Aggregation of market and credit risk capital requirements via integrated scenarios

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1 Introduction

Many financial institutions are exposed to a combination of market and credit (default and rating migration) risks. Measurement and management of these risks is often dealt with using different models and software systems, with each system providing its own standalone measures of risk and capital. To calculate overall measures of risk and capital (accounting for both market and credit risks), firms often combine these standalone measures using a simple correlation matrix based aggregation formula. The limitations of such an approach are well-known (Chen, Kaplin, Levy, & Wang, 2010; McNeil, Kretzschmar, & Kirchner, 2009). In particular, it ignores the effects of any interaction between risks on the institution’s asset and liability values.

A more satisfactory approach would be to generate joint scenarios for market and credit risk factors, thus allowing for their joint impact on the institution’s assets and liabilities. This requires integration of the two different scenario generators, one for market risks and the other for credit risks, so as to create joint scenarios that reflect appropriate levels of dependency between these two different types of risk factor.

In this note, we describe a method for integrating market and credit risk scenarios that have been generated by separate scenario generators. Dependency is captured by extending the credit risk scenario generator to include a small number of important market risk factors. These market risk factors are not used as a replacement for those produced by the market risk scenario generator, but simply used to describe the dependency between credit and market risks. Joint scenarios are created by combining scenarios for individual risks in such a way as to capture key dependencies, using simple reordering algorithms. Such reordering algorithms make use of existing credit and market risk models and software systems. Similar techniques have recently been proposed to capture “wrong way risk” in the assessment of Counterparty Credit Risk and calculation of CVA (Garcia Cespedes, de Juan Herrero, Rosen, & Saunders, 2010).

The rest of the note presents two case studies, describing different ways of applying the technique to a set of market risk scenarios produced by the B&H Economic Scenario Generator and credit risk scenarios produced independently using separate credit risk models. We describe and compare two choices of reordering algorithm, firstly using a single key market risk factor, and then adding a second market risk factor. We conclude with some thoughts on how the technique extends further, and the resulting choices and considerations that are faced by the user.

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1 Here we will refer to credit risk as specifically relating to defaults and migrations. Risk relating to movements in credit spreads will be considered a market risk.
2 Case study - single common factor

To illustrate the method, we consider calculation of economic capital for an asset portfolio consisting of US corporate bonds (50%), US equities (30%), and US real estate (20%). Economic capital will be defined as a one year 99.5% portfolio VaR, and will be estimated using 10,000 scenarios generated using two different scenario generators – one for market risk scenario and another for credit risk scenarios.

Specifically, the B&H Economic Scenario Generator has been used to generate joint scenarios for the following market risk factors:

- S&P 500 index (as a proxy for the equity assets in the portfolio)
- US real estate index (as a proxy for the real estate assets in the portfolio)
- US Government bond yield curve
- US Corporate credit spread curves (Aaa, Aa, A, Baa, Ba, B, Caa ratings)

Independently, a separate credit risk model has been used to generate scenarios for the credit ratings of each bond in the corporate bond portfolio.

The two scenario generators, B&H Economic Scenario Generator and the credit risk model, have been run independently from each other and so we expect to see no dependency between the scenarios produced by them. Exhibit 1 shows a selection of joint scenarios produced by simply pairing the two scenario sets according to the order that they are produced by the two different generators. Exhibit 2 shows a selection of sample Spearman’s rank correlations. Green coloured scatterplots and correlations indicate results produced by the B&H Economic Scenario Generator in isolation, while blue coloured scatterplots and correlations indicate results induced by joining the two scenario sets.

Exhibit 1
Sample scatter plots (1,000 representative scenarios)
Note that since the scenario sets are independent, the sample rank correlations between credit losses and all market risk factors are zero (to within statistical error). However, we may believe that there is a dependency between credit ratings produced by the credit risk model and the market risk factors produced by the ESG that is not captured by simply joining the scenario sets together according to the order that they are produced by the two different scenario generators. What we would like to do is to generate joint market and credit risk scenarios which retain the existing distributional features (marginal distributions and dependencies between different market risk factors), while additionally reflecting our view of the dependency between market and credit risks.

Here we will consider an approach whereby existing scenarios for all individual risk factors are retained, thereby retaining existing marginal distribution assumptions. We will however combine scenarios for individual risk factors in such a way as to change certain dependencies between them. In particular we will combine market and credit risk scenarios so as to reflect target dependency relationships between these.

