Valuation and Risk Assessment of Participating Life Insurance in the Presence of Credit Risk

Colloquium of the International Actuarial Association
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Nadine Gatzert und Michael Martin
Friedrich-Alexander-University (FAU) Erlangen-Nürnberg
Introduction

Motivation

- Credit risk has long not been in the focus of most insurance companies

- Due to the long-term and guaranteed liabilities, life insurers invest a large part of their capital in long-term assets such as corporate and government bonds

- As a consequence of financial and sovereign crises, credit risk became increasingly important in the recent years

- In participating life insurance, capital investment management decisions can substantially affect
  - Policyholders’ surplus participation
  - Insurers’ risk situation
Introduction

Aim of paper

- Analyze the impact of market and credit risks in participating life insurance from the equityholders’ perspective

- Focus on the asset side and the induced market risk for
  - Stocks
  - Bonds (corporates and governments)

- Insurers’ assets are simultaneously affected
  - Equity risk
  - Interest rate risk
  - Credit risk

- Compare the differences for fair contract parameters and risk measures when taking credit into account or not
Model framework

Company overview

- At $t = 0$
  - Policyholders’ upfront premium: $P(0) = k \cdot A(0)$
  - Initial equity capital: $E(0) = A(0) - P(0)$

- Dividend payment (see Bohnert, Gatzert, and Jørgensen, 2012)
  $$D(t) = \mathbb{I}_{\{A(t) - P(t) > \beta E(0)\}} \cdot \beta \cdot E(0)$$

Modeling the liabilities (see, e.g., Gatzert, 2008)

- Cliquet-style guarantee
  $$P(t) = P(t-1) \cdot (1 + r_p(t)) = P(t-1) \cdot \left[ 1 + \max \left( r_g \cdot \frac{A(t)}{A(t-1)} - 1 \right) \right]$$

- Terminal surplus participation
  $$B(T) = \max \left( A(T) - P(T), 0 \right)$$
Model framework

Asset dynamics (1)

Equity risk

- Stocks follow a geometric Brownian motion

\[ dA_S(t) = \mu_S \cdot A_S(t) \, dt + \sigma_S \cdot A_S(t) \, dW^P_S(t) \]

Interest rate risk

- Non-defaultable zero bond price is defined by the Cox, Ingersoll, and Ross (1985) (CIR) model

  - Short rate \( r(t) \): 
  \[ dr(t) = \kappa \cdot (\theta - r(t)) \, dt + \sigma_r \cdot \sqrt{r(t)} \, dW^Q_r(t) \]

  - Closed affine form for non-defaultable zero bond price:

\[ p(t,h) = E_t^Q \left( e^{-\int_t^h r(s) \, ds} \right) = e^{F(t,h) - G(t,h)r(t)} \]
Credit risk

- Defaultable zero coupon bond price is based on the reduced-form credit risk model by Jarrow, Lando, and Turnbull (1997) (JLT)
  - Time-homogenous Markov process: \( X = (x(t), t \in \mathbb{N}_0) \)
    \[
    \begin{pmatrix}
    \psi_{1,1}(t,h) & \cdots & \psi_{1,m}(t,h) \\
    \vdots & & \vdots \\
    \psi_{m-1,1}(t,h) & \cdots & \psi_{m-1,m}(t,h) \\
    0 & 0 & \cdots & 1
    \end{pmatrix}
    \]
  - Distribution: \( \Psi(t,h) = \{\psi_{i,j}(t,h)\} \)
  - Stopping time: \( \tau^B = \inf\{t \in \mathbb{N} : x(t) = m\} \) and recovery rate of face value: \( \delta \)

- Defaultable zero coupon bond price with rating \( x(t) = i \)
  \[
  \hat{p}(t,h) = E^Q_t \left( \mathbb{I}_{\{\tau^B > h\}} \cdot e^{-\int_t^h r(s)ds + \mathbb{I}_{\{\tau^B \leq h\}} \cdot \delta_R \cdot e^{-\int_t^h r(s)ds} } \right)
  = p(t,h) \cdot \left( \delta_R + (1 - \delta_R) \cdot (1 - \psi(t,h)) \right)
  \]
Model framework

Asset dynamics (3)

Market value at time $t$

- Stock portfolio comprising $N_S$ exposures
  \[ A_S(t) = \sum_{i=1}^{N_S} A_{S,i}(t) \]

- Bond portfolio comprising $N_B$ exposures
  \[ A_B(t) = \sum_{j=1}^{N_B} A_{B,j}(t) \]

with \[ A_{B,j}(t) = \mathbb{I}_{\{\tau_j^B > t\}} \cdot \sum_{h=t+1}^{T_j} \left( CF_j(h) \cdot \hat{p}_{x_j(t)=i}(t,h) \right) + \mathbb{I}_{\{\tau_j^B = t\}} \cdot \delta_R \cdot A_{B,j}(t-1) \]
Management decisions regarding the asset allocation

