Understanding the Death Benefit Switch Option in Universal Life Policies

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Motivation

• Universal life policies are the most popular insurance contract design in the U.S.
  - Lifelong policies with flexible premium payments (frequency, amount) as long as cash value remains positive
  - Death benefit either
    - level: fixed face amount, or
    - increasing: pays available cash value in addition to fixed amount
  - Embed the option to switch from one death benefit scheme to the other: „death benefit switch option“
    - Switch from increasing to level without costs / evidence of insurability (unlike switch from level to increasing)
Motivation

- Option of no concern for insurers?
  - Death benefit is fixed at the current level, does not affect net amount at risk
  - But, crucial: dependence on *premium payment behavior* after switch (not prescribed by insurer)

> Combination of two options, can be very valuable

> Has not been investigated to date
Aim

- Enhance understanding of this feature
- Develop model framework of increasing universal life policies
- Incorporate switch probabilities and stochastic interest rates
- Investigate effects of adverse exercise behavior depending on health status of insureds
  - Consider mortality heterogeneity using a frailty factor
  - Assume different switch probabilities for different health status
- Account for modified premium payment behavior after switch
Aim

• Based on this model: quantify net present value of option from the insurer’s perspective
  - Conduct simulation analysis
  - Sensitivity analysis with respect to frailty distribution
  - Derive policy recommendations for life insurers
The model of an increasing universal life policy

- Pool of increasing lifelong universal life policies
- Cash value (policy reserve) $V_t$
- Increasing death benefit $Y_t = Y + V_t$, $t = 1, \ldots, T$
- One-year table probability of death at age $x+t$ $q^t_{x+t}$, $t = 0, \ldots, T - 1$
- Constant annual interest rate $i$
- Constant annual premiums $B$ (equivalence principle)

$$B \cdot \sum_{h=0}^{T-1} (1+i)^{t-h} = Y \cdot \sum_{h=0}^{T-1} q^t_{x+h} (1+i)^{t-h-1}$$
Death benefit switch option

- Death benefit at time $t$ given exercise of the switch option at time $\tau$

\[ Y^{(\tau)}_t = \begin{cases} 
Y_t, & t = 1, \ldots, \tau \\
Y_\tau, & t = \tau + 1, \ldots, T 
\end{cases} \]

- Switch before peak of cash value: original premiums too high, need to be reduced
- Switch near, at, after peak of cash value: higher premiums needed due to higher death benefit
Premium payment scenarios

- Cannot analyze death benefit switch option alone: we need to make assumptions about premium payment
  - Consider two viable scenarios after switch:
    - Minimum *constant* premium (level premium scenario)
    - Minimum *flexible* premium (risk premium scenario)
- Ensure positive cash value throughout the contract term
- Any other constant or flexible premiums need to exceed these premium amounts

\[
B_{t}^{(\tau)} = \begin{cases} 
B, & t = 0, \ldots, \tau - 1 \\
B^{(\tau)}, & t = \tau, \ldots, T - 1
\end{cases}
\]
Model framework: Premium scenario

- Minimum constant premium (level premium scenario)

\[
B^{(r)} = \sum_{t=0}^{T-r-1} p'_{x+t} (1+i)^{-t} + V = \sum_{t=0}^{T-r-1} p'_{x+t} q'_{x+t+1} (1+i)^{-(t+1)} \Rightarrow B^{(r)} = \max \left\{ \frac{Y \sum_{t=0}^{T-r-1} p'_{x+t} q'_{x+t+1} (1+i)^{-(t+1)} - V}{\sum_{t=0}^{T-r-1} p'_{x+t} (1+i)^{-t}} \right\}, 0
\]

a) Switch before peak of cash value curve

b) Switch at peak of cash value curve
Model framework: Premium scenario

- Minimum flexible premium (risk premium scenario)

\[
B_t^{(r)} = \begin{cases} 
B, & t = 0, \ldots, \tau - 1 \\
\max \left\{ 0, q'_{x+t} Y^{(r)}_t (1+i)^{-1} - V_t^{(r)} \right\}, & t = \tau, \ldots, T - 1
\end{cases}
\]

a) Switch before peak of cash value curve

b) Switch at peak of cash value curve

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Contract valuation

- Valuation depends on mortality
- Consider mortality heterogeneous insureds
  - Individual frailty factor $d$ specifies individual's state of health

