OPTIMAL ASSET-ALLOCATION WITH MACROECONOMIC CONDITIONS AND LABOR INCOME UNCERTAINTY

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Abstract

This paper analyzes optimal asset-allocation strategies over the life-cycle of an individual by taking into account both: the labor income risk profile of the individual, and the macroeconomic dynamics of the assets included in the portfolio. Changes on the states of the economy are characterized by a discrete-time, discrete-space Markov chain aiming to capture the dynamics of the market. Since the dynamics of the financial market affects the labor income of the individual, labor income risk is characterized by the correlation level it exists between the market and the labor income process. Under this model, the optimal asset-allocation problem is solved numerically by backwards induction. Numerical simulations show that the traditional advice of "hold stocks during the early years and gradually change them to bonds as the individual gets older", is not optimal when the labor income of the individual is imperfectly correlated to the market and/or the economy exhibits change of regimes. These results should be considered when designing pension plans.

Keywords: asset allocation; labor income risk; life-cycle models; regime switching; pension plans.
JEL Classification: D91; G11; J32

1. INTRODUCTION

This paper analyzes the optimal asset allocation over the life-cycle of an individual when his labor income is uncertain during his working life period due to the nature of his profession or due to the economic prevailing conditions. Workers with same individual characteristics such as age, education level, geographical location and same average labor income, they may be exposed to very

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different labor income risks: individuals with safe jobs are less sensitive to market movements and macroeconomic conditions, while individuals with uncertain wages are much more sensitive to them. Research work on optimal assset allocation over the life-cycle is huge. The seminal work of Merton (1969) analyzes optimal asset-allocation under uncertainty by assuming that labor income follows a stochastic process that is perfectly correlated to the stock process. If the individual is allowed to take short positions, the individual can hedge his labor income risk with the available financial assets. Bodie et al. (1992), extend the optimal porfolio analysis by incluiding labor flexibility, that is, the individual faces the decision about the amount of hours he allocates to work and to leisure. El Karoui and Jeanblanc-Picque (1998) and Koo (1999) derived properties of optimal asset allocation by incorporating the fact that individuals can not borrow against future labor income, therefore liquidity constrains are imposed into the model. Henderson (2005) studies optimal asset allocation when market and labor income are imperfectly correlated. See Bodie et al. (2009) for a review of recent scientific literature.
The contibution of this paper is to characterize optimal asset-allocation over the life-cycle of an individual that is liquid constrained while incorporating both: the correlation of the labor income with the market, and the change of states or macroeconomic conditions on the market (regime-switching). The next section describes the model, Section 3 states the optimal asset-allocation problem, Section 4 provides the numerical implementation and results, while Section 5 concludes.

2. THE MODEL

The states of the economy. Assume that the economy activities take place on discrete time. Let $\mathcal{T}$ be the time index set $\{0, 1, 2, ..., T\}$, where $T < \infty$. Let $(\Omega, \mathcal{F}, P)$ be a complete probability space and $\mathbf{m} = \{\mathbf{m_t} \,|t \in \mathcal{T}\}$ be a discrete-time finite-state Markov chain on $(\Omega, \mathcal{F}, P)$ that describes the evolution of the different states of the economy $\mathbf{s} = \{\mathbf{s_1}, \mathbf{s_2}, ..., \mathbf{s_K}\}$. The states are identified by standard unit vectors $\{\mathbf{e_1}, \mathbf{e_2}, ..., \mathbf{e_K}\}$, where $\mathbf{e_i} = (0, ..., 1, ..., 0)'$ and $'$ denotes the transpose. Assume the Markov chain $\mathbf{m}$ is time homogeneous. Let $p_{i,j} = \Pr(\mathbf{m_{t+1}} = \mathbf{e_j} \,|\mathbf{m_t} = \mathbf{e_i})$; $i, j = 1, ..., K$ be the transition probabilities and denote by $\mathbf{A}$ the transition probability matrix $[p_{i,j}]_{i,j=1,...,K}$ of the chain $\mathbf{m}$ under P. Let $\pi = (\pi_1, \pi_2, ..., \pi_K,)'$ be the initial distribution of the chain, where $\pi_i = \Pr(\mathbf{m_0} = \mathbf{e_i})$. Assume that the process is stationary. Elliott et al. (1994) proved that the chain $\mathbf{m}$ admits the following decomposition under P:

$$\mathbf{m_{t+1}} = \mathbf{A}\mathbf{m_t} + \mathbf{M_{t+1}}$$

where $\mathbf{M_{t+1}}$ is a martingale with respect to the filtration generated by $\mathbf{m}$ under P.

