Aggregation of market risk capital and credit risk capital assessments via integrated scenarios

AFIR / ERM Colloquium, Lyon
Background
Background

» Growing use of 1-year VaR as a measure of insurance firms’ capital requirements

» Capital requirement should allow for joint impact of all risks on the firm’s year balance sheet at 1 year:
  – Market risk
  – Credit (default and migration) risk
  – + mortality risk, lapse risk, operational risk etc

» To calculate overall measures of risk and capital (accounting for all risks) firms often estimate standalone capital requirements and combine using a formula e.g.

\[ EC_{total} = \sqrt{EC_{market}^2 + EC_{credit}^2 + 2\rho_{market,credit}EC_{market}EC_{credit}} \]

  – Formula derived under simplifying assumptions (normality, linearity)
  – Under more realistic assumptions, how do we choose the correlation input to the formula?
Background

» A more general approach would be to create joint scenarios for all risk factors, calculate total impact on balance sheet and resulting overall capital measure

» In practice, scenarios for different risk types are often generated using separate, specialised, models and software systems
  – e.g. Specialised credit models typically model credit risk at a far more granular level than broad (market + credit) risk scenario generators

» Integration of specialised models at a „fundamental“ level may be complicated due to differences in model structure
Capturing dependency via scenario reordering

» Can we create joint scenarios by combining scenarios for individual risks in such a way as to capture key dependencies?

» Our paper explores the use of reordering algorithms as a method for combining scenarios

– Technique previously applied to credit-market risk dependency in the context of counterparty credit risk: Effective Modeling of Wrong Way Risk, Counterparty Credit Risk Capital and Alpha in Basel II, Garcia Cespedes, de Juan Herrero, Rosen & Saunders (2010)
Capturing dependency via scenario reordering

» Retain original scenarios for individual risk factors
  – Marginal distribution assumptions unchanged

» Change order of risk factors relative to each other
  – Changes dependency between such risk factors

» How to choose new order?
  – Proposed approach uses common “bridging” factors to define the reordering

MARKET SCENARIOS

<table>
<thead>
<tr>
<th>Sim</th>
<th>Market Factor 1</th>
<th>MF2 / CF2</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.20</td>
<td>0.32</td>
<td>...</td>
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<tr>
<td>2</td>
<td>0.23</td>
<td>0.15</td>
<td>...</td>
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<tr>
<td>3</td>
<td>0.18</td>
<td>0.24</td>
<td>...</td>
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<tr>
<td>4</td>
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</table>

CREDIT SCENARIOS

<table>
<thead>
<tr>
<th>Sim</th>
<th>Credit Factor 1</th>
<th>MF2 / CF2</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>3</td>
<td>0.26</td>
<td>0.17</td>
<td>...</td>
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<tr>
<td>4</td>
<td>0.20</td>
<td>0.23</td>
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<tr>
<td>...</td>
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<td>...</td>
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</table>

COMBINED SCENARIOS

<table>
<thead>
<tr>
<th>Sim</th>
<th>Market Factor 1</th>
<th>Market Factor 2</th>
<th>Credit Factor 1</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.20</td>
<td>0.32</td>
<td>0.20</td>
<td>...</td>
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<tr>
<td>2</td>
<td>0.23</td>
<td>0.15</td>
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<td>0.26</td>
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<tr>
<td>4</td>
<td>0.15</td>
<td>0.12</td>
<td>0.31</td>
<td>...</td>
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<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
Credit model structure

» We consider a granular credit model where credit quality of individual firms depends on systematic country and industry factors.

\[ r_k = \sqrt{RSQ_k} \phi_k + \sqrt{1 - RSQ_k} \epsilon_k \]

- **Firm/Counterparty**
- **Systematic Factor**
- **Idiosyncratic Factor**

49 Country Factors

61 Industry Factors
Expansion of the credit model

» Idea is to expand the credit model to include additional random variables representing market risk factors:

- Estimation by measuring correlation between historical credit factors and market risks
- Chosen to be consistent with market risk model assumptions

» See *Modeling Credit Correlations Using Macroeconomic Variables*, Moody’s Analytics Modeling Methodology (December 2012)
Example: single bridging factor
Example

» Aim to calculate economic capital requirements (1-year VaR) for an asset portfolio
  – Corporate Bonds (50%)
  – Equities (30%)
  – Real Estate (20%)

» 10,000 scenarios generated for market risk factors at one year horizon:
  – Equity returns
  – Real estate returns
  – Risk-free interest-rates
  – Corporate credit spreads

» Independently 10,000 scenarios generated for credit risk factors at one year horizon:
  – Ratings of each issuer in the corporate bond portfolio
Example reordering scheme: Align credit and market scenarios using single bridging factor

» Expand credit risk model to include a single bridging factor: an equity index representing the firms underlying the corporate bond portfolio

» Reorder market risk scenarios so that ranks of equity index are aligned between market and credit risk scenario sets