$$q_x = \begin{cases} 
  d \cdot q'_x, & d \cdot q'_x < 1 \\
  1, & x = \min \left\{ 0, \ldots, \omega \right\} : d \cdot q'_x \geq 1 \\
  0, & \text{otherwise}
\end{cases}$$

for $x \in \left\{ 0, \ldots, \omega \right\}$ and $q_\omega := 1$ for $d < 1$

- $d \geq 1$ Insureds with average or below-average life expectancy
- $d < 1$ Insureds with above-average life expectancy

$$f^\Gamma_{(\alpha, \beta, \gamma)}(d) = \frac{1}{\Gamma(\alpha) \beta^\alpha} (d - \gamma)^{\alpha-1} e^{-\frac{d-\gamma}{\beta}}, \text{ for } d \geq \gamma, \gamma \in [\underline{0}], \alpha, \beta > 0.$$
Contract valuation

- Switch probabilities $s(t,d)$

$$F_r(k) = P(\tau \leq k) = \sum_{h=1}^{k} s(h) \prod_{\nu=1}^{h-1} (1 - s(\nu))$$

- Short-rate process: Vasicek model

$$dr(t) = \kappa (\theta - r(t)) dt + \sigma dW^Q(t)$$

- Affine term structure, zero bond price formula:

$$P(0,t) = E^Q \left( e^{-\int_0^t r(u) du} \right) = \exp \left\{ \left( 1 - e^{-\kappa t} \right) \left( \theta - \frac{\sigma^2}{2\kappa^2} - r \right) - t \left( \theta - \frac{\sigma^2}{2\kappa^2} \right) - \frac{\sigma^2}{4\kappa^3} \left( 1 - e^{-\kappa t} \right)^2 \right\}$$
Contract valuation

- Net present value (NPV) of the increasing policy

\[
NPV(d) = E^Q \left( \sum_{t=0}^{T-1} B_{t+1} \cdot 1_{\{X_{t+1} \geq t\}} \cdot e^{-\int_{0}^{t} r(u)du} \right) - E^Q \left( \sum_{t=0}^{T-1} Y_{t+1} \cdot 1_{\{X_{t+1} = t\}} \cdot e^{\int_{0}^{t+1} r(u)du} \right) = \sum_{t=0}^{T-1} B_t \cdot p_x \cdot P(0, t) - \sum_{t=0}^{T-1} Y_{t+1} \cdot p_x \cdot q_{x+t} \cdot P(0, t+1).
\]

- NPV of the increasing policy with death benefit switch option

\[
NPV^{(r)}(d) = E^Q \left( \sum_{t=0}^{T-1} B_t^{(r)} \cdot 1_{\{X_{t+1} \geq t\}} \cdot e^{-\int_{0}^{t} r(u)du} \right) - E^Q \left( \sum_{t=0}^{T-1} Y_{t+1}^{(r)} \cdot 1_{\{X_{t+1} = t\}} \cdot e^{\int_{0}^{t+1} r(u)du} \right) = \sum_{t=0}^{T-1} \left( \sum_{k=1}^{T} B_t^{(k)} \cdot s(k) \prod_{h=1}^{k-1} \left(1 - s(h)\right) \right) \cdot p_x \cdot P(0, t) - \sum_{t=0}^{T-1} \left( \sum_{k=1}^{T} Y^{(k)}_t \cdot s(k) \prod_{h=1}^{k-1} \left(1 - s(h)\right) \right) \cdot p_x \cdot q_{x+t} \cdot P(0, t+1).
\]
Contract valuation

- Expectation
  \[ NPV = E^Q \left( NPV(D) \right) \]
  \[ NPV^{(\tau)} = E^Q \left( NPV^{(\tau)}(D) \right) \]

- Value of the death benefit switch option: Difference between value of policy with switch option and value of policy without the switch option

  \[ NPV^{Opt} = NPV^{(\tau)} - NPV \]
Numerical examples: Input parameters

- Policy face value \( Y = $100,000 \)
- Age at inception: \( x = 45 \) years
- Actuarial minimum interest rate \( i = 3.5\% \)
- U.S. 1980 CSO male ultimate composite mortality table
- Frailty distribution \( D \sim \Gamma(2.0; 0.25; 0.5) \).
- Constant annual premium \( B = $5,937 \) (calibrated)
- NPV for increasing policy (without switch) from the insurer’s perspective is \( NPV = $2,866 \)
Value of the death benefit switch option by constant switch probability (insurer perspective)