The financial assets. Assume there are two financial assets available on the economy: a risk-free asset or bond traded at price B_t an a risky asset or stock traded at price S_t at time t. Let r_i be the return of the risk-free asset when the economy is in state i and let $\mathbf{r} = (r_1, ..., r_K)$. The return of the risk-free asset at time t depends on the state of the economy at time $t - 1$, then $r_t =< \mathbf{r}, \mathbf{m_{t-1}} >$, where $<, >$ denotes the product of vectors. The evolution of the price of the risk-free asset over time is $B_{t+1} = B_t \exp(r_{t+1})$. Assume that the dynamics of the price of the risky asset is given by

$$S_{t+1} = S_t \exp(\mu_t^S - \frac{1}{2}\sigma_t^{2S} + \sigma_t^S \epsilon_{t+1}^S) \tag{1}$$

where μ_t^S and σ_t^S denote, respectively, the return and the volatility of the risky asset and ϵ_{t+1}^S follows a standard normal distribution. The parameters vary across $i = 1, ..., K$ regimes driven by the chain process $\mathbf{m}$. The vectors containing the return and the volatility of the risky asset for the K states of the economy are, respectively, $\boldsymbol{\mu}^S = (\mu_1, ..., \mu_K)$ and $\boldsymbol{\sigma}^S = (\sigma_1, ..., \sigma_K)$. The parameters of the equation (1) can be rewritten as

$$\mu_t^S =< \boldsymbol{\mu}^S, \mathbf{m_{t-1}} >; \sigma_t^S =< \boldsymbol{\sigma}^S, \mathbf{m_{t-1}} > . \tag{2}$$

Labor income. Assume that individuals live until date T and they receive a labor income amount $L_t > 0$ during their working life period for years $\{0, 1, 2, ..., T^*\}$, where $T^* < T$ denotes the retirement date [2].

Let μ^L be the annual growth rate of the labor income and σ^L be its deviation. Assume that the dynamics of the labor income process is given by

$$L_{t+1} = L_t \exp(\mu_t^L + \sigma_t^L \epsilon_{t+1}^L) \tag{3}$$

where ϵ_{t+1}^L follows a standard normal distribution and the parameters of the equation (3) may switch across the different K states of the economy over time, thus

$$\mu_t^L =< \boldsymbol{\mu}^L, \mathbf{m_{t-1}} >; \sigma_t^L =< \boldsymbol{\sigma}^L, \mathbf{m_{t-1}} > . \tag{4}$$

The processes given in equations (1) and (3) are correlated, reflecting the fact that the dynamics of the financial market affects the labor income of the individual. Thus,

$$\epsilon_{t+1}^L = \rho \epsilon_{t+1}^S + \sqrt{1-\rho^2}\epsilon_{t+1} \tag{5}$$

where ρ denotes the correlation between the labor income process and the risky asset price process and ϵ_{t+1} follows a standard normal distribution independent of ϵ_{t+1}^S.

Equations (3) and (5) capture the labor income risk profiles across individuals that may share same personal characteristics but that differ on their labor income risk. If $\sigma^L = 0$ the labor income of the individual behaves as a risk-free bond, while if $\rho = 1$ the labor income behaves as a stock. Section 4 will compare optimal asset-allocation results for different labor risk profiles.

Human capital. Let H_t be the value of human capital at time t, that is, the present value of the future labor income of the individual during his remaining working life period $\{t+1, t+2, ..., T^*\}$. H_t is calculated by[3]

$$H_t = \sum_{i=t+1}^{T^*} L_i \exp(-\sum_{j=t+1}^{i} d_j) \tag{6}$$

where d_j denotes the stochastic rate at which labor income is discounted. The discount rate includes the risk-free rate r_j plus the risk premium for the labor income process κ_j, i.e. $d_j = r_j + \kappa_j$. According to the CAPM, κ_j can be evaluated as $\kappa_j = \rho\sigma_t^L/\sigma_t^S(\mu_t^S + 1 - \exp(r_t))$.

[2]In a more realistic framework, survival probabilities can be incorporated to the model to allow T be stochastic. See for example Blake et al. (2003). For simplicity, this paper assumes that T^* and T are known in advance to isolate the effect that labor income risk and change of states in the economy have over optimal asset-allocation.

