# Market Consistent Valuation of Cash Balance Liabilities

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April 26, 2013

**Abstract**

Cash balance (CB) pension benefits are accumulated at guaranteed crediting rates, usually based on interest rates on government securities. Viewed as a financial liability, the benefit is a form of interest rate derivative, which can be valued using financial models and theory. In this paper, we consider the value using the Hull White interest rate model, and explore the risks associated with different crediting rate choices.

The results indicate that the CB liability is rather larger than emerges from traditional actuarial methods, and, further, that the most popular choices for crediting rates are also the most risky, from a perspective of volatility in the liability value.

## 1 Introduction

Cash Balance (CB) pension plans play a highly significant role in US employer-sponsored pension provision. According to the US Department of Labor Employee Benefits Security Administration (EBSA, 2012), in 2010 there were over 12 million participants in Cash Balance pension plans, representing around 9% of the total number of participants in any employer sponsored plan. The Cash Balance design started to become popular in the US during the late 1990's, when several large employers shifted their final salary DB plans to Cash Balance plans.

The attraction of Cash Balance plans for employers is very similar to the attraction of Defined Contribution plans (DC). Both ostensibly involve the payment of employer/employee contributions into a fund for each member; the fund accumulates over time to create a lump sum benefit at the member's retirement. The benefit may be annuitized, but the rates for conversion are determined at retirement. The difference between the CB plan and the DC plan is that the DC contributions are invested in assets, and the interest on the assets is passed back to the employee. In the CB plan, the interest earned on the employee's account is fixed in advance, either in absolute terms, or in terms of a market rate applying at each crediting date. So, for example, the employer may specify that all member accounts will earn 6% per year until retirement, or that the interest rate applied at each year end will be the published yield on 5-year government bonds, with an additional margin of, say, 0.25%. This specification of the crediting rate results in a benefit that is classified under US regulation as a Defined Benefit plan, not a Defined Contribution plan. This regulatory situation has been a motivating factor in some transitions from traditional DB to Cash Balance, as the requirements for changing from one type of DB plan to another are much less onerous in many cases, than switching from DB to DC, with apparently similar benefits to the employer.

Much has been written about Cash Balance plans, in the financial and accounting literature, and in actuarial notes and monographs. In the 1990s and early 2000s, the literature focussed on the impact on employers and plan members of the decision to move from traditional DB to CB. Kopp and Sher (1998) compared the benefits from traditional DB and CB plans, and concluded that CB was more favourable to younger participants with shorter service, while traditional DB provided better benefits for the older members. Clark and Munzenmaier (2001) subsequently reported on a number of case studies of early traditional DB to CB transitions. Coronado and Copeland (2003), who are very positive about the CB design, reviewed the case for switching from traditional DB to CB or DC, and proposed that the tax regulations would encourage overfunded plans to switch to CB, and underfunded plans to switch to DC. However, there have been significant CB transitions from underfunded plans, so this does not fully explain the phenomenon. Thomas and Williams (2009) review the costs and benefits to plan members, with a critique of the accounting approach to the pension decision, which, in their view, fails to recognize employees as stakeholders. Clark and Schieber(2000) also review the 'winners and losers' in the transition to CB, addressing some controversial techniques used for transferring benefits.

Gold (2001) considers the CB design from the perspective of maximizing shareholder value. His conclusion, that the optimal design should be based on equity yields (and that the optimal asset allocation should include no equities) is derived from tax arbitrage between fund and sponsor. Gold's paper is not concerned with valuation or risk management, although he does make some pertinent observations on valuation; notably, that a vested compensation credit of 5% of pay cannot cost less than 5% of pay, although it may appear to do so through the filter of a subjective valuation.

Actuarial valuation of CB plans is addressed by McMonagle(2001), Lowman(2000) and Murphy (2001), with different conclusions. Both Murphy and McMonagle comment on the problems arising from applying valuation techniques designed for traditional DB plans to the CB design. Murphy, for example, shows that traditional actuarial methods generate a loss on termination – that is, the account value, which is the termination benefit for a vested participant, is generally greater than the actuarial valuation. This should be alarming; it is not consistent with accounting principles. However, Lowman sees the issue differently, proposing that a valuation method that generates values higher than the account value are the problem, and that a lower value is preferred, indicating that the value of benefits to members who continue participation is less than the value to members who withdraw their funds and self-manage. In practice, this could lead to perverse incentives, as well as unanticipated losses (for example, on large layoffs) and potential solvency problems for the sponsor. Brown et al(2001) use a similar approach to ours, coming from the perspective of financial analysis. Their results are less dramatic than ours, as they considered the value based on the economic conditions of the time, and did not look at the sensitivity to time varying factors in the economy.

Our approach will not take the corporate finance approach, nor the traditional actuarial approach. We view the CB benefit as a financial liability of the sponsoring employer. This benefit can be analyzed using the models and paradigms of financial economics and risk management. In this context, the valuation of the liability would be the same whether the plan sponsor retains the assets/liabilities, or whether all payments are transferred to a third party, apart from any difference in default risk. The result is therefore an objective, market consistent valuation of the CB liability. It is not, currently, consistent with the actuarial valuation techniques in use, but we contend that it is a better measure of cost and risk, and that developments in market consistent accounting may lead to the current actuarial approaches being superseded. In addition, we believe that a market consistent valuation should be a benchmark for reporting purposes, even if the traditional actuarial approach is still used for funding.