- Market value of assets (stocks and bonds) at time $t$ before (-) and after (+) rebalancing the assets (based on Gerstner et al., 2008)
  - Market value of assets at time $t$:
    \[ A(t) = A_S^-(t) + A_B^-(t) + CF(t) - D(t-1) \]
  - Free capital at time $t$:
    \[ F(t) = A(t) - A_B^-(t) - D(t-1) = A_S^-(t) + CF(t) - D(t-1) \]
  - Market value of stocks at time $t$ after rebalancing:
    \[ A_S^+(t) = \max \left( \min \left( \alpha \cdot A(t), F(t) \right) \right), 0 \]
  - Market value of bonds at time $t$ after rebalancing:
    \[ A_B^+(t) = A_B^-(t) + F(t) - A_S^+(t) \]
Model framework

Fair valuation from the equityholders’ perspective and risk measurement

- Time of company default
  \[ \tau^C = \inf \{ t \in \{1, \ldots, T \} : A(t) < P(t) \} \]

- Equity capital at time \( t \)
  \[ E(t) = \begin{cases} A(t) - L(t), & \tau^C > t \\ 0, & \text{else} \end{cases} \quad \text{with} \quad L(t) = \begin{cases} P(t), & (t < T_j) \cap (\tau^C > t) \\ P(t) + \delta_L \cdot B(t), & (t = T_j) \cap (\tau^C > t) \\ A(t) \cdot (1 - c), & \tau^C = t \\ 0, & \text{else} \end{cases} \]

- Equityholders’s claim at time \( t \) and fair condition
  \[ \Pi^E = E^Q \left( \mathbb{1}_{\{\tau^C > T\}} \cdot e^{-\int_0^T r(s) ds} \cdot E(T) + \sum_{i=1}^T e^{-\int_0^{T_i} r(s) ds} \cdot D(t-1) + \mathbb{1}_{\{\tau^C \leq T\}} \sum_{i=1}^{\tau^C - 1} e^{-\int_0^{T_i} r(s) ds} \cdot D(t-1) \right) = E(0) \]

- Net present value and shortfall probability
  \[ NPV^P = E^Q \left( \mathbb{1}_{\{\tau^C > T\}} \cdot e^{-\int_0^T r(t) dt} \cdot L(T) + \mathbb{1}_{\{\tau^C \leq T\}} \cdot e^{-\int_0^{\tau^C} r(t) dt} \cdot L(\tau^C) \right) - P(0) \quad \text{and} \quad SP = LPM_{0} = P(\tau^C \leq T) \]
Numerical results

Input parameters

- Company and contract parameters
  - $T = 15$
  - $P(0) = 85$, $E(0) = 15$, $A(0) = 100$
  - $\delta_L = 0.3$
  - $\beta = 0.06$

- Stock portfolio

<table>
<thead>
<tr>
<th>$i$</th>
<th>Stock</th>
<th>$m_S$</th>
<th>$\sigma_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DAX 30</td>
<td>0.0637</td>
<td>0.2164</td>
</tr>
<tr>
<td>2</td>
<td>FTSE 100</td>
<td>0.0436</td>
<td>0.1658</td>
</tr>
<tr>
<td>3</td>
<td>Dow Jones Industrial</td>
<td>0.0755</td>
<td>0.1784</td>
</tr>
</tbody>
</table>

- CIR model

<table>
<thead>
<tr>
<th>$r(0)$</th>
<th>$\sigma_r$</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0369</td>
<td>0.0342</td>
<td>0.1810</td>
</tr>
</tbody>
</table>

- Correlations ($S_i$, $r$)

- Bond portfolio

<table>
<thead>
<tr>
<th>$j$</th>
<th>Type</th>
<th>Rating</th>
<th>Maturity (years)</th>
<th>Coupon p.a. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Corporate</td>
<td>AA</td>
<td>15</td>
<td>2.800</td>
</tr>
<tr>
<td>2</td>
<td>Corporate</td>
<td>A</td>
<td>15</td>
<td>4.125</td>
</tr>
<tr>
<td>3</td>
<td>Corporate</td>
<td>BBB</td>
<td>15</td>
<td>7.125</td>
</tr>
<tr>
<td>4</td>
<td>Government</td>
<td>AAA</td>
<td>16</td>
<td>2.275</td>
</tr>
<tr>
<td>5</td>
<td>Government</td>
<td>A</td>
<td>15</td>
<td>3.000</td>
</tr>
<tr>
<td>6</td>
<td>Government</td>
<td>BB</td>
<td>17</td>
<td>7.750</td>
</tr>
</tbody>
</table>

- JLT model

<table>
<thead>
<tr>
<th>$\delta_R$</th>
<th>$\psi$</th>
<th>Transition rates from rating agencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.55</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Numerical results

Fair contracts with and without credit risk

**Bond portfolio 1**

<table>
<thead>
<tr>
<th>α (%)</th>
<th>1: r_g = 1.25%</th>
<th>2: r_g = 1.75%</th>
<th>3: r_g = 2.25%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** Bond portfolio 1 (higher grade): B_1, B_2, B_4, B_5.

