**Valuation and Risk Assessment of Participating Life Insurance in the Presence of Credit Risk**

Nadine Gatzert, Michael Martin∗

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**Abstract**

In participating life insurance, management decisions regarding the asset composition can substantially impact the value of a policy from the policyholders’ perspective as well as the insurer’s risk situation. Due to the long-term guarantees often embedded in these contracts, life insurers typically invest a considerable portion of their capital in long-term assets such as corporate and government bonds. Besides interest rate risk, the value of these bond investments is thus particularly influenced by credit spread risk. Thus, the aim of this paper is to examine the impact of the market risk associated with the asset composition on fair valuation and risk assessment with focus on credit risk and the interaction with equity risk and interest rate risk. In our analysis we emphasize the tradeoff between higher coupon payments of lower grade bond portfolios and increased credit risk as well as the interaction with equity risk arising from stock investments.

**Keywords:** Participating life insurance; credit risk; interest rate risk; risk-neutral valuation; asset management

**JEL Classification:** G22, G32

1. **Introduction**

Corporate and government bond exposures typically constitute a major part in life insurance companies’ asset portfolios as a result of the long-term guarantees regularly embedded in traditional life insurance contracts.1 Besides interest rate risk, the assets class of bonds is particularly affected by credit spread risk, i.e. the risk of default by the issuer of the bond. As a con-

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1 In Germany, for instance, the average portion invested in fixed-income assets in the end of 2011 amounted to almost 90% of the total capital investments in the life insurance sector (see GDV, 2012).
sequence of the recent financial and sovereign crises, apart from interest rate risk especially
the risk of corporate or sovereign default and thus credit risk became increasingly important.
The results of the quantitative impact study (QIS) 5 for the European supervisory system Sol-
vency II confirmed for the German insurance industry the high relevance of considering credit
risk (including spread risk). Following interest rate risk, the solvency capital requirements for
credit risk (quantified by the spread risk sub-module of the Solvency II framework) represents
the second largest part of market risk in the life insurance sector when solvency capital re-
quirements are calculated according to the standard model of Solvency II (see BaFin, 2011, p.
16). The aim of this paper is to examine the impact of the asset portfolio composition in the
context of participating life insurance contracts with respect to fair valuation and risk mea-
surement. We thus contribute to the existing literature by explicitly including and focusing on
credit risk associated with the substantial portions of bond investments in addition and in inte-
raction with interest rate risk and equity risk.

The fair valuation and risk measurement of participating life insurance contracts has been
subject to extensive research (see, e.g. Briys and de Varenne 1997, Grosen and Jørgensen,
2000 and 2002, Ballotta, Haberman, and Wang, 2006). In addition, several studies combine
both approaches (e.g., Barbarin and Devolder, 2005, Gatzert and Kling, 2007, and Graf,
Kling, and Russ, 2011). The impact of stochastic interest rates on the valuation of participat-
ing life insurance contracts is examined by, e.g., Bernard, Le Courtois, and Quittard-Pinon
(2005), while dynamic asset management decisions of the insurer are studied in Kleinow and
Willder (2007), Gatzert (2008), and Gerstner et al. (2008), whereby the latter examine the
financial risk of a life insurer in a general asset-liability management framework assuming
that the insurers’ asset composition consists of non-defaultable bonds and stocks that are pe-
riodically rebalanced. Other work focuses on the impact of different surplus appropriation
schemes in participating life insurance, thereby also taking into account mortality risk (Bohn-
ert and Gatzert (2012)), while Bohnert, Gatzert, and Jørgensen (2012) extend this work and
examine management strategies regarding the asset and liability composition by taking into
account participating life insurances and annuities in addition to different asset portfolios.

2 The standard model in Solvency II calculates the basic SCR (BSCR) by a bottom-up approach through six
risk modules (life, non-life, health, market, and default risk as well as intangibles). According to the results
of QIS 5 from the Federal Financial Supervisory Authority in Germany (BaFin) for the German insurance in-
dustry, market risk module represents the key risk driver in the life insurance sector with 82% of the BSCR.
Regarding the market risk module for life insurers, the interest rate risk dominates (64%), followed by the
spread risk (including spread and credit risk) (31%) and equity risk (13%) (see BaFin, 2011, p. 16). All val-
ues without diversification.
Hence, in the context of participating life insurances, the presence of credit risk and in particular the valuation of defaultable bonds has not been focused yet, even though bond exposures typically constitute a major part of an insurer’s assets and may have a considerable impact, especially against the background of long contract durations and an increasing credit risk. In this paper, we thus aim to contribute to previous literature by examining the impact of credit risk associated with bond investments on the fair valuation and risk assessment of participating life insurance contracts at the company level. The underlying participating life insurance contract is thereby assumed to feature a guaranteed interest rate, an annual cliquet-style surplus participation rate, and a terminal bonus. The asset model is based on Gerstner et al. (2008) assuming that the insurer invests in stocks and bonds, which is extended by integrating the credit risk model used in Gatzert and Martin (2012), who study the impact of default risk associated with corporate and government bonds under Solvency II (with focus on the asset side only). Thus, the valuation of market risk takes into account interest rate risk and credit risk as well as equity risk. While the risk of changes in the term structure of interest is quantified by the short term interest rate model from Cox, Ingersoll, and Ross (1985) (CIR) (see, e.g., Gerstner et al., 2008), the default risk is included by the reduced form credit risk model from Jarrow, Lando, and Turnbull (1997) (JLT). In addition to default risk, the JLT model quantifies spread risk that specifies rating class movements. Based on this model, we then calibrate the contracts to be fair from the equityholders’ perspective and calculate the net present value and the shortfall risk from the policyholder’s viewpoint with and without credit risk.

Our results demonstrate the consideration of credit risk associated with bonds has a considerable impact on the fair valuation in the context of participating life insurance contracts, even in case of higher grade bond exposures. However, the dimension of underestimating the insurers’ shortfall risk when ignoring credit risk depends on the allocation of the underlying asset portfolio, including the quality of the bond portfolio and the targeted stock portion. In particular, the numerical examples illustrate the tradeoff between higher credit risk generally associated with higher coupon payments and the insurer’s shortfall risk.

The remainder of the paper is organized as follows. The model framework of a life insurance company along with the liability and asset dynamics, contract valuation and risk measurement is presented in Section 2. Section 3 contains the numerical results, and Section 4 summarizes the results.