# 2 Framework, assumptions, notation

## 2.1 The CB benefit

We develop valuation formulae, and illustrate the method with numerical results for a simple but reasonably representative CB benefit. We value the pension obligation of a hypothetical participant whose benefit is fully vested. We will ignore (for the moment) mortality and other demographic considerations.

To value a contingent payment using market consistent, or fair value approach, we evaluate the expected discounted value of the payment, using a market consistent, arbitrage free model for interest rates. The objective is a valuation that is equal to the cost of replicating the payouts using market instruments. Even though we may value the liabilities using expected discounted values, the underlying theory is not really probabilistic, it is market based. In particular, the sponsor's intentions with respect to the investment of the plan assets is not pertinent to the market based valuation.

We start from the Traditional Unit Credit (TUC) valuation principle, which means that we value the contributions accrued up to the valuation date, allowing for future credited interest, but with no allowance for future contributions or salary increases. Unit credit methods are accruals-based methods. We define the accrued benefit at $t$, assuming a retirement date at $T > t$, to be the benefit based on the participant's account value at $t$, with interest at the guaranteed crediting rate up to $T$. We make no adjustment for future salary increases, as the benefit funded by past contributions is not affected by future salary increases – that will only change the future contributions, which are not yet part of the plan liabilities. There are other ways to allocate the ultimate benefit to accrual years, but this to us is the most natural, and is consistent with Enderle et al (2006), who conclude that the TUC valuation is the only approach consistent with economic principles. Our accruals basis is also consistent with the conclusions of Chen and Hardy (2009) which developed a market consistent valuation formula for a floor offset pension, which is another example of a hybrid benefit design.

Let $F_t$ denote the participant's account value at $t$. The account value is the total of the past contributions, accumulated to $t$ at the appropriate credited interest rates. The valuation date is $t = 0$, at which time the notional account value is $F_0$.

Assuming, first, annual interest rate crediting, let $i^c(t)$ denote the credited interest rate declared at $t$ for the year $t$ to $t + 1$. The final lump sum benefit under for the plan participant, at her retirement date $T$, say, is

$$B_T = F_0 \prod_{t=0}^{T-1} (1 + i^c(t))$$

Assuming continuous crediting, at a rate at $t$ of $r^c(t)$ per year, compounded continuously, we have

$$B_T = F_0 \, e^{\int_0^T r^c(t)dt} .$$

The frequency of interest crediting varies between plans, with annual crediting being most common (58% of plans, Hill et al (2010)), and with monthly, quarterly and daily crediting also commonly utilized. It is convenient for our illustrative calculations in the following sections to work with continuous crediting, but it is straightforward to adapt the results for less frequent updating. For the rest of this paper, we use continuously compounded rates for crediting and discount, unless specified otherwise.

The specification of the crediting rate varies in practice. According to Hill *et al* (2010), 7% of CB plans offer a fixed crediting rate, with values of between 3% and 8% per year. More typically, $r^c_t$ is a market dependent random variable. The most common design is to use the quoted par yield on 30-year Treasury Bonds (used by 41% of CB plans, Hill et al, 2010). The next most common is to set $r^c(t)$ at the 1-year treasury bond yield at $t$, with an additional margin of (typically) 1% (used by 19% of CB plans). Treasury bond yields at other durations, usually with additional margins, account for a further 20% of CB plans. The remainder may use a CPI rate, or an equity based rate, or a discretionary rate. Most rates are based on the 'safe harbor' list from IRS(1998). In this paper we consider fixed and treasury-bond crediting rates

Where the crediting rate is constant, $B_T$ is also constant. Where the crediting rates depend on future market rates, $r^c(t)$ is a random process for $t > 0$, and hence $B_T$ is a random variable.

## 2.2 The Yield Curve

It is often important for CB valuation to model the yield curve of interest rates through time. We denote by $r(t)$ the continuously compounded, risk free rate of interest at $t$, (this could be interpreted as the annualized rate on overnight government securities) so $\{r(t)\}$ is a stochastic process. At the valuation date, time $t = 0$, say, we know $r(0)$, and we also observe a yield curve of spot rates for different durations. Let $r_n(t)$ denote the $n-$year spot rate observed at $t$ (continuously compounded). Then, from the initial yield curve, we observe $r_k(0)$ for different durations $k$, and $r(0) = \lim_{k\to 0} r_k(0)$.

We will also utilize zero coupon bond prices . Let $p(t,T)$ denote the price at $t$ of \$1 due at $T$. These prices are observable at $t$, and are used to construct the yield curve. Recall (see, eg Dickson et al (2009)) that for $t \leq T$,

$$p(t, t+k) = e^{-k\, r_k(t)}\,.$$

The zero coupon bond price, $p(t,t+k)$ gives the market consistent discount factor at $t$ for a $k$-year investment. That is, the present value at $t$ of \$1 due at $t+k$ is $p(t,t+k)$.