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Numerical results

Fair contracts with and without credit risk

Bond portfolio 2

\[ r_g = 1.25\% \]
\[ r_g = 1.75\% \]
\[ r_g = 2.25\% \]

Notes: Bond portfolio 2 (lower grade): \( B_2, B_3, B_5, B_6 \).
### Numerical results

#### Risk measurement with and without credit risk (1)

<table>
<thead>
<tr>
<th>Bond portfolio 1</th>
<th>$\alpha = 0%$</th>
<th>$\alpha = 5%$</th>
<th>$\alpha = 10%$</th>
<th>$\alpha = 15%$</th>
<th>$\alpha = 20%$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With credit risk</td>
<td>$-0.69\text{ pp}$</td>
<td>$-1.08\text{ pp}$</td>
<td>$-1.58\text{ pp}$</td>
<td>$-0.60\text{ pp}$</td>
<td>$-0.78\text{ pp}$</td>
</tr>
<tr>
<td>Without credit risk</td>
<td>$-0.76\text{ pp}$</td>
<td>$-0.93\text{ pp}$</td>
<td>$-0.69\text{ pp}$</td>
<td>$-0.59\text{ pp}$</td>
<td>$-0.68\text{ pp}$</td>
</tr>
<tr>
<td></td>
<td>$-0.49\text{ pp}$</td>
<td>$-0.68\text{ pp}$</td>
<td>$-0.33\text{ pp}$</td>
<td>$-0.42\text{ pp}$</td>
<td>$-0.50\text{ pp}$</td>
</tr>
<tr>
<td></td>
<td>$-0.34\text{ pp}$</td>
<td>$-0.42\text{ pp}$</td>
<td>$-0.23\text{ pp}$</td>
<td>$-0.34\text{ pp}$</td>
<td>$-0.34\text{ pp}$</td>
</tr>
</tbody>
</table>

**Notes:** Bond portfolio 1 (higher grade): $B_1, B_2, B_4, B_5$.

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Numerical results

Risk measurement with and without credit risk (2)

**Bond portfolio 2**

<table>
<thead>
<tr>
<th>α</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>7.75 pp</td>
<td>-9.33 pp</td>
<td>5.58 pp</td>
<td>6.59 pp</td>
<td>6.91 pp</td>
<td>5.08 pp</td>
<td>6.69 pp</td>
<td>5.77 pp</td>
<td>5.00 pp</td>
<td>3.28 pp</td>
<td>3.71 pp</td>
<td>6.57 pp</td>
<td>3.20 pp</td>
<td>2.54 pp</td>
</tr>
</tbody>
</table>

**Notes:** Bond portfolio 2 (lower grade): B₂, B₃, B₅, B₆.
Numerical results

Impact of bonds’ recovery (with $r_g = 1.75\%$)

<table>
<thead>
<tr>
<th>Stock Portion $\alpha$</th>
<th>Shortfall Probability SP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>5%</td>
<td>4%</td>
</tr>
<tr>
<td>10%</td>
<td>8%</td>
</tr>
<tr>
<td>15%</td>
<td>12%</td>
</tr>
<tr>
<td>20%</td>
<td>16%</td>
</tr>
<tr>
<td>25%</td>
<td>20%</td>
</tr>
<tr>
<td>30%</td>
<td>24%</td>
</tr>
</tbody>
</table>

Notes: Bond portfolio 1 (higher grade): $B_1, B_2, B_4, B_5$; bond portfolio 2 (lower grade): $B_2, B_3, B_5, B_6$. 

[Graph showing the impact of bonds' recovery on shortfall probability for two bond portfolios with different grades and varying stock portions.]

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Summary

- Focus on participating life insurance contracts: Study the interaction between equity risk, interest rate risk, credit risk

- Impact of credit risk on the insurers’ shortfall risk strongly depends on the insurer’s asset allocation
  - Tradeoff between higher coupon payments (→ offered by lower grade assets to compensate investors for taking higher risk) and increase in shortfall risk: a “turning point” may exist
  - Tradeoff between equity risk and credit risk
    - Shortfall risk can even decrease for an increasing stock portion in case of high credit risk exposure in (low-grade) bond portfolios

- Consideration of interaction effects between different risks imposed by asset portfolio is vital for an adequate risk assessment
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Thank you very much for your attention!

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