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3 See, e.g. Gatzert and Kling (2007) for a comparison of different types of participating life insurance contracts presented in previous literature and their impact on pricing and risk measurement.
2. Model Framework

2.1 Company overview

In the following, we consider a life insurer offering participating life insurance contracts and concentrate on the company’s investment decisions with particular focus on credit risk associated with bond investments. Hence, we assume that the insurer invests in two types of assets, bonds and stocks, and additionally distinguish between corporate and government bonds. Table 1 shows the life insurer’s balance sheet at time $t$, where $E(t)$ denotes the equityholders’ account and $L(t)$ represents the book value of the liabilities, which are equal to the policy reserves $P(t)$ during the contract term and in case the company is not insolvent. The contract term is denoted with $T$ and represents the date where the company is closed down.

Table 1: Balance sheet at time $t$

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_E(t)$</td>
<td>$E(t)$</td>
</tr>
<tr>
<td>$A_S(t)$</td>
<td>$L(t)$</td>
</tr>
</tbody>
</table>

The market value of the assets $A(t)$ at time $t$ is divided into two accounts,

$$A(t) = A_E(t) + A_S(t),$$

where $A_E(t)$ denotes the bond account and $A_S(t)$ the stock account. At inception of the contract, the initial assets base $A(0)$ consists of the equityholders’ contribution $E(0)$ and the initial policy reserves, which are equal to the value of liabilities at time 0 and given by the upfront premium $P(0) = L(0)$ paid by the policyholders. The upfront premium is given by $P(0) = k \cdot A(0)$ with positive real parameter $k$, describing the leverage of the life insurer. Hence, the initial equity capital is given by $E(0) = (1-k) \cdot A(0)$. The initial capital is then invested in a portfolio of stocks and bonds with stock portion $\alpha$,

$$A_S(0) = \alpha \cdot A(0) \quad \text{and} \quad A_E(0) = (1-\alpha) \cdot A(0).$$

(1)

To compensate equityholders for proving capital to ensure guarantees offered to the policyholders, they receive an annual dividend payment $D(t)$ given by

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The parameter $k$ can be interpreted as the wealth distribution coefficient (see Grosen and Jørgensen, 2002).
\[ D(t) = \mathbb{I}_{[A(t) - P(t) > \beta E(0)]} \cdot \beta \cdot E(0), \]  

(2)

where the dividend payment is defined as a constant fraction \( \beta \) of the initial equity capital \( E(0) \) (see Bohnert, Gatzert, and Jørgensen, 2012) and is only paid out if the surplus is sufficient to cover the dividend payments, i.e. if \( A^*(t) - P(t) > \beta \cdot E(0) \) (and zero otherwise).

### 2.2 Modeling the liabilities

Regarding the liabilities, we assume a participating life insurance contract with a cliquet-style guarantee.\(^6\) Hence, the development of the policy reserves \( P(t) \) are given by

\[
P(t) = P(t-1) \cdot (1 + r_p(t)) = P(t-1) \cdot \left(1 + \max \left( r_g, \gamma \cdot \frac{A(t)}{A(t-1)} - 1 \right) \right),
\]

(3)

where \( r_p(t), t = 1,2,\ldots,T \), denotes the annual policy interest rate, which at least amounts to a guaranteed interest rate \( r_g \) or an annual surplus participation \( \gamma \) in the insurer’s annual investment return. Once the policy interest rate is credited to the reserves, it becomes part of the guarantee and in the next year is again at least compounded with the guaranteed interest rate, thus causing cliquet-style interest rate effects.

In addition to the annual policy interest rate, the policyholders optionally participate in the terminal bonus \( B(T) \), defined as

\[
B(T) = \max \left( k \cdot A(T) - P(T), 0 \right).
\]

(4)

Therefore, at time \( T \), the policyholder receives

\[
L(T) = P(T) + \delta_L \cdot \max \left( k \cdot A(T) - P(T), 0 \right),
\]

in case the insurer has not become insolvent during the contract term, where \( \delta_L \) denotes the terminal surplus participation coefficient.

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\(^5\) While \( A^*(t) \) denotes the assets before dividend payment, the market value of the assets after the dividend is paid out is denoted by \( A(t) \).

2.3 Asset dynamics

Bond investments

Concerning the insurer’s investments in bonds, two risk drivers have to be taken into account, interest rate risk and credit risk (including spread risk). Hence, the underlying short term interest rate process is defined following Cox, Ingersoll, and Ross (1985) by

$$dr(t) = \kappa \cdot (\theta - r(t)) \, dt + \sigma_r \cdot \sqrt{r(t)} \, dW^Q_r(t).$$

on the probability space \((\Omega_r, \mathcal{F}_r, \mathbb{Q})\) with filtration \(\mathcal{F}_r\) generated by the Brownian motion under the risk-neutral probability measure \(\mathbb{Q}\) where \(\kappa\) defines the speed of mean reversion, \(\theta\) the long-term mean, and \(\sigma_r\) the volatility of the process. Using the affine term structure representation, zero coupon bond prices \(p(t,h)\) at time \(t\) and for maturity \(h\) are then given by (see Björk, 2009)

$$p(t,h) = E_t^Q \left( e^{\int_0^h r(s) \, ds} \right) = e^{F(t,h) - G(t,h) r(t)},$$

where

$$F(t,h) = \frac{2 \cdot \kappa \cdot \theta}{\sigma_r^2} \cdot \ln \left( \frac{2 \cdot a \cdot e^{\frac{(\kappa + a) \cdot h}{2}}}{(\kappa + a) \cdot \left( e^{a(h-t)} - 1 \right) + 2 \cdot a} \right), \quad G(t,h) = \frac{2 \cdot \left( e^{a(h-t)} - 1 \right)}{(\kappa + a) \cdot \left( e^{a(h-t)} - 1 \right) + 2 \cdot a},$$

$$a = \sqrt{\kappa^2 + 2 \cdot \sigma_r^2}.$$

Under the real-world probability measure \(\mathbb{P}\), the short rate process changes to

$$dr(t) = \left( \hat{\kappa} \cdot \hat{\theta} - (\hat{\kappa} - \lambda_{r,0} \cdot \sigma) \cdot r(t) \right) \, dt + \sigma \cdot \sqrt{r(t)} \, dW^P_r(t)$$

with a market price of interest rate risk \(\lambda_r(t, r(t))\) that is proportional to the short rate and given by \(\lambda_r(t, r(t)) = \lambda_{r,0} \cdot \sqrt{r(t)}\) (see Brigo and Mercurio, 2007).