» Key properties of reordered scenario set:
  1. No change to marginal (market and credit) scenarios
  2. No change to joint market risk scenarios
  3. Changes credit-equity dependency to agree with that produced by expanded credit model
  4. Other credit-market risk dependencies *induced*
Dependency: reordered scenario sets

» Original:

<table>
<thead>
<tr>
<th>Credit loss</th>
<th>S&amp;P500</th>
<th>Real Estate</th>
<th>US Govt 10yr rate</th>
<th>US A 10yr rate</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.00</td>
<td>0.00</td>
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<td>S&amp;P500</td>
<td>0.31</td>
<td>-0.13</td>
<td>-0.09</td>
<td>-0.15</td>
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<tr>
<td>US Real Estate</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>US Govt 10yr rate</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Dependency controlled within ESG
Dependency induced

S&P 500 Index

US Real Estate Index

» Reordered:

<table>
<thead>
<tr>
<th>Credit loss</th>
<th>S&amp;P500</th>
<th>Real Estate</th>
<th>US Govt 10yr rate</th>
<th>US A 10yr rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.56</td>
<td>-0.17</td>
<td>0.07</td>
<td>0.18</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>0.31</td>
<td>-0.13</td>
<td>-0.09</td>
<td>-0.15</td>
</tr>
<tr>
<td>US Real Estate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Govt 10yr rate</td>
<td></td>
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</tr>
</tbody>
</table>

Dependency controlled within credit risk model
Dependency controlled within ESG
Dependency induced

S&P 500 Index

US Real Estate Index
Economic capital

- Use joint scenarios to calculate Economic Capital defined as 99.5% VaR of losses on the asset portfolio (less expected loss)

<table>
<thead>
<tr>
<th>Economic Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint scenarios*</td>
</tr>
<tr>
<td>Credit risk scenarios only</td>
</tr>
<tr>
<td>Market risk scenarios only</td>
</tr>
</tbody>
</table>

* Return on corporate bond portfolio calculated using credit ratings from credit risk model and risk-free rates + credit spreads from market risk model

- Compare with formula approach:

\[ EC_{total} = \sqrt{EC_{market}^2 + EC_{credit}^2 + 2\rho_{market,credit}EC_{market}EC_{credit}} \]

- In this example, the “implied” correlation \( \rho_{market,credit} = 0.29 \)
Example: multiple bridging factors
Reordering using multiple common bridging factors

» So far have considered aligning credit and market scenarios using single bridging factor: an equity index

» Suppose our asset portfolio contains two corporate bond sub-portfolios (US and UK) and two equity sub-portfolios (US and UK)
  – How to choose the common equity factor?

» For example, suppose we choose the S&P500:

<table>
<thead>
<tr>
<th></th>
<th>Credit loss (US)</th>
<th>S&amp;P500</th>
<th>FTSE100</th>
<th>US 10yr Rate</th>
<th>UK 10yr Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit loss (UK)</td>
<td>0.66</td>
<td>-0.73</td>
<td>-0.50</td>
<td>0.09</td>
<td>0.09</td>
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<tr>
<td>S&amp;P500</td>
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<td>-0.33</td>
<td>0.69</td>
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<tr>
<td>FTSE100</td>
<td>-</td>
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<td></td>
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</tr>
<tr>
<td>US 10yr Rate</td>
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<td></td>
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<td>0.64</td>
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<tr>
<td>UK 10yr Rate</td>
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</tr>
</tbody>
</table>

Dependency controlled within expanded credit risk model
Dependency controlled within ESG
Dependency induced

– In this case, the induced correlation between the FTSE 100 and losses on the US bond portfolio is larger than the induced correlation between the FTSE 100 and losses on the UK bond portfolio
Reordering using multiple common risk factors

» Reordering technique extends to multiple bridging factors:

1. Expand and calibrate the credit model to include multiple bridging factors
e.g. S&P 500 and FTSE 100
2. Group market risk factors according to which bridging factor they are attached to
   » In this case, align all risk factors with the S&P500, except the FTSE100
3. Separately reorder each market risk group

» Key properties of reordered scenario set:

1. No change to marginal (market and credit) scenarios
2. No change to joint market risk scenarios within groups
   » But dependencies between market risks across groups are induced
3. Changes dependency between credit and all chosen common risk factors to agree with that produced by expanded credit model
4. Other credit-market risk dependencies induced
Dependency: reordered scenario sets

» Single bridging factor (S&P 500):

![Dependency control within expanded credit risk model](image1.png)

» Two bridging factors (S&P 500 and FTSE 100):

![Dependency control within expanded credit risk model](image2.png)
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Summary
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» Formula based approaches to risk aggregation are based on simplifying assumptions about interaction between different risks

» A more general approach to risk aggregation involves creating joint scenarios for all risk factors, calculating total impact on balance sheet and resulting overall capital measure

» Proposed technique uses output of existing credit and market risk scenario generators

» Technique uses „bridging“ factors to define alignment of credit and market risk scenarios in a way that captures key dependency relationships
References

» S. Morrison: *Aggregation of market and credit risk capital requirements via integrated scenarios*, Barrie & Hibbert Research report (2013)

