Non Gaussian yields: which impact on default options retirement plans?

Stéphane Hamayon (Harvest), Florence Legros (Université Paris-Dauphine), Yannick Pradat (Harvest)
An empirical study on American and French financial yields on the long run

• Yields must be considered « dividends included » and in real terms
• Duration lowers risk: equities appear as the less risky asset in the LR, the risk decreasing quicker than in the Gaussian case
• Equities prices (DS) are characterised by mean reversion and strong synchronisation with economic growth.
• This calls for a inter temporal diversification and a life cycle allocation for default options retirement plans.
Yields must be considered « dividends included » and in real terms

If nominal yields show similar performances in both the countries ...

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Money</strong></td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
</tr>
<tr>
<td></td>
<td>4.3%</td>
<td>2.5%</td>
<td>2.2%</td>
<td>3.1%</td>
<td>4.6%</td>
</tr>
<tr>
<td><strong>Bonds</strong></td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
</tr>
<tr>
<td></td>
<td>3.1%</td>
<td>2.2%</td>
<td>3.5%</td>
<td>3.4%</td>
<td>6.0%</td>
</tr>
<tr>
<td><strong>equities</strong></td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
</tr>
<tr>
<td></td>
<td>7.7%</td>
<td>6.7%</td>
<td>7.1%</td>
<td>10.8%</td>
<td>10.6%</td>
</tr>
<tr>
<td><strong>Inflation rate</strong></td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
</tr>
<tr>
<td></td>
<td>1.9%</td>
<td>0.5%</td>
<td>2.5%</td>
<td>13.5%</td>
<td>3.6%</td>
</tr>
</tbody>
</table>

Real yields are much lower in France, which drives us to cancel war periods.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Money</strong></td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
<td>France</td>
</tr>
<tr>
<td></td>
<td>2.4%</td>
<td>2.1%</td>
<td>-0.3%</td>
<td>-10.4%</td>
<td>1.0%</td>
<td>1.2%</td>
</tr>
<tr>
<td><strong>Bonds</strong></td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
<td>France</td>
</tr>
<tr>
<td></td>
<td>1.2%</td>
<td>1.8%</td>
<td>1.1%</td>
<td>-10.1%</td>
<td>2.3%</td>
<td>2.6%</td>
</tr>
<tr>
<td><strong>equities</strong></td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
<td>France</td>
<td>United-States</td>
<td>France</td>
</tr>
<tr>
<td></td>
<td>5.8%</td>
<td>6.3%</td>
<td>4.6%</td>
<td>-2.8%</td>
<td>7.0%</td>
<td>5.8%</td>
</tr>
</tbody>
</table>
Duration lowers risk: equities appear as the less risky asset in the long run.
Duration lowers equities risk: it decreases quicker than in the Gaussian case.
After 30 years, equities yields show a risk decrease which is quicker than in the Gaussian case.
Stationarity tests show that equities prices (DS) are characterised by mean reversion.
1 - An empirical study on American and French financial yields on the long run

• Yields must be considered « dividends included » and in real terms
• Duration lowers risk: equities appear as the less risky asset in the LR, the risk decreasing quicker than in the Gaussian case
• Equities prices (DS) are characterised by mean reversion and strong synchronisation with economic growth.

➤ This calls for a inter temporal diversification and a life cycle allocation for default options retirement plans.

2 – Which performances with a semi parametric allocation strategy?

• The non Gaussian distribution pleads for a CF VaR as the risk to be minimized in a life allocation
• A higher place for equities with the CF VaR criterium
• Higher replacement rates as well

Conclusion
The non Gaussian distribution pleads for a CF VaR as the risk to be minimized in a life allocation strategy

• **Usual problem:**
  
  \[
  \text{Max } \mu - \lambda_1 \sigma^2
  \]
  
  – with: \( \mu \), expected return
  – \( \lambda_1 \), risk aversion parameter
  – \( \sigma^2 \), return variance

• **Non gaussian distribution problem:**
  
  \[
  \text{Max } \mu - \lambda_1 \sigma^2 + \lambda_2 S - \lambda_3 K
  \]
  
  – with: \( \lambda_1 \), \( \lambda_2 \), \( \lambda_3 \), respectively: variance aversion, asymmetry preference, kurtosis aversion.
The non-Gaussian distribution pleads for a CF VaR as the risk to be minimized in a life allocation strategy.

<table>
<thead>
<tr>
<th>Optimisation programme for a parametric VaR</th>
<th>Optimisation programme for a Cornish-Fisher (CF) VaR</th>
</tr>
</thead>
</table>
| \[
\begin{align*}
\text{min}_w(\text{VaR}_\alpha(w)) \\
\text{S. C.: } & \sum_{i=1}^{3} w_i = 1 \\
& 0 \leq w_i \leq 1
\end{align*}
\] |
| \[
\begin{align*}
\text{min}_w(\text{VaR}_\alpha(w)) \\
\text{S. C.: } & \sum_{i=1}^{3} w_i = 1 \\
& 0 \leq w_i \leq 1
\end{align*}
\] |
| With: \[\text{VaR}_\alpha = w' \cdot \mu + z_\alpha \cdot \sigma\] | With: \[\text{VaR}_\alpha = w' \cdot \mu + Z_{\alpha} \cdot \text{Cornish - Fisher} \cdot \sigma\]  |
| \[\approx z_\alpha + \frac{1}{6} (z_\alpha^2 - 1) \cdot S + \frac{1}{24} (z_\alpha^3 - 3z_\alpha) \cdot K\]  |
| \[z_\alpha \approx \frac{\mu_3}{\sigma^3} : \text{Skewness}\]  |
| \[\tilde{K}(X) = \frac{\mu_4}{\sigma^4} : \text{Kurtosis}\]  |
| \[K = \tilde{K} - 3 : \text{excess Kurtosis}\]  |
A higher place for equities with the CF VaR strategy: simulations with a 3% VaR in the US case ...
... and in the French case
A strategy which provides better replacement rates: US case

Parametric VaR

CF VaR
Conclusion

• It’s worth taking into account the non Gaussian characteristics of equity distribution
• Non Gaussian yields deserve semi parametric VaR strategies
• Monte Carlo simulations must be avoided