To account for credit risk in the valuation of defaultable bond exposures, the reduced-form credit risk model by Jarrow, Lando, and Turnbull (1997) is used (see also Gatzert and Martin, 2012). The model extends the non-defaultable zero coupon prices given by the model of Cox, Ingersoll, and Ross (1985) by accounting for potential credit rating transitions and default of bond investments, described by a Markov process. Furthermore, in the case of default, a constant and exogenously given recovery rate \(\delta_R\) is paid out at maturity (recovery of treasury val-
ue). Jarrow, Lando, and Turnbull (1997) assume independency between interest rate and transition and default process, such that the price of a defaultable zero coupon \( \hat{p}(t,h) \) is given by

\[
\hat{p}(t,h) = E^Q \left( \mathbb{I}_{[t^B \leq h]} \cdot e^{-\int_{t}^{t^B} r(s) ds} + \mathbb{I}_{[t^B > h]} \cdot \delta_k \cdot e^{-\int_{t}^{t^B} r(s) ds} \right)
\]

\[= E^Q \left( e^{-\int_{t}^{t^B} r(s) ds} \cdot \left( \mathbb{I}_{[t^B \leq h]} + \mathbb{I}_{[t^B > h]} \cdot \delta_k \right) \right)
\]

\[= p(t,h) \cdot (\delta_k + (1-\delta_k) \cdot (1 - \psi(t,h))) \]  

(6)

where the indicator function \( \mathbb{I}_{[t^B \leq h]} \) is equal to one if the default time \( t^B \) is at or before maturity \( h \). The expression \( \psi(t,h) \) denotes the risk-neutral probability of default at time \( t \) with maturity \( h \), i.e. for \( h-t \) periods, and is defined by the distribution \( \Psi(t,h) \) of the underlying Markov process in discrete time (Markov chain), \( X = (x(t), t \in \mathbb{N}_0) \), on the risk-neutral probability space \( (\Omega, \mathcal{F}, \mathbb{Q}) \) and discrete state space \( E = \{1,...,k\} \) with

\[
\Psi(t,h) = \begin{pmatrix}
\psi_{1,1}(t,h) & \cdots & \psi_{1,k}(t,h) \\
\vdots & \ddots & \vdots \\
\psi_{k-1,1}(t,h) & \cdots & \psi_{k-1,k}(t,h) \\
0 & \cdots & 1
\end{pmatrix}
\]  

(7)

Here, the state \( x(t) = k \) determines the (absorbent) default state and the time of default is defined by a filtration \( \mathcal{F}_t = (\mathcal{F}_{t,t})_{t \in \mathbb{N}_0} \) with stopping time \( t^B \) that is adapted to the Markov process \( X \):

\[\tau^B = \inf \{ t \in \mathbb{N} : x(t) = k \} \].

By distinguishing different credit ratings \( x(t) = i \), the defaultable zero coupon price \( \hat{p}(t,h)_{x(t)=i} \) is specified by the rating-specific default transition probability \( \psi_{i,k}(t,h) \) in a risk-neutral world. Finally, we follow Gatzert and Martin (2012), where the market value of a bond exposure \( j \), which is not defaulted until time \( t \), \( A_{b,j}(t) \), is defined by the sum of all future cash

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7 The last row in Equation (7) describes the absorbent state of default. To obtain risk-neutral probabilities for the rating transitions, Jarrow, Lando, and Turnbull (1997) propose to adjust the real world probabilities \( \Psi(t,h) \) by a time-depending and rating specific risk premium \( \pi_{i,k}(t) \) by \( \Psi(t,h) = \Pi(t) \cdot (\Psi(t,h) - I) + I \) with \( k \times k \) matrix \( \Pi(t) = \text{diag}(\pi_{i,1}(t),...,\pi_{i,k-1}(t),1) \).
flows $CF_j(h)$ for $h > t$ multiplied by the defaultable zero coupon price evaluated at time $t$ and given by

$$A_{B,j}(t) = \sum_{h=t+1}^{T_j} \left( CF_j(h) \cdot \hat{p}_{s_j(t)=h} \right) + \mathbb{1}_{\{\tau_j^B = 0\}} \cdot CF_j(t),$$

where $T_j = \max\{t \mid CF_j(t) \neq 0\}$ describes the maturity of the bond asset. The cash flows $CF_j(t)$ of bond $j$ are determined by

$$CF_j(t) = \begin{cases} CP_j(t) \cdot FV_j \cdot n_j(t-1), & (t < T_j) \land (\tau_j^B > t) \\ (1 + CP_j(T_j)) \cdot FV_j \cdot n_j(t-1), & (t = T_j) \land (\tau_j^B > t) \\ \delta_r \cdot A_{B,j}(t-1), & \tau_j^B = t \\ 0, & \text{else,} \end{cases} \quad (8)$$

depending on time $t$ and the $j$-th bonds’ time of default $\tau_j^B$. The first two cases in Equation (8) describe the non-default state ($\tau_j^B > t$) where either the annual coupon $CP_j(t)$ (in % of the face value $FV_j$) is paid out ($t < T_j$) or, at maturity, the face value plus the coupon payment is paid ($t = T_j$). Both depending on the bonds’ face value $FV_j$ and the number of bonds $n_j(t-1)$ at time $t-1$. Finally, the third case in Equation (8) specifies the default state of the issuer ($\tau_j^B = t$), where the investors receive the recovery rate of the bonds’ market value before default, $A_{B,j}(t-1)$ (recovery of market value, see, e.g., Duffie and Singleton, 1999). For a bond portfolio consisting of $N_B$ bonds, the market value is given by

$$A_B(t) = \sum_{j=1}^{N_B} A_{B,j}(t).$$

**Stock investments**

With respect to the market value of stocks, we assume a geometric Brownian motion, i.e.

$$dA_{S,i}(t) = \mu_{S,i} \cdot A_{S,i}(t) \, dt + \sigma_{S,i} \cdot A_{S,i}(t) \, dW_{S,i}^P(t), \quad (9)$$

with constant drift $\mu_{S,i}$ and volatility $\sigma_{S,i}$ over time for stock $i$ and $W_{S,i}^P$ a standard $\mathbb{P}$-Brownian motion on the probability space $(\Omega_S, \mathcal{F}_S, \mathbb{P})$ with filtration $\mathcal{F}_S$ and real-world probability measure $\mathbb{P}$. Hence, the solution of the stochastic differential equation for stock $i$ at time $t$, $A_{S,i}(t)$, is given by
with independent standard normally distributed random variables $Z_{S,i}(t)$ (see Björk, 2009). When changing the probability measure to the risk-neutral measure $\mathbb{Q}$, the drift of the stochastic differential equation in Equation (9) changes to the risk-free short rate $r(t)$,

$$A_{S,j}(t) = A_{S,j}(t-1) \cdot e^{\left(\mu_{s,j} - \frac{\sigma_{s,j}^2}{2}\right) + \sigma_{s,j} Z_{s,j}(t)}$$

The market value of a portfolio containing $N_S$ stock exposures $A_S(t)$ at time $t$ is then given by

$$A_S(t) = \sum_{i=1}^{N_S} A_{S,i}(t).$$

Furthermore, the stock price processes (see Equation (9)) are assumed to be correlated by $dW_{S,i}^p dW_{S,j}^p = \rho_{i,j} dt$, $\forall i, j$, and the short rate process in Equation (5) is correlated with the stocks with $dW_{S,i}^p dW_r^p = \rho_{i,r} dt$, $\forall i$.