[3]For $K = 1$ and $\rho = 1$, i.e. no regime switching and perfect correlation between market and labor income, H_t is the present value of an annuity that increases as a geometric progression with initial payment L_t and growth rate $\mu^L - \sigma^L(\mu^S - r)/\sigma^S$.

Individual preferences. Let $U(y)$ be the utility function of the individual over his whole life period. Two different phases of his life-cycle are distinguished: the accumulation period, where the individual works from date 0 until retirement date T^* and the distribution period from retirement until his death at date T. The utility function U can be written as

$$U(y) \equiv E\left[\sum_{0}^{T^*} u(y) + \sum_{T^*}^{T} u^*(y)\right] \tag{7}$$

where $u(y)$ is the utility of the individual during the accumulation period and $u^*(y)$ is the utility of the individual during the distribution period. For numerical purposes assume that the individual has CRRA utility function that takes the form $u(y) = \frac{y^{1-\gamma}}{1-\gamma}$ for $y > 0$, $\gamma \neq 1$ and $u(y) = \ln(y)$ for $y > 0$, $\gamma = 1$; where γ denotes the risk aversion parameter.

3. THE OPTIMAL ASSET-ALLOCATION PROBLEM

During the accumulation period, at each period t, the individual works and receives in exchange a labor income amount L_t, then he has to decide: how much to consume $c_t > 0$; the proportion α_t of his remaining wealth that he will allocate on stocks, and the proportion $(1 - \alpha_t)$ that he will allocated on bonds, where $0 \leq \alpha_t \leq 1$. The evolution of wealth is given by

$$W_{t+1} = (W_t + L_t - c_t)[\alpha_t \exp(\mu_t^S - \frac{1}{2}\sigma_t^{2S} + \sigma_t^S \epsilon_{t+1}^S) + (1 - \alpha_t)\exp(r_t)] \tag{8}$$

To prevent that the individual borrow against future labor income, the constrains $B_t > 0$ and $S_t > 0$ are imposed at each time period.

The problem is to find optimal α_t and c_t that maximizes the individual's utility of total wealth W_t and human capital H_t over his life period. The Bellman equation is written as

$$V(W_t + H_t) = \max\left[U(c_t) + E_t V_{t+1}(W_{t+1} + H_{t+1})\right] \tag{9}$$

where V_t is the value function, W_t is the state variable described in equation (8) and $\{c_t, \alpha_t\}$ are the control variables of the dynamic optimization problem.

Equation (9) has no analytical solution so it will be solved numerically by backwards induction.

4. NUMERICAL IMPLEMENTATION AND RESULTS

The numerical procedure is the following: first, generate the discrete Markov chain $\mathbf{m_t}$ that describes the different states of the economy by using the initial probability distribution of the chain π and the transition probability matrix $\mathbf{A}$. Then simulate the values of the financial assets by using equations (1) and (2). Then compute ϵ_{t+1}^L by using equation (5) and simulate the labor income process L_t by using equation (3). Calculate the human capital H_t by using equation (6). Simulate this procedure N times and evaluate the objective function as an average of equation (7). Solve equation (9) by backwards induction.

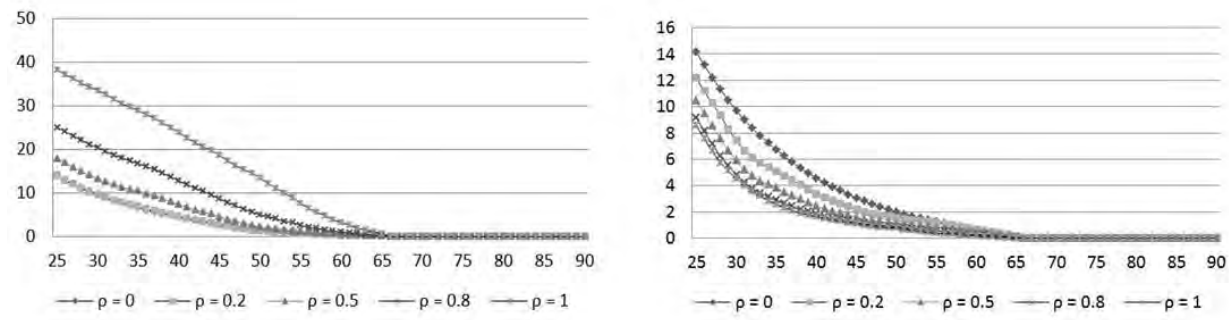

Figure 1: Human capital for different correlation levels. Left: market mostly on expansion. Right: market on recession followed by expansion periods.

