Convergence of Capital and Insurance Markets: Pricing Aspects of Industry Loss Warranties

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Introduction

Motivation

- Increasing risk of extreme losses caused by value concentration and climate change

- Limited (and volatile) capacity of traditional reinsurance and retrocession markets

- Alternative risk transfer (ART) intends to provide additional (re)insurance (or retrocession) coverage by transferring insurance risks to the capital market

- One of the most traded ART instruments are industry loss warranties
Introduction

Industry Loss Warranties

- Defining feature is the dependence on an industry loss index
- Underlying indices are e.g., PCS and PERILS
- Binary and indemnity-based ILWs:
  - Binary ILWs depend only on the underlying industry loss index
  - Indemnity-based ILWs additionally depend on the company loss
- Several configurations: aggregated, occurrence, …
- In our paper: binary and indemnity-based aggregated ILWs
Industry Loss Index

Aggregated Industry Loss Index

- For every occurring catastrophe industry loss estimations are provided (e.g., by PCS or PERILS)

- The index does not instantly display the real insured damage: first an estimation is given, which is adjusted several times.

- Exemplary development of an industry loss index
Industry Loss Index

Aggregated Industry Loss Index

- We model the underlying aggregated industry loss index related to the risk-exposure period $T$ as:

$$I_t^T = \sum_{i=1}^{N_t^T} X_t^i$$

- $N_t^T$ counts all catastrophes that occurred until time $t$ and within the risk-exposure period $(T)$.

- $X_t^i$ denotes the (time-dependent) estimation of the insured loss through the $i$-th catastrophe at time $t$. 
Pricing ILW

Binary ILWs

- The payoff of ILWs depends on the first time $\tau_Y^T$ the industry loss index exceeds the trigger level $Y$:

$$\tau_Y^T = \inf \left\{ t \mid I_t^T \geq Y \right\}$$

- The payoff at maturity of a binary ILW (denoted “$b$”) with trigger level $Y$, risk-exposure period $T$ and maturing at time $T'$ is given by

$$P_{ILW,T'}^{b,Y,T} = D \cdot 1_{\{\tau_Y^T \leq T'\}}$$
Pricing ILW

Binary ILWs

- For arbitrage-free valuation, one has to calculate the expectation of the discounted cash flow under a certain risk-neutral measure $Q$.

- Crucial requirement is the existence of a liquid market.

- The industry loss index cannot be traded itself, but one can assume a liquid market for derivatives of the industry loss index.

- We assume a liquid market of binary zero coupon index linked cat bonds, i.e., with payoff (nominal w.l.o.g. 1)

$$P_{\text{cat},T,T'}^{b,Y,T} = 1_{\{T_Y^T > T'\}}$$
Pricing ILW

Binary ILWs

- In the case of a liquid market the price is the expectation of the discounted cash flow under a risk-neutral measure $Q$

$$V_{cat,T'}^{b,0,Y,T}(t) = E^{Q} \left( e^{-r(T'-t)} 1_{\{\tau_{Y}^{T} > T'\}} | F_t \right)$$

- The price of a binary ILW is also the expectation of the discounted cash flow under the same risk-neutral measure $Q$

$$V_{ILW,T'}^{b,Y,T}(t) = E^{Q} \left( e^{-r(T'-t)} 1_{\{\tau_{Y}^{T} \leq T'\}} | F_t \right)$$
Pricing ILW

Binary ILWs

- The prices are related through

\[ V_{\text{cat},T'}^{b,0,Y,T}(t) = E_t^Q \left( e^{-r(T'-t)} 1_{\{\tau_Y > T'\}} \right) \]

\[ = e^{-r(T'-t)} - E_t^Q \left( e^{-r(T'-t)} 1_{\{\tau_Y \leq T'\}} \right) \]

\[ = e^{-r(T'-t)} - V_{\text{ILW},T'}^{b,Y,T}(t) \]

- So the price of an binary ILW is (Replication of ILW)

\[ V_{\text{ILW},T'}^{b,Y,T}(t) = e^{-r(T'-t)} - V_{\text{cat},T'}^{b,0,Y,T}(t) \]
Pricing ILW

Binary ILWs

- This holds under no further distribution assumption, since the cash flow of an ILW can be replicated by buying a normal bond and selling a cat bond.