**Management decisions regarding the asset allocation**

Regarding the insurer’s decisions on the asset allocation over time, we follow the assumption in Gerstner et al. (2007) by rebalancing the capital investments at the beginning of each time period. Hence, we distinguish between the market value of each asset class (stocks and bonds) at time $t$ before and after rebalancing the assets using superscript (-) and (+). Thus, the total value of assets at time $t A(t)$ is given by the market value of stocks $A_S^-(t)$ and bonds $A_B^-(t)$ before rebalancing plus the cash flows $CF(t)$ received by the bond investments and minus the dividend payments to the equityholders $D(t-1)$ from period $t-1$ (see Equation (2)), which are paid out at time $t$, i.e.,

$$A(t) = A_S^-(t) + A_B^-(t) + CF(t) - D(t-1) = A_S^+(t) + A_B^+(t).$$

Bond exposures are assumed not to be sold before maturity such that the available capital for rebalancing (and new investments) $F(t)$ at time $t$ is given by

$$F(t) = A(t) - A_B^-(t) - D(t-1) = A_S^-(t) + CF(t) - D(t-1)$$

(11)
In addition, a constant stock portion $\alpha$ is assumed for each time period (see Equation (1)). Thus, the market value of stocks after rebalancing is determined by

$$A_s^+(t) = \max\left( \min\left( \alpha \cdot A(t), F(t) \right), 0 \right). \quad (12)$$

Stocks are assumed to be invested in equal proportions in the $N_S$ available assets with $A_{s_j}^+(t) = A_s^+(t)/N_S$. As a result of the rebalancing and the stock investment given in Equation (12), the bond investment can then be residually derived by

$$A_b^+(t) = A_b^-(t) + F(t) - A_s^+(t),$$

where $A_b^-(t)$ denotes the fixed investment in bonds that cannot be sold before maturity, i.e. we assume that all bonds have maturity $T$. Considering a portfolio of $N_B(t)$ bonds, where $N_B(t)$ denotes the number of available bonds that are not defaulted until time $t$, the market value of the bond portfolio at time $t$ is calculated by

$$A_B^+(t) = \sum_{j=1}^{N_B(t)} n_j(t) \cdot A_{B,j}^{FV=1}(t), \quad (13)$$

where $n_j(t)$ denotes the number of bonds from issuer $j$ with face value equal to one ($FV_j = 1$) and the corresponding market value $A_{B,j}^{FV=1}(t)$ at time $t$. By setting the face value to one, the number of bonds $n_j(t)$ is standardized with respect to the face value. Thus, the number of bonds in Equations (8) and (13) is given by the respective number from the previous period $n_j(t-1)$ and the number of bonds bought by means of newly available capital for new investments at time $t$, expressed by

$$n_j(t) = n_j(t-1) + \frac{F(t) - A_s^+(t)}{N_B(t)} \cdot \frac{1}{A_{B,j}^{FV=1}(t)},$$

where the newly available capital is equally invested in all $N_B(t)$ bonds that are not defaulted until time $t$.\(^8\)

Note that in case of a situation where during the contract time all bonds in the insurers’ asset portfolio are defaulted until time $t < T$, i.e. $N_B(t) = 0$, the company cannot reallocate its assets

\(^8\) At contract inception, the number of assets bought from issuer $j$ is set to $n_j(0) = A_B(0)/\left( N_B \cdot A_{B,j}^{FV=1}(0) \right)$. 
to bonds. In this situation, we thus assume that the insurer invests the recovered amount in a riskless asset according to Equation (5). Accordingly, the bond account at time $t$ is given by

$$A_b(t) = \delta_R \cdot A_b(\tilde{\tau}^b) \cdot e^{\int_0^t r(s) \, ds},$$

where $\tilde{\tau}^b$ denotes the time where the last bond in the portfolio defaulted ($N_B(\tilde{\tau}^b) = 0$), i.e. $\tilde{\tau}^b = \inf \{ t \in \mathbb{N} : N_B(t) = 0 \}$. Furthermore, all stock exposures from the time of the last bonds’ default $\tilde{\tau}^b$ are hold until maturity $T$ and are not further reallocated.

Isolating the impact of credit risk

To isolate the impact of credit risk associated with bonds, we first only consider interest rate risk. This implies that the bond evaluation process in Equation (6) is reduced to the valuation of non-defaultable zero coupon bonds by setting

$$\hat{p}(t,h) = p(t,h).$$

In the non-defaultable case in Equation (14), the distribution $\Psi(t,h)$ of the underlying Markov process under the risk-neutral measure $\mathbb{Q}$ (see Equation (7)) is thereby set to an identity matrix, thus disregarding the probability of rating transitions. Hence, when accounting for all risks (including credit risk) in the bond valuation framework, the valuation approach in Equation (6) is used, while in the case without credit risk, we revert to Equation (14).

2.4 Fair valuation from the equityholders’ and the policyholders’ perspectives

To evaluate the claims by equityholders and policyholders, risk-neutral valuation is used. One thereby has to take into account that the insurer may become insolvent during the contract term in case assets are not sufficient to cover the liabilities, i.e. in this case the policy reserves. The time of the default $\hat{\tau}$ is accordingly defined by

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9 Note that in general but depending on the stock portion in the portfolio, a default of all bonds in the asset portfolio would also induce the default of the insurance company as a whole.

10 An alternative interpretation for setting $\hat{p}(t,h) = p(t,h)$ is the assumption of a total recovery with $\delta_R = 1$, where the total market value of the exposure is paid out if the bond defaults and the bond exposure still exists. However, in our model, the assumption of the identity matrix for the distribution $\Psi(t,h)$ is adopted, such that every bond pays its coupons until maturity.
\[ \tau^c = \inf \left\{ t \in \{1,...,T\} : A(t) < P(t) \right\}. \]