The benchmark parameters for the numerical simulations are the following: $N = 10000$ simulations; $K = 2$ states of the economy; $T^* = 40$; $T = 65$ years[4]; annual risk-free rate $\mathbf{r} = (0.04, 0.02)$; annual mean stock return $\boldsymbol{\mu}^S = (0.10, 0.06)$; std. dev. stock return $\boldsymbol{\sigma}^S = (0.12, 0.20)$; annual growth rate labor income[5] $\boldsymbol{\mu}^L = (0.01, 0.01)$; std. dev. labor income $\boldsymbol{\sigma}^L = (0.03, 0.03)$; correlation between market and labor income $\rho = \{0, 0.2, 0.5, 0.8, 1\}$.

Optimal asset-allocation for individuals that exhibit different labor income risk profiles are analyzed. Although labor income parameters remain the same over the two states of the economy, the regime switching on the market evolution affects the dynamics of the labor income by the correlation parameter. Human capital for different levels of correlation is depicted in Figure 1.

Is it clear that human capital decreases with the age, but when two states on the economy are considered, human capital may decrease faster or slower according to the prevalent state of the economy.

Figure 2 (left) depicts optimal asset-allocation results when the labor income is not correlated to the market and the market is mostly in expansion state. At the early years, the individual should invest mostly on stocks because his labor income is bond-likely, then he reduces it gradually as his human capital also decreases.

If the economy is in a recession state during the period the individual is young, and if his labor income is highly correlated with the market, a very different optimal portfolio is derived. This is depicted in Figure 2 (right): at the early years, since his labor income is highly correlated with the market and the market is mostly on recession, it is optimal for him to hold mostly bonds. Then, when the economy switches to an expansion state, he is better off by holding mostly stocks, but later, as his human capital decreases with the age, he will hold mostly bonds before retirement date.

[4]It corresponds, for example, to an individual that starts working at age 25, retires at age 65 and dies at age 90.
[5]For simplicity, assume that labor income parameters are the same across states.

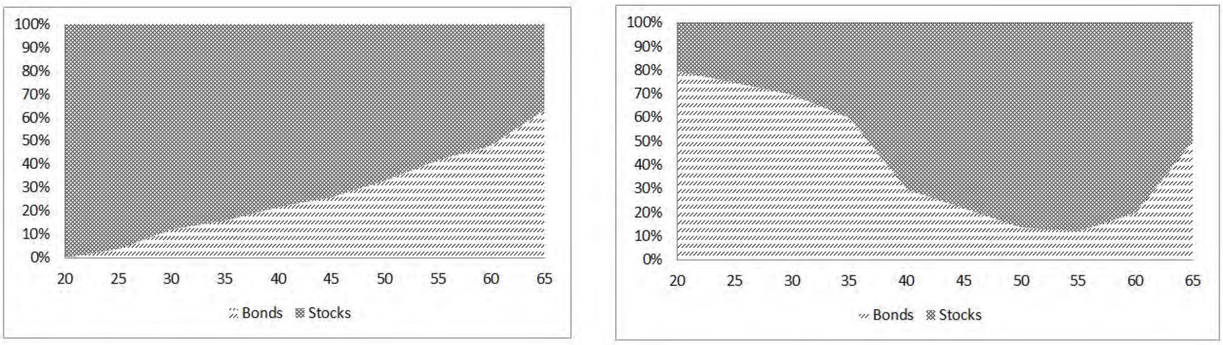

Figure 2: Optimal asset-allocation. (left) safety labor income profile, market mostly on expansion (right) risky labor income profile, market on recession followed by expansion periods

5. CONCLUSIONS

By considering a Markov regime switching model, this paper analyzes optimal asset-allocation over the life-cycle of an individual considering both: the evolution of the macroeconomic conditions that affect financial assets and the evolution of the labor income the individual receives. Numerical simulations show that the traditional advice of "hold stocks during the early years and gradually change them to bonds as the individual gets older", is not optimal if the labor income of the individual is imperfeclty correlated to the market and the economy exhibits change of regimes. In an extension of this paper, these results are considered for designing pension plans.

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