We will assume an arbitrage free model for the future term structure of interest rates. For any such model, the expected discounted value at time $t = 0$ of of a fixed payment due at $t = T$, say, must match the known market value of that payment. That is, from the zero coupon bond rates, at time $t = 0$ we observe that the present value of \$1 due at $T$ is $p(0,T)$ by replication – that is, an investment in the zero coupon bond of $\$p(0,T)$ will exactly meet the \$1 due at $T$. By matching this to the expected discounted value of \$1 due at $T$, allowing for the unknown path for the future force of interest $r(s)$, we have

$$p(0,T) = E_0^Q\left[e^{-\int_0^T r(s)ds}\right]. \tag{1}$$

We use the $Q$ superscript to indicate that this is a risk neutral valuation, and the 0 subscript to indicate that we are taking values at time 0.

Forward rates at $t$ can be derived from the curve of zero coupon bond prices or the spot rate curve. The continuously compounded forward rate contracted at $t$ applying at time $t+k$ is $f(t,t+k)$, where

$$p(t,t+k) = e^{-\int_t^{t+k} f(t,s)\,ds} \tag{2}$$

$$\Rightarrow f(t,t+k) = -\frac{d}{dk}\log p(t,t+k) = -\frac{1}{p(t,t+k)}\,\frac{d}{dk}p(t,t+k). \tag{3}$$

At any time $t$, the market yield curve is known, and may be expressed in terms of the zero coupon bond prices, the spot rates or the forward rates. That is, at $t$, for all $k > 0$, we observe any of $p(t,t+k)$, $r_k(t)$ and $f(t,t+k)$, and

$$p(t,t+k) = e^{-k\, r_k(t)} = e^{-\int_0^k f(t,t+u)du} = E_t^Q\left[e^{-\int_t^T r(s)ds}\right]$$

Note also that $r(t) = f(t,t)$.

## 2.3 The valuation formula

The value at $t = 0$ of the (random) payoff of $B_T$ at $T$ is

$$_0V = E^Q \left[ B_T \, e^{-\int_0^T r(t)dt} \right] .$$

Both $B_T$ and $r(t)$ are random variables in this expectation, and they are dependent, in general. Assuming continuous interest crediting, we can write the valuation formula as

$$_0V = E^Q \left[ F_0 \, e^{\int_0^T (r^c(t) - r(t))dt} \right] . \tag{4}$$

This demonstrates that the key factor in the cost of the CB benefit is the relationship between the crediting rate and the risk-free discount rate. In the general case we will need a joint model for $r^c(t)$ and $r(t)$ to determine the market consistent benefit value. However, there are two cases where we are able to determine the value of the cash benefit without specifying a model, and we consider these in the first part of the Section 3.

In the numerical illustrations in the following sections we will report the valuation factors, which we define as

$$v(0,T) = E^Q \left[ e^{\int_0^T (r^c(t) - r(t))dt} \right] . \tag{5}$$

The valuation factor $v(u, u+T)$ is the market consistent value at $u$ of a CB benefit due at $u+T$, expressed per \$1 in the participant's hypothetical account balance at $u$.

# 3 Valuation results

## 3.1 Fixed crediting rate

Suppose that $r^c(t)$ is constant – for example, say $r^c(t) = 0.05$, compounded continuously, for all $t$, and that the retirement date for the participant whose benefit we are valuing is in 20-years. Then $B_{20} = F_0 e^{20\,(0.05)} = 2.7183\, F_0$ is not a random variable, and from equations (1) and (4),

$$_0V = E^Q \left[ 2.7183\, F_0 \, e^{-\int_0^{20} r(t)dt} \right] = 2.7183\, F_0 \, p(0,20) \, .$$

Recall that $p(0,20)$ is the price at the valuation date of a 20-year zero-coupon bond, which is available from published market data. For illustration, suppose that the yield on 20-year zero coupon bonds at the valuation date is, say, 3.5% per year compounded continuously, then $v(0, 20) = 1.35$, which means that the valuation liability is 135% of the participant's account balance at the valuation date.

### 3.2 Crediting rate $r(s)$ plus a margin

The second case is where crediting is continuous (daily or weekly, in practical terms), and the crediting rate each day is based on the same short term treasury rates that are used to discount the future cash flow. For example, suppose the crediting rate is

$$r^c(t) = r(t) + m$$

where $r(t)$, as defined above, is the instantaneous rate of interest on government bonds, and $m$ is a pre-specified margin – around 1.75% would be consistent with the range of practice. In this case, the valuation formula (4) becomes

$$\begin{aligned} {}_0V &= E^Q\left[F_0\, e^{\int_0^T r(t)+m-r(t)dt}\right] \\ &= F_0\, e^{mT} \end{aligned}$$

Assuming a 20-year horizon to retirement, and a margin of $m = 1.75\%$ for the crediting rate over the government short-term rate, the valuation factor in this case is $v(0, T) = e^{mT} = 1.41$, which means that the value of the CB benefit in this case would be 141% of the participant's account balance at the valuation date.