In this case, costs for the insolvency settlement may arise that reduce the remaining assets by a factor \( c \) (see Bohnert, Gatzert, and Jørgensen, 2012). Hence, the book value of liabilities \( L(t) \) is given by

\[
L(t) = \begin{cases} 
P(t), & (t < T_j) \wedge (\tau^c > t) \\
\frac{P(t) + \delta_L \cdot B(t)}{A(t) \cdot (1 - c)}, & (t = T) \wedge (\tau^c > T), \\
A(t) \cdot (1 - c), & \tau^c = t \\
0, & \text{else}
\end{cases}
\] (15)

and the equityholders’ account \( E(t) \) is residually derived by

\[
E(t) = \begin{cases} 
A(t) - L(t), & \tau^c > t \\
0, & \text{else}
\end{cases}
\]

i.e. equityholders receive nothing in case of default at time.\(^{12}\) Hence, the value of the equityholders’ claim \( \Pi^E \) is given by

\[
\Pi^E = \mathbb{Q} \left[ \mathbb{1}_{\{\tau^c > t\}} \left( e^{-\int_{0}^{\tau^c} r(s)ds} \cdot E(T) + \sum_{j=1}^{T} e^{-\int_{0}^{\tau^c} r(s)ds} \cdot D(t-1) \right) + \mathbb{1}_{\{\tau^c = t\}} \sum_{j=1}^{T} e^{-\int_{0}^{\tau^c} r(s)ds} \cdot D(t-1) \right] \quad (16)
\]

Either the company remains solvent during the whole contract term, in which case the equityholders receive their annual dividend payments \( D(t) \) and the terminal payout \( E(T) \), or the insurer defaults before maturity, implying a zero payoff for the equityholders. To ensure a fair situation for the equityholders and thus an adequate compensation for the risk taken, the value of the payoff must be equal to their initial contribution, i.e.

\[
\Pi^E = E(0). \quad (17)
\]

\(^{11}\) Note that in our setting, three different times of default have to be distinguished: 1) The default time \( \tau^p \) of a single bond exposure \( j \), 2) the default time \( \bar{\tau} \) of the last bond in the bond portfolio, and 3) the time \( \bar{\tau} \) where the life insurer as a whole defaults.

\(^{12}\) Grosen and Jørgensen (2002) use knockout barrier options to evaluate the policyholders’ claim using closed-form solutions, which we cannot achieve in the present setting, as the asset returns are not normally distributed due to taking into account credit risk (see Equations (6) and (7)).
Therefore, for a given set of contract parameters, for instance, the fixed dividend payment $b$ can calibrated to ensure that (17) holds. Based on these fair contracts, we further calculate the net present value (NPV) from the policyholders’ perspective, which is analogously given by

$$NPV^P = E^Q \left( \mathbb{1}_{\{\tau^C > T\}} \cdot e^{-\int_0^{\tau^C} r(t) \, dt} \cdot L(T) + \mathbb{1}_{\{\tau^C \leq T\}} \cdot e^{-\int_0^{\tau^C} r(t) \, dt} \cdot L(\tau^C) \right) - P(0).$$

(18)

### 2.5 Risk measurement

For the fairly calibrated situation from the equityholders’ perspective, in addition to the NPV of the policyholders, we further calculate the shortfall probability ($SP$) and the mean loss ($ML$) as the lower partial moment of degree zero ($LPM_0$) under the real-world probability measure $\mathbb{P}$, which are given by

$$SP = LPM_0 = P(\tau^C \leq T)$$

and

$$ML = LPM_1 = E \left( -\left( A(\tau^C) - P(\tau^C) \right) \cdot \mathbb{1}_{\{\tau^C \leq T\} \}} \right),$$

whereby the latter also takes into account the extent of the default instead of only the probability.

### 3. Numerical Results

This section investigates the effect of market risk comprising equity, interest rate and credit risk on the fair value and risk of participating life insurance contracts with specific focus on the impact of credit risk. All numerical results are based on Monte Carlo simulation with 100,000 sample paths, based on latin hypercube sampling (see Glasserman, 2010).

#### 3.1 Input parameters

Regarding the participating life insurance contract, we assume a contract term of $T = 15$ years. The upfront premium is set to $P(0) = 85$, i.e. $k = 0.85$, and the initial contribution of the equi-

---

13 To ensure that the numerical results remain stable, different sets of random numbers were tested for robustness.
tyholders is $E(0) = 15$ leading to an initial asset base of $A(0) = 100$. Furthermore, the terminal surplus participation coefficient for the policyholders’ participation at maturity is assumed to be $\delta_L = 0.6$. The fraction for the constant dividend payment as defined in Equation (2) is set to $\beta = 0.03$, while the costs of insolvency are assumed to be $c = 0.05$.

The insurer invests in a portfolio of stocks consisting of three price indices. Table 2 presents the stocks with estimated annualized parameters of the expected return $m_S = \mu_S - 0.5 \cdot \sigma^2_S$ and volatility $\sigma_S$ based on monthly data from 01/1988 to 07/2011 from the Datastream database.

**Table 2: Stock portfolio (with annualized parameters)**

<table>
<thead>
<tr>
<th>$S_i$</th>
<th>$m_S$</th>
<th>$\sigma_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 DAX 30</td>
<td>0.0637</td>
<td>0.2164</td>
</tr>
<tr>
<td>2 FTSE 100</td>
<td>0.0436</td>
<td>0.1658</td>
</tr>
<tr>
<td>3 Dow Jones Industrial</td>
<td>0.0755</td>
<td>0.1784</td>
</tr>
</tbody>
</table>

Notes: The parameters from the Datastream database are estimated based on monthly data from 01/1988 to 07/2011 with $S_1$: DAX 30 (price index), $S_2$: FTSE 100 (price index) and $S_3$: Dow Jones Industrials (price index).

The input parameters of the Cox, Ingersoll, and Ross (1985) interest rate model are calibrated based on the six months “Euro Interbank Offered Rate” (EURIBOR) using monthly data for a ten year time horizon (from 01/1999 to 12/2008). Using maximum likelihood estimation, the annualized long-term mean and the speed of mean reversion are given by $\theta = 0.0369$ and $\kappa = 0.181$, respectively, with a volatility of $\sigma_r = 0.0342$. Furthermore, the market price of risk is assumed to be zero ($\lambda_r,0 = 0$) and the initial value corresponds to the long-term mean with $r(0) = \theta$.  

Concerning the insurer’s bond investment, three fixed income corporate bonds and three fixed income government bonds with different credit quality are considered. Table 3 displays the bond exposures with credit quality (rating), maturity, and corresponding fixed annual coupon payment.

---

14 See Brigo et al. (2009). The solution of the stochastic integrals $\int_{t=0}^{T} r(s) ds$ is approximated by applying numerical methods (composite trapezoidal rule of the Newton-Cotes formulas, see Press et al., 2007).