- Negative for “level premium scenario”
- Positive but decreasing for “risk premium scenario”, due to high risk premiums, but: turns negative in case of policy lapse
- High s implies more negative values (early switch, long lifetime)
Value of the death benefit switch option by switch probability and health status: Level premium

Switch probability if $d<1$
(above-average life expectancy)

Switch probability if $d\geq1$
(average or below-average life expectancy)

NPV (€)

$0$

$-4'000$

$-3'500$

$-3'000$

$-2'500$

$-2'000$

$-1'500$

$-1'000$

$-500$

$0$

$500$

$1'000$

$1'500$

$2'000$

$2'500$

$3'000$

$3'500$

$4'000$

$500$

$1'000$

$1'500$

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$3'500$

$4'000$

$500$

$1'000$

$1'500$

$2'000$

$2'500$

$3'000$

$3'500$

$4'000$
Value of the death benefit switch option by switch probability and health status: Risk premium

Switch probability if $d<1$ (above-average life expectancy)

Switch probability if $d\geq 1$ (average or below-average life expectancy)
### Switch option value for specific exercise scenarios depending on health status

<table>
<thead>
<tr>
<th></th>
<th>(s=100%) at (t=41) (peak)</th>
<th>(s=10%) at (t=25) to (t=41)</th>
<th>(s=10%) to (s=100%)* (linear) at (t=25) to (t=41)</th>
<th>(s=10%) at (t=5) to (t=15)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Below LE</td>
<td>Above LE</td>
<td>All</td>
</tr>
<tr>
<td><strong>Level premium</strong></td>
<td>All</td>
<td>-365</td>
<td>12</td>
<td>-750</td>
</tr>
<tr>
<td></td>
<td>Risk premium</td>
<td>1’324</td>
<td>-89</td>
<td>1’413</td>
</tr>
<tr>
<td><strong>Risk premium</strong></td>
<td>Risk premium (lapse)</td>
<td>808</td>
<td>10</td>
<td>799</td>
</tr>
</tbody>
</table>

\(d \geq 1\) Insureds with average or below-average life expectancy

\(d < 1\) Insureds with above-average life expectancy

Lapse scenario: lapse as soon as risk premium exceeds 10% of new level death benefit

*Note: \(s\) denotes the death benefit switch option value, and \(t\) denotes the time at which the switch is exercised.*
Results

• Strong adverse effects can be observed depending on premium payment method and health status

• Scenarios that are intuitively rational pose greatest threat to insurers, namely
  - If insureds with above-average life expectancy switch early and thus save risk premiums by making level payments
  - If impaired insureds set out premium payments after switch, being aware of possibly not surviving until high risk premiums have to be paid
Policy implications for life insurers

• Analysis allowed identification of four key factors of relevance for the switch option value:
  - insureds’ life expectancy
  - premium payment method after switch
  - switch probabilities (time of switch)
  - lapsation
  ➢ Combination of these factors can make switch option either valuable or risky for insurer

• Problem: Option can be valuable when exercised early as well as late during the contract term
Policy implications for life insurers

• Switch as an alternative to surrender
• Impose new evidence of insurability (tradeoff: costs, penalizes healthy insureds)
• Prescription of premium payments after switch, combined with charges for group that causes adverse effects
• Restrict switch exercises to predefined time ranges
• In summary: death benefit switch option can pose a threat to insurers in case of adverse exercise behavior with respect to insureds health status
  ➢ Careful monitoring is crucial
Thank you very much for your attention!

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Deterministic results: Level premium
Deterministic results: Risk premium

**Diagram Description:**
- The diagram represents the relationship between time of death, switch exercise time, and the risk premium.
- The x-axis indicates the time of death, ranging from 0 to 50.
- The y-axis shows the switch exercise time, with values 11, 21, 31, 41, and 51.
- The z-axis measures the risk premium, ranging from $-40,000 to $100,000.
- The graph visualizes the risk premium's behavior over time and switch exercise times.

**Referenced Material:**
- N. Gatzert, G. Hoermann: „Understanding the Death Benefit Switch Option“