We assume that, within the credit risk model, randomness in corporate bond credit ratings comes about due to exposure of bond issuers’ credit quality to random credit risk factors. Any dependency between corporate bond portfolio values and market risk factors fundamentally arises from dependency between the credit model’s risk factors and the ESG’s market risk factors. The method we will use to capture this dependency is to extend the credit risk model to include some market risk factors in common with the ESG. These common market risk factors will used to provide the link between the credit and market risk scenario sets.

For example, within Moody’s Analytics RiskFrontier, corporate bond credit ratings are driven by a set of credit risk factors with a correlation structure defined by the GCorr model. Recent research by Moody’s Analytics shows how the GCorr model can be expanded to include additional risk factors, for example market risk factors (Pospisil, Patel, & Levy, 2012). The expanded GCorr model can be calibrated by estimating correlations between historical data for the additional market risk factors and existing credit risk factors. The addition of market risk factors within GCorr can be used in several applications including stress testing and reverse stress testing. Here we will consider the use of additional market risk factors as a link to facilitate integration with a market risk scenario generator such as the B&H Economic Scenario Generator.

To illustrate the technique, we have expanded the credit model in our example to include a single additional market risk factor – the S&P500 index. Of all the market risk factors considered here, this is the one that we might expect to have the strongest level of dependency with corporate bond credit quality, given the fundamental link between corporate debt and equity and given that our sample portfolio is largely exposed to US reference entities.

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2 Credit losses here account for migrations and defaults but assume interest-rates and spreads are static.
With this extension, the credit risk scenarios now contain a factor representing the S&P 500 index in addition to the existing credit risk factors. Now, our ESG scenarios also contain the S&P 500 index. What we would like to do is use the ESG’s S&P 500 index scenarios, but use the credit model’s S&P 500 factor to define the dependency structure between this and credit risk factors. This can be achieved using the following algorithm:

1) Order the expanded credit risk scenarios according to the level of the S&P 500 factor (as generated by the credit risk model); Order the ESG scenarios according to the level of the S&P 500 (as generated by the ESG).
2) Align the reordered credit risk and ESG scenarios.
3) Discard the credit risk model’s scenarios for the S&P 500.

On application of this algorithm, we obtain a joint scenario set with the following properties:

- Scenarios for each individual risk factor in isolation are unchanged and so marginal distributions of all risk factors are unchanged. Note in particular that we retain the ESG scenarios for the S&P 500. The expanded credit risk model’s scenarios for the S&P 500 are not used to describe marginal distributions, just dependency.
- Joint scenarios for market risk factors are unchanged and so dependencies between market risk factors are unchanged.
- The process of reordering scenarios means that the dependency between credit scenarios and S&P 500 is the same as that produced by the expanded credit risk model, in the sense that their ranks have the same joint distribution\(^3\).
- Dependencies between credit risk and other market risk factors are induced rather than explicitly controlled. For example, the dependency between the US real estate index and the credit loss arises as a by-product of (1) the assumed dependency between the US real estate index and the S&P 500 (controlled within the ESG), and (2) the assumed dependency between the S&P 500 and credit risk factors (controlled within the expanded credit model).

Exhibit 3 shows a selection of joint scenarios produced by this algorithm, while Exhibit 4 shows sample Spearman’s rank correlations.

\(^3\) Technically speaking, they have the same empirical copula.
Exhibit 3
Sample scatter plots (1,000 representative scenarios)

Exhibit 4
Sample rank correlations (Spearman’s ρ)

As before, green coloured scatterplots and correlations indicate dependencies controlled within the B&H Economic Scenario Generator. Comparison with Exhibits 1 and 2 indicate that these have not changed during the process of reordering.

The red coloured scatterplot and correlation indicate the dependency between the S&P 500 and credit losses, controlled within the credit risk model. As expected the correlation here is negative (and relatively strong). This reflects the historically observed correlation between S&P 500 returns and the risk factors driving corporate credit quality, and is consistent with the idea that corporate debt and equity are fundamentally linked.

As before, blue coloured scatterplots and correlations indicate results induced by joining the two scenario sets. While a simple pairing of scenarios according to the order in which they are produced by the two different generators gives rise to zero correlation between the bond portfolio value and all market risk factors (recall Exhibit 2). These induced correlations are now non-zero due to the process of pairing scenarios in a systematic way. For example, the rank correlation between credit losses and the US real estate index is negative (-0.17) reflecting the strong negative rank correlation (-0.56) between credit losses and the S&P 500 arising from the expanded credit risk model, plus the positive correlation (0.31) between the S&P 500 and the US real estate index arising from the ESG.