15 Due to ensure coupon payments for the entire contract time, we consider bond exposures where the maturity is approximately on a level with the contract time ($T = 15$). Here, asset-liability management (e.g. duration matching or cash-flow matching) are not in the focus of the analysis.
Table 3: Bond portfolio considered in the numerical analysis

<table>
<thead>
<tr>
<th>Bj</th>
<th>Type</th>
<th>Rating</th>
<th>Maturity (years)</th>
<th>Coupon p.a. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Corporate</td>
<td>AA</td>
<td>15</td>
<td>2.800</td>
</tr>
<tr>
<td>2</td>
<td>Corporate</td>
<td>A</td>
<td>15</td>
<td>4.125</td>
</tr>
<tr>
<td>3</td>
<td>Corporate*</td>
<td>BBB</td>
<td>15</td>
<td>7.125</td>
</tr>
<tr>
<td>4</td>
<td>Government</td>
<td>AAA</td>
<td>16</td>
<td>2.275</td>
</tr>
<tr>
<td>5</td>
<td>Government</td>
<td>A</td>
<td>15</td>
<td>3.000</td>
</tr>
<tr>
<td>6</td>
<td>Government</td>
<td>BB</td>
<td>17</td>
<td>7.750</td>
</tr>
</tbody>
</table>

* To ensure a maturity for the total contract time of T=15 years, the bonds’ original maturity set from 14 up to 15 years.


In the numerical analysis, two bond portfolios are composed and compared: 1) the higher grade portfolio consisting of higher rated corporate and government bonds (bond portfolio 1: B1, B2, B4, B3) and 2) a portfolio that presents the riskier bond investment by lower rated exposures (bond portfolio 2: B2, B3, B5, B6). At contract interception in t = 0, all stock and bond exposures are invested in equal proportions in consideration of the fixed stock portion alpha with

\[ A_{S,i}(0) = A_S(0) \cdot \alpha / N_S \quad \text{and} \quad A_{B,j}(0) = A_B(0) \cdot (1 - \alpha) / N_B(0). \]

To take correlations between the stock processes (see Equation (9)) as well as the stock and the interest rate process (see Equation (5) and (9)) into account, we apply the Gauss copula (see McNeil, Frey, and Embrechts, 2005). Table 4 shows the estimated pairwise correlations with risk-free interest rate rf and stock price Si of stock i. The parameters for stocks are calibrated on monthly data from 01/1988 to 07/2011 and for interest rate from 01/1999 to 12/2008.

Table 4: Asset correlations

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>0.68</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S3</td>
<td>0.65</td>
<td>0.76</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>rf</td>
<td>-0.19</td>
<td>-0.27</td>
<td>-0.10</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Notes: The correlations in row Si, Sj, S3 and r are estimated by monthly data from 01/1988 to 07/2011 with Si: DAX 30 (price index), S2: FTSE 100 (price index), S3: Dow Jones Industrials (price index) and r: EURIBOR (6 month offered rate, data from 01/1999 to 12/2008).
The distribution $\Psi(t,h)$ of the Markov chain in Equation (7) for modeling rating transitions in the credit risk model from Jarrow, Lando, and Turnbull (1997) is assumed to be time-homogenous. Accordingly, the distribution is constant and can be exhibited by $\Psi(t,h) = \Psi(h-t) = \left(\psi_{i,j}(h-t)\right)_{i,j\in\mathcal{E}}$ (see McNeil, Frey, and Embrechts, 2005). The constant distribution is estimated and published by rating agencies based on historical observations. For corporate bonds, we revert to the report from Standard & Poor’s (see Vazza, Aurora, and Kraemer, 2010), presenting the average one-year transition rates for global corporate bonds and based on transition data from 1981 to 2009 (see Appendix, Table A.1). A further report from Standard & Poor’s (see Chambers, Ontko, and Beers, 2011) presents the average one-year transition rates for foreign currency ratings of government bonds (see Appendix, Table A.2). Bangia et al. (2002) suggest eliminating the class of not rated (“NR”) issuers by a proportional partitioning to all rating classes (see Appendix, Table A.3, and Appendix, Table A.4). Following Gatzert and Martin (2012), the risk premium is assumed to be constant for all ratings and times (see Cairns, 2004): $\pi_{x(t)=i}(t) = 1.4, \forall t \in \mathbb{N}_0^*, \forall i \in \{1,\ldots,k-1\}$. Regarding the recovery rate in the Jarrow, Lando, and Turnbull (1997) credit risk model that is paid out in the case of default, the assumption of a constant recovery rate for all exposures (corporate and government) is made. The recovery rate is set to $\delta_R = 0.55$ which is in line with the internal ratings based (IRB) approach for senior claim credit risks of Basel II (see BIS, 2006) and will be subject to variation in the numerical analysis.\footnote{The European banking supervisory system was reformed in 2006 with the Basel II framework by introducing an international standardized regulatory system. A reformation of the current regulations, Basel III, will come into force from 2013.}

### 3.2 Fair contracts and risk measurement: The impact of credit risk

We first determine fair contract parameters for different asset portfolios and conduct risk measurement. Solving Equation (17), the fair annual surplus participation parameter can be derived depending on the stock portion for a fixed guaranteed interest rate and terminal surplus participation coefficient (see Equations (3) and (4)). To identify the influence of credit risk, two different bond portfolios compared. While the stock portfolio, as mentioned in Table 2, remains the same, the bond investment portfolio is either composed of higher grade bonds (bond portfolio 1) or lower grade bonds (bond portfolio 2), both comprising corporate and government bonds, respectively (see Table 3). Thus, Figure 1 displays results for the higher grad bond portfolio and Figure 2 shows results for the lower grade portfolio. We first examine the bond portfolio comprising higher grade assets (bond portfolio 1) and thus inducing a lower shortfall risk.
Figure 1: Fair annual surplus participation rate $\gamma$ with corresponding shortfall probability $SP$ for various stock portions $\alpha$ and guaranteed interest rates $r_g$ based on the higher grade bond portfolio

a) Fair contract parameters

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$r_g$</th>
<th>With credit risk</th>
<th>Without credit risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>1.25%</td>
<td>-0.41 pp</td>
<td>-0.30 pp</td>
</tr>
<tr>
<td>5%</td>
<td>1.75%</td>
<td>-0.81 pp</td>
<td>-0.52 pp</td>
</tr>
<tr>
<td>10%</td>
<td>2.25%</td>
<td>-2.95 pp</td>
<td>-1.47 pp</td>
</tr>
<tr>
<td>15%</td>
<td>1.25%</td>
<td>-0.30 pp</td>
<td>-0.27 pp</td>
</tr>
<tr>
<td>20%</td>
<td>1.75%</td>
<td>-0.52 pp</td>
<td>-1.05 pp</td>
</tr>
<tr>
<td>25%</td>
<td>2.25%</td>
<td>-1.47 pp</td>
<td>-1.05 pp</td>
</tr>
</tbody>
</table>

b) Shortfall probability (extent of misestimating credit risk given as percentage points)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$r_g$</th>
<th>With credit risk</th>
<th>Without credit risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>1.25%</td>
<td>0.04 pp</td>
<td>0.04 pp</td>
</tr>
<tr>
<td>5%</td>
<td>1.75%</td>
<td>0.08 pp</td>
<td>0.06 pp</td>
</tr>
<tr>
<td>10%</td>
<td>2.25%</td>
<td>0.14 pp</td>
<td>0.12 pp</td>
</tr>
<tr>
<td>15%</td>
<td>1.25%</td>
<td>0.04 pp</td>
<td>0.04 pp</td>
</tr>
<tr>
<td>20%</td>
<td>1.75%</td>
<td>0.08 pp</td>
<td>0.06 pp</td>
</tr>
<tr>
<td>25%</td>
<td>2.25%</td>
<td>0.14 pp</td>
<td>0.12 pp</td>
</tr>
</tbody>
</table>

