Variable Annuities –
issues relating to dynamic hedging strategies
Who we are

The Mazars group is one of the world’s leading audit firms. Our presence spans five continents and 50 countries, where we have more than 10,500 staff working in 200 offices.

Revenues and staff levels have grown in a strong and sustained manner, enabling the Mazars group to confirm its position as a key player in the audit market and internationally.
Mazars Actuarial Services

Our actuarial teams at Mazars are made up of technical experts who specialize in:

- Financial engineering,
- Insurance actuarial work,
- Compensation and employee benefits,
- Statistics and modeling.

Our team will help you:

- Define and implement your growth strategy,
- Develop your approach to assets/liability management
- Manage all your financial risks,
- Manage and implement accounting regulations
- Evaluate your technical insurance activities,
- Identify and evaluate any company debts/liabilities,
## Variable Annuities

Nature and amount of given guarantee determine hedging strategy

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<th>GMDB</th>
<th>GMAB</th>
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<tr>
<td>Return of premium</td>
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<td>Premium roll-up at  x%</td>
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<td>Anniversary lock-in (ratchet)</td>
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<td>Combination of roll-up and ratchet</td>
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*Diagram: A large oval labeled 'hedge' with a smaller oval connected to it.*
Static and Dynamic hedge

**Static**

Hedge Portfolio is built at the beginning of the contract

- **Plus:**
  - easy to implement

- **Minus:**
  - only suitable for simple products,
  - Deviations from initial assumptions (e.g. mortality, lapse) will lead to a mismatch between liabilities and hedge portfolio
Static and Dynamic hedge

**Dynamic**

Hedge Portfolio is adapted to the risk profile of the liabilities over time

- **Plus:**
  - Very flexible
  - Allows some corrections of initial assumptions

- **Minus:**
  - Increased operational risk
  - Challenge to internal models for capital requirements
Dynamic Hedging: dealing with the operational risk

- Documentation of the hedging strategy needed:
  - Description of the hedged risks
  - Used financial instruments
  - Parameters of the strategy (outstanding, strikes,…)

- Governance: endorsement of the strategy and regular monitoring of the implementation

- Sufficient technical and human resources

- Internal model has to be capable to quantify the effect of the strategy
  - In particular model has to include the parameters that have an impact on the hedge instruments and strategy (e.g. implied volatility)
Risks related to Dynamic Hedging

- Risk inherent to dynamic hedging are complex and varied
- In addition to the risks listed (e.g. in Ledlie et al., see on the right), one should consider **liquidity risk**, usually not included in market risk, yet it is a real risk
Liquidity Risk

- Treasurer: being short of cash
- Portfolio Manager: risk for a market to be inactive
- Central Bank: risk of drainage of the liquidity circulating in the economy

In the context of dynamic hedging, we are looking at the second perspective of liquidity risk: market inactivity.
Simulating a Liquidity Crisis

Two steps (cf. Basle committee on banking supervision):

- Choice of a **liquidity horizon**
- Estimate the **liquidity cost**
Liquidity Horizon

The liquidity horizon is the time needed to liquidate a position without affecting dramatically the market.

Constraints:
1. Increasing with the outstanding amount
2. Consider the depth of the market
3. Possibility to mishedge the risk with a proxy: use of a deeper market with similar behaviour to the original market.

The Basle committee defined an arbitrary lower bound of 3 months – to be discussed in the insurance context.
Liquidity Cost

Once the liquidity horizon is determined, several methods can be used to estimate the liquidity cost, for example:

1. Impact of a scenario in which the hedging strategy is not implemented
   - Simple to implement
   - But: in scenarios where the P+L effect of hedging is negative, the entity would benefit from non-implementation of hedging – which is inconsistent with market conditions of a liquidity crisis

2. VaR approach (unhedged position), defined by
   - Time horizon: might be chose equal to liquidity horizon
   - Level of the VaR-quantile: matter of calibration, e.g. 5 %
Bid-Ask-Spread: a risk premium

- Bid-ask spread reflects the uncertainty of the market value, like does the volatility
- Original model would represent mid-prices, bid-ask spread is added to model transaction prices
- Model could link the bid-ask spread to volatility
- But: this link may have a strong impact on the tails of the distribution
A Model for the Market Structure

- Neglecting the liquidity risk, the value of the portfolio is the product of the bid price and the sold quantity.

- Implicit assumption: constant liquidation price independent from the quantity sold, i.e. sufficiently deep market.
A Model for the Market Structure

- In the case of insufficient market depth, the liquidation price will decrease with the quantity sold.
- For a given instrument, we describe the market condition at any time by:
  - A list of decreasing prices $p_1 > p_2 > \ldots > p_n$
  - A list of quantities $q_1 > q_2 > \ldots > q_n$
- Each index corresponds to an offer on the market:
  - Someone on the market wants to buy a quantity $q_i$ at price $p_i$
A Model for the Market Structure

- The process of liquidation is to match the orders in a decreasing order of price as long as the sum of the quantities sold is equal to the quantity to liquidate.
A Model for the Market Structure

- The liquidation price is now the sum of $q_i p_i$ as long as the sum of the $q_i$ is lower than the quantity to liquidate.
- The average price is now decreasing with increasing quantity to liquidate.
- Consistent with the idea that doubling an illiquid portfolio more than doubles the risk.
Modelling future Market Condition

- The framework of market condition allows in principle for changing market depth and liquidity over time.
- However, the granularity of the model leads to huge needs of calculation resources.
- One way to reduce complexity: parameterize behaviour of asset prices, for example by:
  - Ratio of volume sold and market depth
  - Calibrated with a history of transactions (cf. Almgren and Chriss)
- Back-testing?
Thank you for your Attention

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