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*Scatterplots shown here are different due to a different sample of 1,000 scenarios being shown.

*Note that -0.17 ≈ -0.56 x 0.31
2.1 Calculation of economic capital

Given a set of joint scenarios, we can proceed to estimate economic capital for the portfolio. This will be defined as a one year 99.5% VaR, after subtracting off the expected loss i.e.:

$$EC_{total} = q_{99.5\%}(Loss(Market, Credit)) - E(Loss(Market, Credit))$$

where $q_{99.5\%}$ denotes the 99.5% quantile. $Loss(Market, Credit)$ denotes the portfolio loss under a combination of credit and market risks.

Exhibit 5 shows estimated economic capital calculated using the joint scenario set, plus standalone economic capital based on credit and market risks in isolation.

<table>
<thead>
<tr>
<th>Economic Capital</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint scenarios ($EC_{total}$)</td>
<td>0.2294</td>
</tr>
<tr>
<td>Market scenarios only ($EC_{market}$)</td>
<td>0.0283</td>
</tr>
<tr>
<td>Credit scenarios only ($EC_{credit}$)</td>
<td>0.2195</td>
</tr>
</tbody>
</table>

In the absence of a joint scenario set we can only approximate the total economic capital by combining standalone capital estimates, for example by using a simple aggregation formula:

$$EC_{total} \approx \sqrt{EC_{market}^2 + EC_{credit}^2 + 2\rho_{market,credit}EC_{market}EC_{credit}}$$

where:

- $EC_{market} = q_{99.5\%}(Loss(Market)) - E(Loss(Market))$ is the standalone economic capital assuming no credit risk (i.e. assuming no defaults or transitions)
- $EC_{credit} = q_{99.5\%}(Loss(Credit)) - E(Loss(Credit))$ is the standalone economic capital assuming no market risk (i.e. just defaults and transitions).
- $\rho_{market,credit}$ is a correlation parameter describing the dependency between market and credit risks.

The challenge in using such a formula is to choose an appropriate value for the correlation parameter, $\rho_{market,credit}$. This parameter is highly sensitive to the composition of the asset (or more generally asset-liability) portfolio in question and cannot be readily identified without access to the ‘true’ value for the total economic capital (in which case no approximation is required). Indeed, $\rho_{market,credit}$ might be best thought of as an ‘implied’ correlation that is required in order for the approximate aggregation formula to agree with the exact answer. In the current example, assuming that our joint scenarios appropriately reflect all important dependencies between underlying risk factors, we find that the implied credit-market correlation parameter is $\rho_{market,credit} = +0.29$. Other portfolio choices will result in different implied correlations.
3 Case study - multiple common factors

In the above example, we combined credit and market risk scenarios using a single common factor - the S&P 500 index. Since a single factor is used to define the reordering, all market risk factors are reordered together. The resulting reordered scenarios have the nice property that joint scenarios for market risk factors are unchanged and so dependencies between market risk factors are unchanged.

The potential drawback of such an approach is that it imposes a relatively strong assumption about the correlation structure between credit and market risk factors. In the above example, though the credit vs S&P 500 dependency is explicitly targeted (within the expanded credit risk model), the dependency between credit losses and other market risk factors is induced as a by-product of other dependency assumptions. What if we believe that there are other common factors linking credit and market risks?

As a second example, we consider an asset portfolio consisting of US equities (closely tracking the S&P 500), UK equities (closely tracking the FTSE 100), US corporate bonds and UK corporate bonds. We assume that our analysis requires us to be able to generate joint scenarios for each of these separate sub-portfolios.

To model this portfolio, the B&H Economic Scenario Generator has been used to generate 10,000 joint scenarios for the following market risk factors:

- S&P 500 index
- FTSE 100 index
- US Government bond yield curve
- US Corporate credit spread curves
- UK Government bond yield curve
- UK Corporate credit spread curves

Separately, Moody’s Analytics RiskFrontier has been used to generate scenarios for the credit ratings of each bond in the two corporate bond portfolios.

Now, since the portfolio contains both US and UK corporate bonds, it is natural to ask which common risk factor should be used to link credit and market risk scenarios: the S&P 500, the FTSE 100, or something else (e.g. a global equity portfolio containing a mix of US and UK equities)?