**Notes:** The exposures in the stock portfolio (S) and the bond portfolio (B) are invested in equal proportions. $S_1$: DAX 30 (price index), $S_2$: FTSE 100 (price index), $S_3$: Dow Jones Industrials (price index), $B_1$: Wal-Mart Stores Incorporated, $B_2$: Electricite de France SA, $B_3$: US Airways Incorporated, $B_4$: Finland Republic of (Government), $B_5$: Poland Republic of (Government) and $B_6$: Hungary Republic of (Government). Bond portfolio 1 (higher grade): $B_1$, $B_2$, $B_4$, $B_5$; bond portfolio 2 (lower grade): $B_2$, $B_3$, $B_5$, $B_6$.

Figure 1 a) shows the fair annual surplus participation coefficient from the equityholders’ perspective for different target stock portions and different guaranteed interest rate. We thereby differentiate between the situation where credit risk is taken into account in the risk assessment and the case where these risks are ignored, which allows an isolated analysis of the effect of credit risk (see Equation (14)). Moreover, the dimension of underestimation by ignoring credit risk in the valuation and risk measurement procedure is quantified.
In Figure 1 a), one can observe that for a given guaranteed interest rate $r_g$ the annual participation coefficient $\gamma$ is decreasing for higher stock portions $\alpha$. This observation results from the fact that an increasing stock portion induces higher asset returns for the insurer and, thus, higher cliquet-style effects for the policyholders (see Equations (3) and (15)). This cliquet-style effect also dominates possible losses if the insurer defaults. Furthermore, the results show a lower annual surplus participation rate for higher guaranteed interest rates, since the guarantee serves as a lower bound for the policyholders’ annual return. When taking credit risk into account, the annual participation coefficient \textit{ceteris paribus} increases as a consequence of the higher shortfall risk, since the coupon payments remain the same and it is assumed that there is no credit rating transition during the bonds’ contract term.

The effect of taking credit risk into account has a major impact on the life insurers’ shortfall probability $SP$ as illustrated in Figure 1 b), which is based on the fair parameters in Figure 1 a) (note that the dimension of underestimating shortfall risk is given as the percentage points (pp)). Figure 1b) first shows that the shortfall risk increases for an increasing guaranteed interest rate for a given stock portion, which holds in both situations with and without credit risk. However, when increasing the target stock portion $\alpha$, the effect is ambiguous. In case credit risk is not considered, the shortfall probability increases for higher stock portions. When taking credit risk into account, in contrast, the shortfall risk is highest for very low stock portions in the considered example and then first decreasing (due to a decreasing impact of credit and interest rate risk) and then increasing again for higher stock portions (due to the impact of equity risk, which exceeds the reduced credit risk). In particular, shortfall risk is considerable underestimated especially for lower stock portions, amounting to 3 percentage points (for a guaranteed interest rate of 2.50%) when ignoring credit risk in the analysis, despite the fact that a higher grade bond portfolio is given. For higher stock portions, we can observe that the shortfall risk with and without credit risk approximates each other as a result of the decreasing impact of credit risk and the increasing relevance of equity risk in the presence of higher stock portions.

Hence, due to the interaction effects between equity risk, interest rate risk and credit risk, depending on the extent of credit risk associated with the bond portfolio, including a certain stock portion in the portfolio may even help reducing the overall risk level, while still ensuring fair contracts.

The situation is similar for a bond portfolio with lower investment grades (bond portfolio 2) as shown in Figure 2. However, we also observe major differences. In particular, the gap between the fair annual surplus participation rate with and without taking credit risk into ac-
count is considerably larger as compared to the case of the higher grade bond portfolio (Figure 1 a), which can be explained by the higher coupon payments that compensate the investor for taking credit risk. Hence, similar to equity risk, the investment in lower grade bonds that implicates higher returns due to higher coupon payments results in lower fair parameters $\gamma$.

**Figure 2**: Fair annual surplus participation rate $\gamma$ with corresponding shortfall probability SP for various stock portions $\alpha$ and guaranteed interest rates $r_g$ based on the lower grade bond portfolio

---

**a) Fair contract parameters**

---

**b) Shortfall probability (extent of misestimating credit risk given as percentage points)**

Notes: The exposures in the stock portfolio (S) and the bond portfolio (B) are invested in equal proportions. S: DAX 30 (price index), S: FTSE 100 (price index), S: Dow Jones Industrials (price index), B: Wal-Mart Stores Incorporated, B: Electricite de France SA, B: US Airways Incorporated, B: Finland Republic of (Government), B: Poland Republic of (Government) and B: Hungary Republic of (Government). Bond portfolio 1 (higher grade): B1, B2, B4, B5; bond portfolio 2 (lower grade): B2, B5, B6.
The results in Figure 2 b) further emphasize the extent of credit risk in terms of shortfall risk, which is considerably higher than in the case of a higher grade bond portfolio, such that the shortfall risk is even underestimated up to almost 11 percentage points for low stock portions and a high guaranteed rate of 2.50% in the considered example.

**Figure 3**: Annual surplus $\gamma$ and stock portion $\alpha$ combinations leading to fair contracts with corresponding shortfall probability $SP$ as a function of stock portion $\alpha$ for various recovery rates $\delta_R$ with guaranteed interest rate $r_g = 1.75$

<table>
<thead>
<tr>
<th>Stock Portion $\alpha$</th>
<th>Annual Surplus Participation $\gamma$</th>
<th>Shortfall Probability $SP$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta_R = 0.35$</td>
<td>$\delta_R = 0.55$</td>
</tr>
<tr>
<td></td>
<td>$\delta_R = 0.75$</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The exposures in the stock portfolio (S) and the bond portfolio (B) are invested in equal proportions. S: DAX 30 (price index), S: FTSE 100 (price index), S: Dow Jones Industrials (price index), B: Wal-Mart Stores Incorporated, B: Electricite de France SA, B: US Airways Incorporated, B: Finland Republic of (Government), B: Poland Republic of (Government) and B: Hungary Republic of (Government). Bond portfolio 1 (higher grade): B, B, B, B, bond portfolio 2 (lower grade): B, B, B, B.
3.3 Further analyses

In addition to the analysis in Figures 1 and 2, we further looked at the impact of the recovery rate as displayed in Figure 3. The results show that this has a major impact, especially for the lower grade portfolio. In particular, for a recovery rate of 75%, the shortfall probability in case of the lower grade bond portfolio becomes almost as low as in case of the higher grade portfolio, as the consequences of a bond’s default is not as severe as in case of a $\delta_R = 35\%$.