In practice, this combination of daily crediting rates, based on daily government rates, does not appear to be a common system for the CB benefit. However, the results will be similar, for example, for quarterly crediting based on 3-month T-Bill rates, which is a combination that is utilized by some plans, with a typical margin of around 1.75%.

### 3.3 Crediting rate based on $k$-year spot rates

In cases other than the two described above, we must specify a market consistent model of the term structure of interest rates to value the benefit using equation (4). We use the one-factor Vasicek model for interest rates (Vasicek, 1977), as extended in Hull and White (1990). This model is reasonably tractable, and using the Hull-White version, can be fitted to the term structure of interest rates at the valuation date to ensure that the process for $r(t)$ is consistent with the starting term structure. This model has been used in a number of actuarial applications, including those described in Boyle and Hardy (2003), Iyengar and Ma (2009) and Jorgenson and Linneman (2012). In future work we will consider the effect of using a two factor model, which allows for more variation in the spread between long and short rates, as well as variability in the level of the projected yield curves.

We describe the one factor model briefly here. See, for example, Bjork(2009) or Brigo and Mercurio (2006) for more details. We assume the following stochastic process for the future short term rates on government bonds, continuously compounded:

$$dr(t) = a\,(\theta(t) - r(t))\,dt + \sigma\,dW_t$$

where $a$ and $\sigma$ are model parameters, $\theta(t)$ is a deterministic function of $t$ derived from the starting yield curve; $dW_t$ is a standard Wiener process.

Under this model the price at $t$ of a zero coupon bond maturing at $t+k$ can be written as

$$p(t, t+k) = E_t^Q\left[e^{-\int_t^{t+k} r(s)\,ds}\right] = e^{A(t,t+k) - B(t,t+k)\,r(t)} \tag{6}$$

where $A(t,T)$ and $B(t,t+k)$ are deterministic functions:

$$\begin{aligned} B(t,t+k) &= \frac{1-e^{-a\,k}}{a} \\ A(t,t+k) &= \log\frac{p(0,t+k)}{p(0,t)} + f(0,t)\,B(t,t+k) - \frac{\sigma^2}{4a}\,B(t,t+k)^2\,(1-e^{-2at}) \end{aligned}$$

The terms $p(0,t)$, $p(0,t+k)$ and $f(0,t)$ are all taken from the yield curve information at $t=0$. Note that $B(t,t+k)$ does not depend on $t$.

As we have demonstrated in equation (4), the value of the CB benefit depends on the relationship between the crediting interest rate process $r^c(t)$ and the instantaneous market rate process $r(t)$. The most common crediting rate is the par yield on 30-year treasury bonds, with no additional margin. Using stochastic simulation, we can simulate the joint process for $r(t)$ and for the 30-year par yields, and we demonstrate this in Section 3.4 below. However, if the crediting rate is based on the $k$-year spot rate, rather than the $k$-year par yield, we can derive analytic results for the benefit valuation. This is valuable as an approximation to the par-yield case, and also as it offers some insight into the valuation results .

The $k$ year spot rate at $t$, $r_k(t)$, is related to the $k$-year zero-coupon bond price at $t$ as

$$\begin{aligned}
p(t, t+k) &= e^{-k\, r_k(t)} \\
&\Rightarrow e^{A(t,t+k)-B(t,t+k)r(t)} = e^{-k\, r_k(t)} \\
&\Rightarrow r_k(t) = \frac{B(t,t+k)\, r(t) - A(t,t+k)}{k}
\end{aligned}$$

Now, suppose that the crediting rate is

$$r^c(t) = r_k(t) + m \tag{7}$$

Then equation (4) gives the valuation formula

$$\begin{aligned}
{}_0V &= E_0^Q \left[ \exp\left( \int_0^T r_k(t) + m - r(t)\, dt \right) \right] &\quad (8) \\
&= E_0^Q \left[ \exp\left( \int_0^T \frac{B(t,t+k)\, r(t) - A(t,t+k)}{k} + m - r(t)\, dt \right) \right] &\quad (9)
\end{aligned}$$

The only random terms in the expectation are those involving $r(t)$, so we can express the valuation formula as

$${}_0V = \exp(mT)\, \exp\left( \int_0^T \frac{A(t,t+k)}{k}\, dt \right) E_0^Q \left[ \exp\left\{ - \int_0^T \gamma\, r(t)\, dt \right\} \right] \tag{10}$$

where

$$\gamma = \left( 1 - \frac{B(t,t+k)}{k} \right) = 1 - \left( \frac{1 - e^{-ak}}{ak} \right) \tag{11}$$

which, conveniently, is not a function of $t$.

