The Impact of Disability Insurance on a Life Insurer’s Risk Situation

AFIR/ERM Colloquium of the International Actuarial Association (IAA)
Lyon, June 24th – 26th, 2013

Nadine Gatzert and Alexander Maegebier
Friedrich-Alexander-University of Erlangen-Nürnberg
Introduction: Motivation

● Disability insurance contracts:
  ● Increasing demand and importance, e.g. in the German insurance market
  ● Special characteristics: recovery, different mortality for disabled policyholders, stochastic start of benefit payments

● Previous literature:
  ● Focus on modeling approaches for disability insurance
  ● Hedging effects for portfolios with life insurance and annuity insurance (see, e.g., Gatzert and Wesker, 2012a; Wang et al., 2010)

➢ Combine two strands of the literature:
  ➢ Potential hedging effects between life, annuity and disability insurance
Introduction: Aim of paper

- Analyze the overall risk situation of an insurer offering annuity insurance, disability insurance and term life insurance
  - Study diversification effects and identify risk-minimizing strategies to reduce the overall risk
  - Examine the impact of shocks to general mortality and to the specific mortality rate for disabled policyholders
  - Assess the effect of disability risk, i.e. changes in the invalidity rate

- Risk measures used:
  - Probability of default
  - Mean loss
  - Standard deviation of liabilities
## Model framework: Insurance company

- **Balance sheet of the insurance company at time** $t$:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A(t)$</td>
<td>$E(t)$</td>
</tr>
</tbody>
</table>

**Book values for:**
- $B^A(t)$ annuity insurance (A)
- $B^D(t)$ disability insurance (D)
- $B^L(t)$ term life insurance (L)

- **Assets yield a return** $\varepsilon_t$ at each time $t$:

$$A(t) = A(t-1) \cdot \exp(\varepsilon_t), \quad \varepsilon_t \overset{\text{iid}}{\sim} N\left(\mu_\varepsilon, \sigma_\varepsilon^2\right)$$
Model framework: Insurance company

- Annual dividend for equityholders as a constant fraction $r_e$ of the positive earnings:

$$div(t) = r_e \cdot \max \left( E^*(t) - E(t-1); 0 \right)$$

- Development of the cash flows in a discrete time environment:

<table>
<thead>
<tr>
<th>$t=0^+$</th>
<th>$t=1^-$</th>
<th>$t=1^+$</th>
<th>$t=2^-$</th>
<th>$t=2^+$</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+E(0)$</td>
<td>- <em>div</em></td>
<td>$+\text{premiums}$</td>
<td>- <em>div</em></td>
<td>$+\text{premiums}$</td>
<td>...</td>
</tr>
<tr>
<td>$+\text{premiums}$</td>
<td>- <em>benefits</em></td>
<td>$+\text{premiums}$</td>
<td>- <em>benefits</em></td>
<td>$+\text{premiums}$</td>
<td>...</td>
</tr>
</tbody>
</table>

Equity before dividend is paid
Model framework: Insurance contracts

- State model for annuity (A) and term life insurance (L):

  active (1)  -->  dead (2)

- Prospective reserves based on a bivariate Markov renewal reward process (see, e.g., Stenberg, Manca, and Silvestrov, 2007):

\[
B_A^1(t) = d_{11}^A(0,t;T) \cdot \sum_{\vartheta=1}^{T-t} \psi_1^A \cdot e^{-r \cdot \vartheta} + \sum_{\vartheta=0}^{T-t} b_{12}^A(0,t+\vartheta) \cdot \left[ \sum_{\vartheta'=1}^{\vartheta-1} \psi_1^A \cdot e^{-r \cdot \vartheta'} \right] \\
B_L^1(t) = d_{11}^L(0,t;T) \cdot \sum_{\vartheta=1}^{T-t-1} -\psi_1^L \cdot e^{-r \cdot \vartheta} + \sum_{\vartheta=0}^{T-t} b_{12}^L(0,t+\vartheta) \cdot \left[ \sum_{\vartheta'=0}^{\vartheta-1} -\psi_1^L \cdot e^{-r \cdot \vartheta'} + \gamma_{12}^L \cdot e^{-r \cdot \vartheta} \right]
\]

- Book values:

\[
B^A(t) = n_1^A(t) \cdot B_A^1(t) \\
B^L(t) = n_1^L(t) \cdot B_L^1(t)
\]
Model framework: Insurance contracts

- State model for disability insurance (see, e.g., Haberman and Pitacco, 1999):

- Prospective reserve based on a bivariate Markov renewal reward process, given an active policyholder:

\[
B_1^D(s, t) = d_{11}^D(t - s, t; T) \cdot \sum_{\vartheta = 0}^{T-t-1} -\psi_1^D \cdot e^{-r \cdot \vartheta} \\
+ \sum_{\vartheta = 1}^{T-t} b_{12}^D(t - s, t; t + \vartheta) \cdot \left[ \sum_{\vartheta' = 0}^{\vartheta-1} -\psi_1^D \cdot e^{-r \cdot \vartheta'} + \psi_2^D \cdot e^{-r \cdot \vartheta} + B_2^D(0, t + \vartheta) \cdot e^{-r \cdot \vartheta} \right] \\
+ \sum_{\vartheta = 1}^{T-t} b_{13}^D(t - s, t; t + \vartheta) \cdot \left[ \sum_{\vartheta' = 0}^{\vartheta-1} -\psi_1^D \cdot e^{-r \cdot \vartheta'} + \gamma_{13}^D \cdot e^{-r \cdot \vartheta} \right]
\]
Model framework: Insurance contracts

