Pension scheme design under short term fairness and efficiency constraints

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IAA LIFE Colloquium

Munich
The problem

- Systematic redistribution of funds between generations in mutually owned with-profits pension schemes.
- Problematic as seen from an altruistic board’s point of view.
- At odds with the principle of contribution.
- Previous studies: Døskeland and Nordahl (Geneva, 2008).
"It’s all about the bonus"

(Bonus given when the scheme is sufficiently solvent.)
"It’s all about the bonus"

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Pension scheme design under short term fairness and efficiency constraints
"It’s all about the bonus"

The impact of investment aggressiveness.

![Graph showing the compound bonus option payoff over time for different funding levels and investment strategies.]

- Entering at low funding (105%)
- Entering at full funding (120%)
- Cautious investment ($s=0.1$)
- Aggressive investment ($s=0.5$)

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Pension scheme design under short term fairness and efficiency constraints
“It’s all about the bonus”

Are higher barriers better?

![Diagram showing compound bonus option payoff over time for different barriers and funding levels.]

- Low barrier (110%)
- High barrier (130%)
- Entering at low funding (105%)
- Entering at full funding

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Pension scheme design under short term fairness and efficiency constraints
Setup

- Black-Scholes model \((\Lambda, \sigma, r)\) for assets \(A\).

- Piecewise deterministic liabilities, \(L\).

- CPPI-strategy with multiplier \(\alpha\), that is: nominal amount of risky assets is \(\alpha (A - L)\).

- Funding ratio, \(F \equiv \frac{A}{L}\).

- Yearly bonus: If \(F_i - \kappa > 0\), all excess assets are used to increase guarantees by \(\exp(b_i) = \frac{F_i - \kappa}{\kappa}\).

- Object of interest is terminal benefit: \(\sum_{n=0}^{\infty} \exp(\sum_{k=j+1}^{n} b_k)\).

- Compare the benefits of members entering at low and full funding respectively.

- Design parameters are the bonus barrier \(\kappa\) and the investment aggressiveness \(s \equiv \alpha \sigma\).
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\sum_{n=0}^{\infty} \exp\left(\sum_{k=j+1}^{\infty} b_k\right)
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$$\sum_{n=0}^{\infty} \exp\left(\sum_{k=j+1}^{\infty} b_k\right)$$

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- Design parameters are the bonus barrier \(\kappa\) and the investment aggressiveness \(s \triangleq \alpha\sigma\).
"It’s all about the bonus"

<table>
<thead>
<tr>
<th>Time</th>
<th>Bonus (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
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- **Member entering at low funding (105%)**
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"It's all about the bonus"

Compound bonus option payoff
Member entering at low funding (105%)
Member entering at full funding (120%)
The impact of investment aggressiveness

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Compound bonus option payoff

Time

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Are higher barriers better?

![Graph showing compound bonus option payoffs for different scenarios.](image)

- **Low barrier (110%)**
- **High barrier (130%)**
- **Entering at low funding (105%)**
- **Entering at full funding**

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Properties of bonus frequency

- Entering at *full* funding: Bonus *frequency* decreases as $s$ increases ($\kappa$ is irrelevant).
- Entering *below* full funding: Bonus *frequency* decreases as $\kappa$ increases. At very low initial funding, bonus frequency is maximised with moderately aggressive investment strategies. At initial funding closer to $\kappa$, bonus frequency is maximised with cautious investment strategies.
Conditional bonus increases as $s$ and $\kappa$ are increased.
## Conflicting interests

<table>
<thead>
<tr>
<th>Lucky entrants</th>
<th>Unlucky entrants</th>
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<tr>
<td>Cautious investment</td>
<td>Aggressive investment</td>
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<tr>
<td>High barriers</td>
<td>Low barriers</td>
</tr>
<tr>
<td>Redistributing design</td>
<td>Inefficient design</td>
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"Almost every outcome should be acceptable to almost every one"
“Almost every outcome should be acceptable to almost every one”

Let $X$ be the benefit.

Fairness:

$$P\left( \frac{X(s, \kappa, f)}{X(s, \kappa, \kappa)} > 1 - \delta \right)$$
Measuring fairness and efficiency

"Almost every outcome should be acceptable to almost every one"

Let $X$ be the benefit.

Fairness:

$$
\mathbb{P} \left( \frac{X(s, \kappa, f)}{X(s, \kappa, \kappa)} > 1 - \delta \right)
$$

Efficiency:

$$
\mathbb{P} \left( \frac{X(s, \kappa, \kappa) + X(s, \kappa, f)}{\max_{\bar{s}, \bar{\kappa} \in S \times \mathcal{K}} CE(\gamma; X(\bar{s}, \bar{\kappa}, \bar{\kappa}) + X(\bar{s}, \bar{\kappa}, f))} > 1 - \beta \right)
$$
Letting the initial low funding depend on the investment strategy implies optimal investment strategies that are much more cautious!
Letting the initial low funding depend on the investment strategy implies optimal investment strategies that are much more cautious!
A solidarity transformation rule

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This overcomes systematic redistribution, if the barrier is low. If, in addition, investment is not too aggressive, redistribution is very modest. Such design is a reasonable compromise between the interests of the involved parties, although a rather low barrier is not long-run desirable.
A solidary transformation rule

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This overcomes *systematic* redistribution, if the barrier is low. If, in addition, investment is not too aggressive, redistribution is very modest. Such design is a reasonable compromise between the interests of the involved parties, although a rather low barrier is not long-run desirable.
"So, it’s NOT all about the bonus!?!"