For the second term in equation (10) we need

$$\begin{aligned}
\int_0^T \frac{A(t,t+k)}{k} dt = &\int_0^T f(0,t) B(t,t+k)\, dt + \int_0^T \log \frac{p(0,t+k)}{p(0,t)}\, dt \\
&- \frac{\sigma^2}{4a} \int_0^T B(t,t+k)^2 \left(1 - e^{-2at}\right)\, dt
\end{aligned}$$

Now $B(t, t+k)$ is a constant (given $k$), and

$$\int_0^T f(0,t)\, dt = -\log p(0,T)$$

from equation (2).

$$\int_0^T \log \frac{p(0,t+k)}{p(0,t)}\,dt = -\int_0^T \left((t+k)r_{t+k}(0) - tr_t(0)\right)\,dt$$

this can be evaluated using numerical integration, given the initial spot rate curve. Finally,

$$\frac{\sigma^2}{4a}\int_0^T B(t,t+k)^2\left(1-e^{-2at}\right)\,dt = \frac{\sigma^2}{4a}B(t,t+k)^2\left(T - \left(\frac{1-e^{-2aT}}{2a}\right)\right)$$

The third term in equation(10) is $E_0^Q\left[e^{-\int_0^T \gamma r(t)\,dt}\right]$, which can be determined by adapting the derivation of the bond equation above. The result is

$$E_0^Q\left[e^{-\int_0^T \gamma r(t)\,dt}\right]$$
$$= p(0,T)^\gamma \left(\exp\left(\frac{\sigma^2\gamma}{2a^2}\left(\left(\frac{1-e^{-aT}}{a}\right)(1-2\gamma) + \frac{(1-e^{-aT})^2}{2a} + \frac{\gamma(1-e^{-2aT})}{2a} - T(1-\gamma)\right)\right)\right)$$

We use this model fitted to the market yield curve on US government bonds as at 1 April 2013. Where necessary, we have interpolated to give spot rates at monthly intervals. We assume the yield curve is flat at durations greater than 30 years, which is the maximum recorded duration.

We assume parameters $a = 0.02$ and $\sigma = 0.01$ for the interest rate process. We also consider three cases for the time horizon, $T = 5$ years, $T = 10$ years and $T = 20$ years to retirement. Exits are ignored.

We value the Cash Balance interest rate guarantee using the following crediting rates. The margins (given in basis points (bp) below, are consistent with the safe harbor recommendations in IRS report 96-08.

- A crediting rate equal to the 30-year spot rate: $r^c(t) = r_{30}(t)$.
- A crediting rate equal to the 20-year spot rate: $r^c(t) = r_{20}(t)$.
- A crediting rate equal to the 10-year spot rate: $r^c(t) = r_{10}(t)$.
- A crediting rate equal to the 5-year spot rate plus 25 bp: $r^c(t) = r_5(t) + 0.0025$.
- A crediting rate equal to the 1-year spot rate plus 100 bp: $r^c(t) = r_1(t) + 0.01$.
- A crediting rate equal to the 6-month spot rate plus 150 bp: $r^c(t) = r_{0.5}(t) + 0.015$.

| Crediting Rate (Spot Rates) | Time $T$ to exit 5-Yrs | 10-Yrs | 20-Yrs |
|---|---|---|---|
| 30-yr rate | 1.176 | 1.263 | 1.484 |
| 20-yr rate | 1.136 | 1.210 | 1.443 |
| 10-yr rate | 1.098 | 1.118 | 1.275 |
| 5-yr rate +0.25% | 1.075 | 1.098 | 1.200 |
| 1-yr rate +1% | 1.063 | 1.121 | 1.255 |
| 0.5-yr rate +1.5% | 1.083 | 1.170 | 1.369 |
| 5% fixed rate | 1.229 | 1.340 | 1.562 |

Table 1: Valuation factors per $1 of account balance at 1 April 2013, using the Hull-White model.

- A fixed crediting rate of 5% per year compounded annually: $r^c(t) = \log(1.05)$.

The results are shown in Table 1. The values in the table represent a market consistent valuation of the interest rate guarantee, per $1 of each member's account balance, as at 1 April 2013. One interpretation of these figures is that if a company offered the payoff from the guaranteed interest CB plan as a commercial contract, these would represent fair market value for the contract.

Another perspective is to consider the difference between the hypothetical fund account balance, and the value of the CB benefit with the guaranteed interest offered under the CB pension. For example, for a member with 10-years to termination, and with a crediting rate based on 30-year yields, the valuation factor in Table 1 is $1.263 per $1 of fund. That means that if the member and/or her employer contribute, say, $100 into the hypothetical account at the valuation date, the value of the additional liability is $126.3. The premium of $26.3 represents the cost of the interest guarantee.

The table shows that the interest rate guarantee has a significant market value under our valuation assumptions, particularly for members with longer horizons to retirement or termination. We note that the most common crediting rate, the 30-year yield, generates the highest values other than the fixed 6% assumption, for all horizons. We also note that none of the crediting rates or time horizons give a valuation of less than 1.0, meaning that the interest rate guarantee in the CB plan has a positive, and in most cases very significant market value.

We might allow for exits. The common approach would be to apply deterministic exit rates, ignoring any dependency between exits and the future, random interest rates. In this case, the valuation would be a weighted average of the values for different $T$. For $T = 0$, that is, for immediate terminations, the liability is \$1 for each \$1 of member account balance. So, even allowing for terminations cannot bring the valuation to or below \$1 per \$1 of fund.