Suppose we choose the S&P 500, expand RiskFrontier to include this, and reorder market risk scenarios accordingly. Exhibit 6 shows selected sample rank correlations. In particular, in this example, we find that the correlation between the credit loss on the US corporate bond portfolio and the S&P 500 is -0.73 while the correlation between the credit loss on the US corporate bond portfolio and the FTSE 100 is -0.47. These correlations arise due to the assumed correlations between credit risk factors and the S&P 500 within the expanded credit risk model, reflecting historically observed correlations between these factors, and intuitively have the expected sign (negative), size (both relatively large) and order (the S&P 500 being more strongly correlated with the US bond portfolio than the UK bond portfolio).

Exhibit 6
Sample rank correlations (Spearman’s ρ)
Now, if we look at the induced correlations between the two corporate bond portfolios and the FTSE 100, these have the expected sign (negative) but the sizes appear to be in the wrong order compared to each other. In particular, the FTSE 100 is more strongly correlated with US credit losses than with UK credit losses, and UK credit losses are more strongly correlated with the S&P 500 than with the FTSE 100. These apparent anomalies are a consequence of the fact that the only factor linking the FTSE 100 scenarios with the credit scenarios is the S&P 500. There are no ‘UK specific’ common factors linking the two scenario sets.

Suppose we now expand RiskFrontier to include two additional market risk factors: the S&P 500 index, and the FTSE 100. The addition of the FTSE 100 risk factor gives us an additional common factor that can be used to align the credit risk and market risk scenarios. The idea is to reorder the ESG’s scenarios for the S&P 500 so that they align with RiskFrontier’s scenarios for the S&P 500, and separately reorder the ESG’s scenarios for the ESG scenarios for the FTSE 100 so that they align with RiskFrontier’s scenarios for the FTSE 100.

However, in doing this we are faced with a couple of complications compared to the single factor case:

- Firstly, since we are separately reordering the ESG’s scenarios for the S&P 500 and the FTSE 100, we change the joint scenarios for these two risk factors. Though the marginal scenarios for the S&P 500 and the FTSE 100 are unchanged, their dependency changes (to agree with that assumed within the expanded credit model). Note however that the impact of this change can be limited by calibrating the expanded credit model so that the S&P 500 vs FTSE 100 correlation agrees with that produced by the ESG.

- We have to decide how to order ESG risk factors other than the S&P 500 and the FTSE 100. In this case, we need to group ESG market risk factors into two sets: one ranked by the value of the S&P 500 and the other ranked by the FTSE 100. The choice of grouping will determine which ESG dependencies are retained during the process of reordering scenarios.

As an example, one simple extension of the previous algorithm is to group all ESG risk factors according to the value of the S&P 500, except the FTSE 100 which will be reordered separately (according to the level of the FTSE 100 as produced by the credit model).

On application of this algorithm, we obtain a joint scenario set with the following properties:

- As before, scenarios for each individual risk factor in isolation are unchanged and so marginal distributions of all risk factors are unchanged. Note in particular that we retain the ESG scenarios for the S&P 500 and the FTSE 100.

- Joint scenarios for credit risk factors (e.g. US credit losses vs UK credit losses) are unchanged.

- Joint scenarios for market risk factors within each group are unchanged. In the current example, joint scenarios for all market risk factors except the FTSE 100 are unchanged. However, joint scenarios for market risk factors in different groups are changed. In the current example, the dependency between the S&P 500 and the FTSE 100 is defined by the credit risk model, not the ESG.

- The process of reordering scenarios means that the dependency between the credit risk scenarios and S&P 500 is the same as that produced by the expanded credit risk model, in the sense that their ranks have the same joint distribution. Similarly, the dependency between the credit risk scenarios and the FTSE 100 is the same as that produced by the expanded credit risk model.
• Dependencies between credit risk scenarios and other market risk factors (except the S&P 500 and FTSE 100) are induced rather than explicitly controlled.

Exhibit 7 shows sample Spearman’s rank correlations after application of this algorithm.