Further analyses that still remain to be done comprise an assessment of the net present value from the policyholders’ perspective for different costs of insolvencies. In addition, we plan to compare different risk measures using the mean loss as an alternative to the shortfall probability. In addition, we plan to study the time of default for the different bond portfolios with higher and lower investment grade.

4. Summary

In this paper, the impact of credit risk for insurance companies selling participating life insurance contracts involving cliquet-style guarantees is analyzed. Besides credit risk, we account for equity risk and interest rate risk on the insurers’ asset side and study the interaction between the different risk factors. Regarding the insurer’s asset base, two asset classes are considered: 1) bonds, which are affected by interest rate and credit risk and 2) stocks, which are exposed to equity risk. Using risk-neutral valuation, we calibrate the contract parameters to ensure a fair contract situation for the equityholders and then measure the associated shortfall risk. We thereby compare the impact of different target stock portions that are achieved by an annual rebalancing of the portfolio and study the impact of credit risk in depth by comparing a bond portfolio with higher and lower grade investments.

The numerical results indicate that taking into account credit risk associated with bonds is of great relevance for the risk assessment of life insurance companies. Both, the fair contract parameter and the associated company risk may be considerably misestimated when ignoring the impact of credit risk. In our analysis, we further point out the tradeoff between higher coupon payments offered by lower grade bond portfolios, which thus compensate the investor for taking a higher risk, and an increase in shortfall risk. Our results also show the strong interaction between credit risk, interest rate risk, and equity risk arising from a mixed asset portfolio. In particular, a portfolio comprising only lower grade bonds may imply a higher risk than a portfolio that includes a certain stock portion. This is even true for bond portfolios with higher grade assets, which may still imply a substantial underestimation of an insurer’s short-
fall risk. Hence, given a fair situation from the equityholders’ perspective, shortfall risk may actually first decrease when increasing the stock portion due to a reduction in credit risk before increasing again due to equity risk.

Thus, especially for life insurers that typically invest major portions of their capital in corporate and government bonds, taking into account credit risk and the interaction with equity and interest rate risk is of high relevance in order to conduct an adequate risk assessment and risk management.
REFERENCES


## APPENDIX

### Table A.1: Global corporates average one-year transition rates (%) (see Vazza, Aurora and Kraemer, 2010, p. 27)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>88.21</td>
<td>7.73</td>
<td>0.52</td>
<td>0.06</td>
<td>0.08</td>
<td>0.03</td>
<td>0.06</td>
<td>0.00</td>
<td>3.31</td>
</tr>
<tr>
<td>AA</td>
<td>0.56</td>
<td>86.60</td>
<td>8.10</td>
<td>0.55</td>
<td>0.06</td>
<td>0.09</td>
<td>0.02</td>
<td>0.02</td>
<td>4.00</td>
</tr>
<tr>
<td>A</td>
<td>0.04</td>
<td>1.95</td>
<td>87.05</td>
<td>5.47</td>
<td>0.40</td>
<td>0.16</td>
<td>0.02</td>
<td>0.08</td>
<td>4.83</td>
</tr>
<tr>
<td>BBB</td>
<td>0.01</td>
<td>0.14</td>
<td>3.76</td>
<td>84.16</td>
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<td>0.70</td>
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<tr>
<td>BB</td>
<td>0.02</td>
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<td>0.18</td>
<td>5.17</td>
<td>75.52</td>
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<td>0.79</td>
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<tr>
<td>B</td>
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<td>0.00</td>
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<td>0.31</td>
<td>0.88</td>
<td>11.28</td>
<td>44.98</td>
<td>27.98</td>
<td>14.36</td>
</tr>
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</table>

### Table A.2: Governments average one-year transition rates (%) (see Chambers, Ontko and Beers, 2011, p. 41)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>NR</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>97.78</td>
<td>2.22</td>
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<td>0.00</td>
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<tr>
<td>AA</td>
<td>3.37</td>
<td>93.64</td>
<td>2.25</td>
<td>0.00</td>
<td>0.37</td>
<td>0.37</td>
<td>0.00</td>
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</tr>
<tr>
<td>A</td>
<td>0.00</td>
<td>3.60</td>
<td>92.80</td>
<td>3.60</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
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</tr>
<tr>
<td>BBB</td>
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<td>0.00</td>
<td>6.75</td>
<td>89.03</td>
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<td>0.84</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
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<td>0.00</td>
<td>0.00</td>
<td>6.14</td>
<td>88.06</td>
<td>4.10</td>
<td>1.02</td>
<td>0.68</td>
<td>0.00</td>
</tr>
<tr>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>86.36</td>
<td>3.41</td>
<td>1.89</td>
<td>1.14</td>
</tr>
<tr>
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<td>0.00</td>
<td>31.82</td>
<td>31.82</td>
<td>36.36</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### Table A.3: Global corporates average one-year transition rates derived from Table A.1 accounting for non-rated corporates (NR) (%), rounded values

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>NR</th>
</tr>
</thead>
<tbody>
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<td>7.99</td>
<td>0.54</td>
<td>0.06</td>
<td>0.08</td>
<td>0.03</td>
<td>0.06</td>
<td>0.00</td>
<td>0.00</td>
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<td>90.21</td>
<td>8.44</td>
<td>0.57</td>
<td>0.06</td>
<td>0.09</td>
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<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
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<td>2.05</td>
<td>91.47</td>
<td>5.75</td>
<td>0.42</td>
<td>0.17</td>
<td>0.02</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
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</table>

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17 To obtain row sums equal to one and thus a cumulative distribution function for each row, we adjust individual values in Table A.2.
Table A.4: Governments average one-year transition rates derived from Table A.2 accounting for non-rated governments (NR) (% rounded values)

<table>
<thead>
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<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>C</th>
<th>D</th>
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</thead>
<tbody>
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<td>1.02</td>
<td>0.68</td>
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<td>31.82</td>
<td>31.82</td>
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</tbody>
</table>