- For cases without perfectly fitting cat bonds we give approximations under additional assumptions.

- E.g., an approximation for the case there is a cat bond with different maturity is

\[
V_{ILW,T'}^{b,Y,T}(0) \approx \frac{T'}{\tilde{T}'} e^{-r(T'-\tilde{T})} \left( e^{-r(T'-t)} - V_{cat,\tilde{T}'}^{b,0,Y,T}(t) \right)
\]
Pricing ILW

Indemnity-based ILWs

- The payoff at maturity of an indemnity-based ILW with attachment point $A$, maximum payoff $M$ and trigger level $Y$ is

$$
P_{ILW,T',T}^{A,M,Y,T} = \min\left(M,\left(L_{T'}^T - A\right)_+\right)1_{\{\tau^T_Y \leq T'\}}$$

$$= \left(\left(L_{T'}^T - A\right)_+ - \left(L_{T'}^T - M\right)_+\right)1_{\{\tau^T_Y \leq T'\}}$$

- Indemnity-based ILWs are knock-in call spreads
Pricing ILW

Indemnity-based ILWs

● Crucial is the treatment of the company Loss $L_t^T$. We give three different approaches:
  ● Independence between industry and company loss. This setting is treated by Møller (2003) and could serve as a lower bound
  ● Functional (monotonic increasing) relationship between company and industry loss, i.e., $L_t^T = g(I_t^T)$
Pricing ILW

Exploiting the dependence

- We assume a functional (monotonic increasing) relationship between company and industry loss, i.e., \( L_t^T = g(I_t^T) \)

- This assumption is motivated by
  - Basis risk arises if the company loss and the industry loss are not fully dependent
  - Therefore, ILWs are typically purchased in case of a sufficient degree of dependence

- An example is \( L_t^T = a \cdot I_t^T \), with \( a \) representing an indication of the market share
Pricing ILW

Functional relationship

- The payoff at maturity of an indemnity-based ILW with attachment point $A$, maximum payoff $M$ and trigger level $Y$ is

$$P_{ILW,T'}^{A,M,Y,T} = \min\left(M, \left(g\left(I_{T'}^T\right) - A\right)_+\right)1_{\{\tau_Y^T \leq T'\}}$$

- The price is given as expectation of the (discounted) cash flow under $Q$

$$V_{ILW,T'}^{A,M,Y,T}(0) = E^Q\left(e^{-rT'} \min\left(M, \left(g\left(I_{T'}^T\right) - A\right)_+\right)1_{\{\tau_Y^T \leq T'\}}\right)$$
Pricing ILW

Functional relationship

\[ V_{ILW,T'}^{A,M,Y,T}(0) = E^Q\left( e^{-rT'} \min\left( M, \left( g\left( I_{T'}^T \right) - A \right)_+ \right) 1 \left\{ \tau_Y^{T'} \leq T' \right\} \right) \]

\[ = \ldots = e^{-rT'} \int_{0<x<M} Q\left( g\left( I_{T'}^T \right) - A \geq x \right) dx \]

\[ = \ldots = \int_{A<x<A+M} g'(x) V_{ILW,T'}^{b,x}(0) dx \]

\[ = \int_{A<x<A+M} g'(x) \left( e^{-r(T'-t)} - V_{cat,T'}^{b,0,x,T}(t) \right) dx \]
Summary

- Calculation of arbitrage-free prices of binary ILWs under the assumption of a liquid cat bond market

- Three approaches to overcome the tradability of the company loss

- Calculation of arbitrage-free prices of indemnity-based ILWs under the assumption of a liquid cat bond market and under the assumptions underlying the three presented approaches

- Our formulas can be used to price ILWs with the help of real cat bond prices or any model providing arbitrage-free prices for cat bonds.
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Thank you for your attention.

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References