**Exhibit 7**
Sample rank correlations (Spearman’s ρ)

<table>
<thead>
<tr>
<th>Credit loss (US)</th>
<th>Credit loss (UK)</th>
<th>S&amp;P500</th>
<th>FTSE100</th>
<th>US 10 Year</th>
<th>UK 10 Year</th>
<th>US A 10-year credit spread</th>
<th>UK A 10-year credit spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.66</td>
<td>-0.73</td>
<td>-0.67</td>
<td>0.11</td>
<td>0.10</td>
<td>0.23</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>-0.47</td>
<td>-0.67</td>
<td>0.08</td>
<td>0.07</td>
<td>0.16</td>
<td>0.16</td>
<td>-0.31</td>
<td>-0.31</td>
</tr>
<tr>
<td>0.69</td>
<td>-0.13</td>
<td>-0.13</td>
<td>-0.31</td>
<td>-0.31</td>
<td>-0.23</td>
<td>-0.22</td>
<td>-0.22</td>
</tr>
<tr>
<td>-0.09</td>
<td>-0.09</td>
<td>-0.09</td>
<td>-0.23</td>
<td>-0.22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.64</td>
<td>-0.19</td>
<td>0.00</td>
<td>0.01</td>
<td>-0.17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.23</td>
<td>0.23</td>
<td>0.77</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comparing with Exhibit 6 shows that the addition of the FTSE 100 as a linking factor between credit and market risk scenarios sets has enabled us to explicitly target correlations between the FTSE 100 and credit losses (within the expanded credit risk model) rather than having these induced, thus avoiding associated anomalies that might otherwise arise.

We also note that the S&P 500 vs FTSE 100 dependency is here controlled within the expanded credit risk model rather than the ESG. However, here we have calibrated RiskFrontier to be consistent with the S&P 500 vs FTSE 100 rank correlation produced by the ESG and so the process of reordering has not significantly changed this correlation.

Since the FTSE 100 scenarios have been reordered separately from all other market risk variables, we see that the resulting correlations changed. For example, the correlation between the FTSE 100 and the UK A 10-year credit spread has fallen (in absolute terms) from -0.31 to -0.22. This is a relatively small change in absolute terms, and may be judged reasonable depending on the sensitivity of results to this particular assumption and depending on the user’s confidence in the accuracy of this particular correlation estimate. If this change is judged unreasonable, alternative ordering schemes may be required.

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*It is worth noting that dependency is not completely specified by rank correlation, and other aspects of dependency may change during the process of reordering. In particular, the dependency structure assumed within RiskFrontier is a Gauss copula, while the ESG may exhibit elevated tail dependency compared to a Gauss copula, depending on the choice of ESG model.*
4 Final thoughts

In this note we have described how we can combine separate market and credit risk scenarios so as to achieve dependency between these two different types of risk factor, via a process of reordering scenarios. This technique makes use of existing credit and market risk scenario generators and offers a relatively simple and pragmatic way of creating joint market and credit risk scenarios from these. Such joint scenarios allow calculation of risk and capital measures which jointly account for both market and credit risk in a way that captures fundamental relationships between these risks, rather than crudely approximating this via a correlation matrix based aggregation formula applied to standalone risks.

Here we have considered two different ways of implementing the technique, but there are many other ways in which we could reorder market and credit risk scenarios so as to reflect other dependency assumptions, depending on our choice of key risk factors included in the extended credit risk scenario set and depending on how risk factors are grouped for the purpose of reordering. Though the reordering algorithms themselves may be relatively simple to understand, communicate and implement, the details of how these are specified requires careful consideration by the user. Also, since certain dependencies are induced rather than specified directly, it is also important to validate the output of the reordering process.

In general, practical implementation of this method involves a number of stages:

Stage 1: The dependency model is specified by two choices:
- Which ‘key’ market risk factors will be used to control dependency with credit risk scenarios, depending on the user’s risk exposures and judgement about where the most important dependencies are likely to arise.
- How to group market risk factors for the process of reordering.

Stage 2: Extend and calibrate the credit risk scenario generator to include these key market risk factors.

Stage 3: Group and reorder scenarios to achieve targets.

Stage 4: Analyse joint scenarios and assess appropriateness of resulting dependency relationships (both explicitly targeted and induced). This final step provides a check on the appropriateness of choices made in stage 1. Depending on the results of this validation, we may return to stage 1 (e.g. adding further key market risk factors) and iterate until resulting scenarios are judged reasonable.

Taken to the extreme, all market risk factors of interest could be included in the extended credit risk scenario generator (so that we have a large number of small groups) allowing a very granular, but complex, specification of dependency between credit and market risks. The other extreme considered here, whereby one or two market risk factors are included in the extended credit risk scenario generator (so that we have a small number of large groups) gives less direct control over credit vs. market risk dependencies, but nevertheless may give a reasonable description of these dependencies using a relatively simple model. As ever in financial risk modelling, the modeller must seek out a parsimonious choice which captures the key risks in the simplest possible way.
References


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If you would like to discuss any issues raised in this paper, please contact us at: http://www.barrhibb.com/contact