### 3.4 Crediting rate based on $k$-year par-yield rates

We have used spot rates in the previous section, because the calculations are then analytic, with no sampling uncertainty. However, CB crediting rates are generally based on par yields (at least where $k > 1$). We use Monte Carlo simulation to generate the valuation factors using par-yields, with the same model and assumptions as used in the section above. This will give more realistic valuation, and will also demonstrate how accurate the use of spot rates is as a proxy for par-yields.

Under this approach, we simulate the short rate of interest at the end of each month up to the horizon $T$. Using the simulated short rate at $t$, we can calculate all the $s$-year zero coupon bond prices at that time, $p(t, t+s)$ using equation (6). We then solve for the par-yield on a $k$-year bond, with $\frac{1}{2}$-yearly coupons, denoted $y_k(t)$, say, using the equation of value:

$$1 = \frac{1}{2}\, y_k(t) \left\{ p(t, t+\tfrac{1}{2}) + p(t, t+1) + p(t, t+1\tfrac{1}{2}) + ... + p(t, t+k) \right\} + p(t, t+k) \tag{12}$$

The results are shown in Table 2. As we might expect, given a rising yield curve, using par-yields generate slightly smaller values than the spot rates, particularly for larger values of $k$ and $T$.

In a real-world application of market consistent valuation, we suggest that the par-yield method be used, with Monte Carlo simulation, but the number of projections required could be reduced by using the spot rate valuation as a control variate, as the values are very highly correlated. For more information on control variates, see, eg Hardy(2003).

### 3.5 Impact of the initial yield curve

A feature of arbitrage-free interest rate models, such as the Hull-White model, is that the model is fitted to the initial yield curve at the valuation date, and the shape and

| Crediting Rate (Par-Yields) | Time $T$ to exit 5-Yrs | 10-Yrs | 20-Yrs |
|---|---|---|---|
| 30-yr rate | 1.156 | 1.230 | 1.464 |
| 20-yr rate | 1.129 | 1.193 | 1.426 |
| 10-yr rate | 1.097 | 1.118 | 1.275 |
| 5-yr rate +0.25% | 1.075 | 1.100 | 1.211 |
| 1-yr rate +1% | 1.063 | 1.124 | 1.267 |
| 0.5-yr rate +1.5% | 1.083 | 1.172 | 1.379 |

Table 2: Valuation per $1 of account balance at 1 April 2013, using the Hull-White model with par-yields; 10,000 simulations

level of that curve can have a significant impact on the valuation. We see from Tables 1 and 2 that the cost of the CB pension liability is very significant, in market consistent terms. The most common CB guarantee, the 30-year yield, costs around 146% of the account balance for a member with a 20-year term to retirement, based on the yield curve on US government bonds (reference) at 1 April 2013. It is interesting to investigate the valuation using yield curves from past dates. We are interested, in particular, in the following questions:

1. How volatile are the valuation factors over time, for different values of $k$ (crediting rate term) and $T$ (horizon to retirement)?
2. Has the cost of the CB pension increased significantly, in terms of the valuation factors, since they first became popular in the late 1990s?
3. Has there been any period for which the valuation factors are less than 1.0, indicating that the pension plan need not be fully-funded with respect to the member account balances?

We have calculated valuation factors (that is, the market value at various dates per $1 balance in the member's account) for five of the crediting rates, and applying yield curves as at 1 November for each calendar year from 1998 to 2012, and for 1 April 2013 (these are the numbers from Table 1 above). The results are summarized in Figures 1, 2 and 3. The rates used are spot rates, but we find very similar results using par yields. Results are shown for crediting rates based on 30-year spot rates, 10-year spot rates, 5-year spot rates with a margin of 25bp, 0.5-year spot rates with a margin of 150bp, and a fixed rate of 5% per year. To illustrate the impact of the time horizon, each graph is shown using the same scale. We find the following answers to the questions posed above.

1. The valuation factors can be very sensitive to the initial yield curve, but the impact is much more significant for larger values of $k$, for fixed rate guarantees, and for longer horizons, $T$.

   Consider Figure 1. The four curves derived from market rates (that is, excluding the 5% p.y. fixed rate curve) are decreasing in volatility as $k$ decreases, with the highest volatility arising when $k = 30$ years, and the lowest for the 6-month crediting rates, $k = 0.5$, which is almost flat. The same pattern of volatility emerges for shorter horizons, but less severely.

   The volatility for the fixed rate guarantee is even greater than the 30-year rate guarantee.

   Initially, this is counterintuitive. The longer rates are less volatile than the short rates; the 5% rate is fixed, reducing the randomness in the valuation. One might expect the crediting rates that are less variable to generate valuations that are less variable. However, the absolute level of the crediting rate is not the major factor in the valuation; it is the difference between the crediting rate and the short rate, which (in general) are both stochastic processes. There is more volatility in the spread between 30-year yields and overnight yields than between 6-month yields and overnight yields. We have noted, in Section 3.2 that, in the limit, there is no uncertainty, regardless of the yield curve, where the crediting rate is equal to the short rate plus a fixed margin.

