006)

Problem:
- Bias in record LE is transferred to specific LE
- Consistency is not ensured: If all countries had the same slope as record LE the record LE were different!
Are other approaches viable?

- More complex approaches have been suggested
  - Lee (2006): Gap is not constant but decreases linearly over time
  - Torri (2008): Gap follows a stochastic process

- But any approach based on record LE is a potential victim of transfer of bias and lack of internal consistency

- Part of any proposed model should be a discussion of the consistency of assumptions between specific LE and measured/forecasted record LE
  - This most likely requires a description/analysis of the full dependency structure of all specific LEs involved

- Taking this complexity into account will limit the appeal of simplicity of the record LE approach.
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Conclusions

- Slope of record LE is a biased estimator of slope of any specific LE.
  - This is a mathematical property of the maximum of random variables and is not particular to human longevity or its temporal development.
  - Strength of bias is influenced by a potentially wide range of parameters.
  - Some of those parameters might only be marginally related to human longevity like e.g. the number of countries in scope.

- When forecasting based on the slope of record LE,
  - Care should be taken to prevent transfer of any bias from the record LE to the specific LE to be forecasted.
  - Assumptions on the relationship between the forecasted specific LEs and their ensuing record LE should be checked for consistency.

- Record LE is not a simple measure and analysing/controlling all influence factors will probably reduce its appeal of simplicity.
Contact details

- You are welcome to send any questions, remarks and your opinion on the matter to

  guidogruetzner@secquaero.com

- You might also want to check out my company’s website (our main focus is somewhat different though)

  http://www.secquaero.com
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Some sources and further reading

- Oeppen&Vaupel


  - Forecasts of Cohort Mortality after Age 50. Working paper, Max Planck Institute for Demographic Research, Rostock, Germany.


- Torri, Tiziana (2008)
Development of some selected LEs

Source of data: Human Mortality Database (www.mortality.org) and own calculations
### Parameters

#### Parameters to vary and their implementation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Implementation</th>
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<tr>
<td>Number of countries</td>
<td>Number of random mortality processes/LEs simulated</td>
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<tr>
<td>Initial dispersion of LE</td>
<td>Factor applied to the LE process</td>
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<tr>
<td>Drift/Slope of LE</td>
<td>Parameter of $\kappa_t$ distribution</td>
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<td>Volatility of LE</td>
<td>Parameter of $\kappa_t$ distribution</td>
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<td>Correlation of LE</td>
<td>Parameter of $\kappa_t$ distribution</td>
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Sample data used for comparison

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<tr>
<td>w/o negative</td>
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</table>

Source of data:
- Mortality tables: Human Mortality Database (www.mortality.org)
- Derived values: own calculations
Lee Carter parameters

- Based on ONS Data
- England and Wales male population
- Calibration on years 1978 - 2007

Drift of kappa: -1.1
Standard deviation of $\epsilon$: 1.4