- Prospective reserve for a disabled policyholder:

\[ B^D_2(s, t) = d^D_{22}(t - s, t; T) \cdot \sum_{\vartheta = 1}^{T-t} \psi^D_2 \cdot e^{-r \cdot \vartheta} \]

\[ + \sum_{\vartheta = 1}^{T-t} b^D_{21}(t - s, t; t + \vartheta) \cdot \left[ \sum_{\vartheta' = 1}^{\vartheta-1} \psi^D_2 \cdot e^{-r \cdot \vartheta'} + B^D_1(0, t + \vartheta) \cdot e^{-r \cdot \vartheta} \right] \]

\[ + \sum_{\vartheta = 1}^{T-t} b^D_{23}(t - s, t; t + \vartheta) \cdot \left[ \sum_{\vartheta' = 1}^{\vartheta-1} \psi^D_2 \cdot e^{-r \cdot \vartheta'} + \gamma^D_{23} \cdot e^{-r \cdot \vartheta} \right]. \]

- Book value: \( B^D(t) = \sum_{s=0}^{t} n^D_1(s, t) \cdot B^D_1(s, t) + \sum_{s=0}^{t} n^D_2(s, t) \cdot B^D_2(s, t) \)

- For all 3 insurance contracts: Premiums and benefits were calculated according to the actuarial equivalence principle
Model framework: Insurance contracts

- Actual Spanish mortality rates for risk measurement derived by Lee-Carter (1992) model:

\[
\ln(\mu_x(t)) = \alpha_x + \beta_x \cdot \kappa_t + \epsilon_x(t) \iff \mu_x(t) = e^{\alpha_x + \beta_x \cdot \kappa_t + \epsilon_x(t)}
\]

- Modification by Brouhns, Denuit, and Vermunt (2002):

\[
D_{xt} \sim \text{Poisson}(E_{xt} \cdot \mu_x(t)), \quad \mu_x(t) = e^{\alpha_x + \beta_x \cdot \kappa_t}
\]

- Waiting time distribution from the active to the disabled state follows a logistic distribution (see D’Amico, Guillen, and Manca, 2009) and fits the disability tables of the German Actuarial Association

- Waiting time distribution from the disabled state to the active state follows a geometric distribution and fits the recovery tables of the German Actuarial Association
Model framework: Risk measures

- Probability of default (liabilities are not fully covered by assets):

\[ PD = P(T_d \leq T), \quad T_d = \begin{cases} \inf \{t \in [0, T] : A(t) < L(t)\} \\ T + 1, \quad \forall t : A(t) \geq L(t) \end{cases} \]

- Mean loss which measures the discounted expected loss in the case of default:

\[ ML = E \left[ (L(T_d) - A(T_d)) \cdot e^{-r \cdot T_d} \cdot I\{T_d \leq T\} \right] \]

- Standard deviation of liabilities:

\[ \sigma(L(t)) = \sigma \left( B^A(t) + B^D(t) + B^L(t) \right) \]
## Numerical analysis: Input parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of contracts sold</td>
<td>10.000</td>
</tr>
<tr>
<td>Age of the policyholder in $t = 0$</td>
<td>35 (disability, life), 75 (annuity)</td>
</tr>
<tr>
<td>Contract volume</td>
<td>10.000</td>
</tr>
<tr>
<td>Contract term</td>
<td>25</td>
</tr>
<tr>
<td>Discount rate</td>
<td>3%</td>
</tr>
<tr>
<td>Linear function for disabled mortality $c_x \cdot \mu_x(t)$</td>
<td>$c_{35} = 1100%$, $c_{60} = 150%$</td>
</tr>
<tr>
<td>Initial investment $E(0)$</td>
<td>10 million</td>
</tr>
<tr>
<td>Fraction $r_e$ of the positive earnings</td>
<td>25%</td>
</tr>
<tr>
<td>Rate of return $\varepsilon_t$</td>
<td>$\mu_\varepsilon = 5%$, $\sigma_\varepsilon = 8%$</td>
</tr>
<tr>
<td>Transition matrix for disability insurance</td>
<td>$P^D(S) = \begin{bmatrix} 0 &amp; 0.2 &amp; 0.8 \ 0 &amp; 0 &amp; 1 \ 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
</tr>
</tbody>
</table>
Numerical analysis: Initial risk situation

- Risk measures as a function of the fractions of the contracts
Numerical analysis: Natural hedging

- Shock $e_t$ to the time trend $\kappa_t$ of the mortality rate
Numerical analysis: Natural hedging

- Natural hedging between term life insurance and disability insurance

- Comparison with results from Gatzert and Wesker (2012b): Disability insurances are a less efficient hedging tool than annuity insurances
Numerical analysis: Disability risk

- Shock to the expected waiting time from active to disabled state
Summary

● Results:
  ● Overall risk is reduced
  ● Shocks to general mortality have a minor impact on disability insurance
  ● Shocks to disability have a major influence on the risk inherent to the portfolio

● Implications:
  ● Disability insurance is a less efficient hedge for mortality risk
  ● Instruments to hedge disability risk need to be identified
Outlook

- Include the possibility of recoveries
  - Examine influence on optimal portfolio composition
  - Study the impact of the disability risk

- Add death benefits to the disability insurance contract
  - Analyze the consequences for optimal portfolio composition
  - Regard the effect of shocks to general mortality and the specific mortality of disabled policyholders
References

References


Nadine Gatzert and Alexander Maegebier
Friedrich-Alexander-University of Erlangen-Nürnberg
Numerical analysis: Liability structure

- Standard deviation of liabilities
Numerical Analysis: Disability risk

- Shock to transition probability

![Diagram showing the impact of disability insurance on a life insurer’s risk situation]
Numerical analysis: Mortality risk

- Shock to specific mortality of disabled policyholders

Shocks have minor impact.