2. There is no evidence that the cost of the CB benefit is generally increasing. However, if we look at the 30-year crediting rate, which is a common design choice, we see that values have risen since the 2008 crisis, and are currently close to their maximum value. The major risk factor (as mentioned above) is the spread risk, not the absolute value of the crediting rates. In 1999, long term rates were high, with 30-year rates hitting 6.3%, but since short rates were also high, the valuation factors are rather lower than for 30-year crediting rates in 2013, even though the long rates today are only around 3.5%.

   Again here, the fixed rate guarantee is the exception. The valuation in this case depends only on the spot rates corresponding to the $T$-year horizon. Since all rates have been very low in the past few years, the cost of funding a 5% guarantee is high.

3. Inglis and Macdonald(2011) claimed that the objective of CB pension valuation is

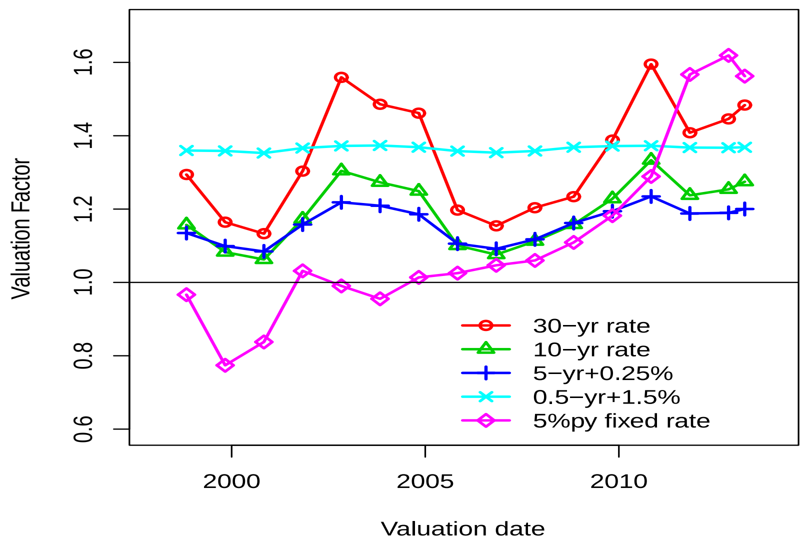


Figure 1: Valuation Factors for $T = 20$-year horizon, as at Nov 1998 to April 2013.

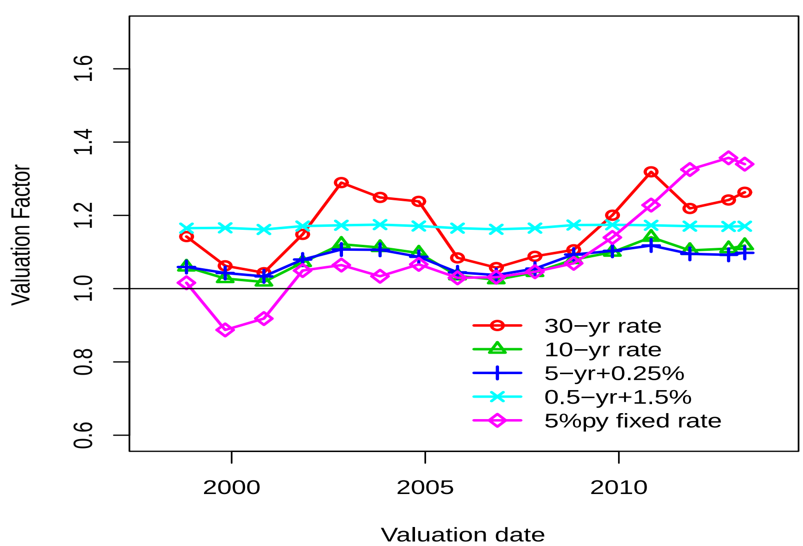


Figure 2: Valuation Factors for $T = 10$-year horizon, as at Nov 1998 to April 2013.

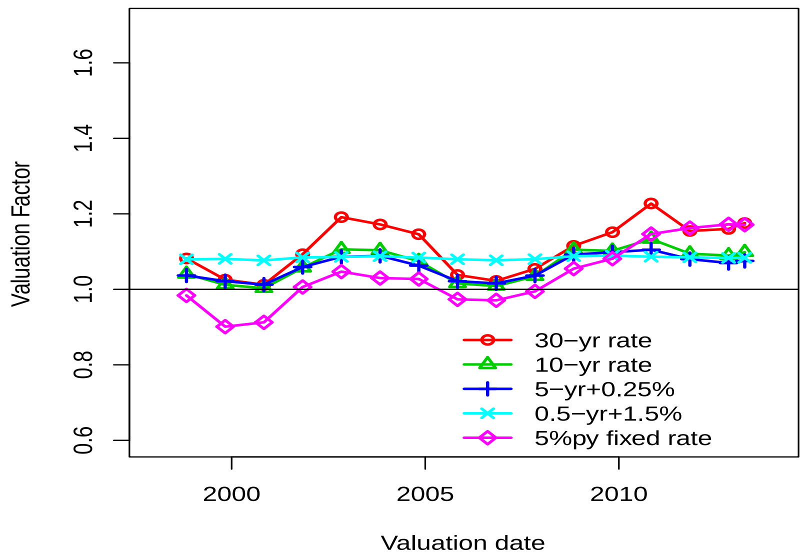


Figure 3: Valuation Factors for $T = 5$-year horizon, as at Nov 1998 to April 2013.

