Longevity risk and hedge effects in portfolios of life insurance products with investment risk

Ralph Stevens, Anja De Waegenaere, and Bertrand Melenberg

Tilburg University and Netspar

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Introduction
Measuring longevity risk

- Future survival probabilities are stochastic $\rightarrow$ uncertainty in payments of life insurance products;

- Longevity risk is often quantified by distributional characteristics of the discounted cash flows, assuming a constant and deterministic interest rate, $r$.

- Problem: interpretation?
  - In real world there is investment risk;
  - Level of future payments is uncertain $\rightarrow$ cannot (fully) hedge against investment risk;
  - When return is as least $r$, this risk measure is a conservative one.

- We use an asset-liability approach to quantify longevity risk.
Longevity risk

- Focus on the effect of systematic longevity risk in a portfolio of life insurance products:
  - Effect of gender composition;
  - Effect of survivor annuities;
  - Effect of death benefits.

- Effect of investment risk when there exists longevity risk:
  - Effect of longevity risk on investment risk;
  - Effect of investment risk on hedge potential.

- Hedge effects of survival swaps:
  - Effect of survivor annuities;
  - Effect of basis risk.
Model
Risk measure: $\mathbb{P}(A_T < 0)$.

Evolution of the assets:

Return, liability payment

\[
A_{s+1} = (A_s - \tilde{L}_s) \cdot (1 + R_{s+1}), \quad \text{for } s = 0, \ldots, T.
\]

Terminal asset value:

\[
A_T = A_0 \cdot \prod_{s=1}^{T} (1 + R_s) - \sum_{s=1}^{T} \tilde{L}_s \cdot \prod_{\tau=s+1}^{T} (1 + R_\tau).
\]

Capital requirement: required initial asset value in excess of the best estimate

\[
A_0 = (1 + c) \cdot BEL.
\]
- Investment portfolio is decomposed in:
  - Best estimate portfolio, with return $r_{s}^{be}$;
  - Buffer portfolio, with return $r_{s}^{bu}$.

- Probability of ruin:
  \[ \mathbb{P}(A_T < 0) = \mathbb{P}(L > (1 + c) \cdot BEL), \]

\[
L \equiv BEL + \sum_{s=1}^{T} \left( \tilde{L}_t - \mathbb{E} \left[ \tilde{L}_s \right] \cdot P^{(s)} \cdot \prod_{\tau=1}^{s} \left( 1 + r_{\tau}^{be} \right) \right) \cdot \prod_{\tau=1}^{s} \left( 1 + r_{\tau}^{bu} \right).
\]

- Effect of longevity risk measured by the capital requirement in excess of best estimate:
  \[ c = \frac{\mathbb{Q}_{1-\epsilon}(L)}{BEL} - 1. \]
Focus on non-hedgeable risks: Best estimate portfolio invested in bonds.
We decompose $L$ into four components:

i) Best estimate of the liabilities; Deterministic, no uncertainty.

ii) Pure longevity risk component; Uncertainty in $L$ given expected returns.

iii) Pure investment risk component; Uncertainty in $L$ given expected cash flows. *Hedgeable risk: generally set equal to zero.*

iv) Interaction investment and longevity risk component; Uncertainty in $L$ which is not captured by the other components. *Additional uncertainty!*
Life insurance products:

i) Single life annuity;
ii) Survivor annuity;
iii) Death benefit.

Uncertainty in forecasting models includes:
Process risk, parameter risk, and model risk.

Longevity risk:
- Variants of Lee-Carter (1992)-model;
- Variants of Cairns-Blake-Dowd (2006)-model;
- P-Splines model (Currie, Durbin, and Eilers; 2004).

Investment risk:
- Bond prices: Vasicek model;
- Stock prices: Brownian motion with drift.
Effect portfolio mix
Buffer requirement depends on investment strategy and liability portfolio of insurer.

Buffer portfolio invested in one-year bonds, best estimate portfolio:
- Only one-year bonds;
- Portfolio of bonds eliminating pure investment risk.

Buffer requirements:

<table>
<thead>
<tr>
<th>Product</th>
<th>$c^{ly,ly}$</th>
<th>$c^{elh,ly}$</th>
<th>$c^{LO}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male single life annuity</td>
<td>27.6%</td>
<td>6.1%</td>
<td>4.4%</td>
</tr>
<tr>
<td>Female single life annuity</td>
<td>33.0%</td>
<td>6.9%</td>
<td>4.9%</td>
</tr>
</tbody>
</table>

We observe:
- Investment risk might be large;
- Liability only approach might underestimate longevity risk.
Solid curve: 100% bonds; dashed curve: 1/3 equity, 2/3 bonds; dotted curve: 100% equity.

- Longevity risk is higher for females than males;
- Investment risk significantly affects required buffer;
- Higher duration typically increases impact of investment risk.
Solid curve: 100% bonds; dashed curve: 1/3 equity, 2/3 bonds; dotted curve: 100% equity.

- Survivor annuities can reduce longevity risk;
- Effect of hedge potential depends significantly on investment risk.
Effect death benefits I

- Investment risk affects hedge potential.
- Death benefit consists of a single payment at the moment the insured dies. → effect of living longer = postponing payment.
- Hedge effects of death benefits are due to discounting effects!
- When there is no investment risk:
  Consider an single life annuity with a yearly payment of 1 and a death benefit with a single payment $\delta$. For $\delta = \frac{1+r}{r}$, the level of the aggregate payments of the portfolio is independent of the number of survivors.
Solid curve: 100% bonds; dashed curve: 1/3 equity, 2/3 bonds; dotted curve: 100% equity.
- As expected: death benefits can reduce longevity risk;
- Survivor annuities reduce hedge effects of death benefits;
- Hedge effects of death benefits are significantly influenced by investment risk.
Effect survivor swaps
Mortality linked assets can reduce longevity risk.

An often proposed product is a survivor swap or longevity bond.

The payments of a survivor swap in year \( s \) are given by:

\[
SS(s, \text{ref}) = S(s, \text{ref}) - K(s, \text{ref}).
\]

Without basis risk: survivor swaps can provide a perfect hedge for single life annuities;

How well do they work:
- In a portfolio of life insurance products?
- When there is basis risk?
Solid curve: no survivor annuities; dashed curve: 35% survivor annuities; dotted curve: 70% survivor annuities.

- Survivor annuities can significantly affect hedge potential of survival swaps;
- Maximum number of swaps depends on product and gender mix.
Mortality rates of insureds and general population differ.

Denuit (2008) uses a Cox-type relational model:

\[
\log(\mu_{x,t}^{(h)}) = \alpha^{(h)} + \beta^{(h)} \cdot \log(\mu_{x,t}^{(g)}),
\]

with \(\alpha^{(m)} = -1.54\), \(\alpha^{(f)} = -1.02\), \(\beta^{(m)} = 0.82\), \(\beta^{(f)} = 0.91\).

Basis risk significantly affects hedge potential of survival swaps.
Conclusions
Conclusions

Liability only approach might underestimate reserve requirements;

Product and gender mix influence required buffer;

Investment risk affects hedge potential:
  - Gender mix and survivor annuities: A higher duration typically leads to a stronger influence of investment risk.
  - Death benefits: More investment risk leads to a weaker hedge potential of death benefits.

Basis risk might significantly reduce the hedge potential of survivor swaps.