to find a method that gives a valuation equal to the total account balance, which in our terms would require all valuation factors to be 1.0. This is not supported by a market consistent valuation. In every case where the crediting rate is based on market yields, the valuation factor is greater than 1.0, which means the benefit value is greater than the account value. The only exception is where the crediting rate is a fixed 5%, when the valuation factor fell below 1.0 in the late 1990s, and for $T = 20$, all the way up to 2003. The reason is that on these dates the yield on $T$-year zero coupon bonds was greater than 5% p.y. This means that the plan sponsor was offering a benefit that could immediately be funded at less than par with no risk, for the accrued account balance at those times.

We note though that actuarial and accounting principles should prohibit the use of a valuation factor of less than 1.0, since in the event of the member leaving, or of a wind-up of the plan, the plan would be immediately liable for the full account balance.

# 4 References

Björk, T. (2009). Arbitrage theory in continuous time. Oxford University Press.

Boyle P, Hardy M R (2003) Guaranteed annuity options. Astin Bulletin, 33 (2) pp. 125152.

Brigo, D., Mercurio, F. (2006). Interest rate models-theory and practice: with smile, inflation and credit. Springerverlag Berlin Heidelberg.

Brown David, Dybvig Philip, Marshall William (2001) The cost and duration of cash balance plans. Financial Analysts Journal Nov/Dec 2001, 50-62.

Chen K. and Hardy M. R. (2009) The DB underpin hybrid pension plan: fair valuation and funding. North American Actuarial Journal 13(4) 407-424.

Clark Robert L. and Fred W. Munzenmaier (2001) Impact of replacing a defined benefit pension with a defined contribution plan or a cash balance plan. NAAJ 5.1 32-56.

Clark, Robert L., and Sylvester J. Schieber,(2000) Taking the Subsidy Out of Early Retirement: The Story Behind the Conversion to Hybrid Pensions. In Innovations in Retirement Financing, edited by Olivia S. Mitchell, Zvi Bodie,.

Coronado, Julia L. and Phillip C. Copeland. 2003. Cash Balance Pension Plan Conversions and the New Economy. Federal Reserve Board Finance and Economics Discussion Series. Working Paper No. 2003-63.

Dickson, D.C.M., Hardy, M.R. and Waters H.R. (2009) Actuarial Mathematics for Life Contingent Risk. Cambridge University Press.

Employee Benefits Security Administration (EBSA) 2012. Private Pension Plan Bulletin: Abstract of 2010 Form 5500 Annual Reports. US Dept of Labor.

Enderle, G, Gold J, Latter G and Peskin M (2006). The Pension Actuary's Guide to Financial Economics. Society of Actuaries Monograph.

Gold, Jeremy. 2001. Shareholder-Optimal Design of Cash Balance Pension Plans. Pension Research Council. Working Paper 2001-7.

Hardy, M. (2003). Investment guarantees: modeling and risk management for equity-linked life insurance . Wiley.

Hill, T, Pang G and Warshawsky M (2010) Hybrid pension plans: a comprehensive look at their history, economics and features. Towers Watson Perspectives Report.

Hull, J. and A. White(1990): Pricing Interest Rate Derivative Securities, Review of Financial Studies , Vol 3, 4, 573-592.

Inglis E and McDonald J (2011) Unique issues for hybrid plans. Society of Actuaries Annual Meeting Papers.

Iyengar, G., Ma, A. K. C. (2009). Cash flow matching: a risk management approach. North American Actuarial Journal, 13(3), 370-378.

Jørgensen, P. L., Linnemann, P. (2012). A comparison of three different pension savings products with special emphasis on the payout phase. Annals of Actuarial Science, 6(01), 137-152.

Kopp, Steve J and Sher, Lawrence J 1998. A benefit value comparison of a cash balance plan with a traditional final average pay BD plan. The Pension Forum, Vol 11.1 1-16.

Lowman, Thomas B(2000) Actuarial Aspects of Cash Balance Plans. Society of Actuaries Monograph.

McMonagle, Daniel R (2001)Cash Balance Plans in a Traditional Defined Benefit World. In Cash Balance Symposium Monograph, Society of Actuaries.

Murphy, Raymond J (2001) The Cash Balance Funding Method. In Cash Balance Symposium Monograph, Society of Actuaries.

Thomas, P and Williams, P. (2009) Cash Balance Pension Plans: A Case of Standard Setting Inadequacy. Critical Perspectives on Accounting, March, 2009 20 (2009) 228-254.

Vasicek. O. A. (1977): An Equilibrium Characterization of the Term Structure, Journal of Financial Economics, 5, 177-188.