Solvency assessment within the ORSA framework
Issues and quantitative methodologies

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Outlines

1. Introduction
2. Notion of multi-year solvency
3. Parametric solutions to assess the Overall Solvency Needs
4. Application
5. Conclusion
Article 45 - Solvency II directive:

As part of its risk-management system every insurance undertaking and reinsurance undertaking shall conduct its own risk and solvency assessment. That assessment shall include at least the following:

(a) the overall solvency needs taking into account the specific risk profile, approved risk tolerance limits and the business strategy of the undertaking;

(b) the compliance, on a continuous basis, with the capital requirements and with the requirements regarding technical provisions;
**Own Risk and Solvency Assessment - Quantitative issues**

Article 45 - Solvency II directive:

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(a) *the overall solvency needs taking into account the specific risk profile, approved risk tolerance limits and the business strategy of the undertaking;*

(b) *the compliance, on a continuous basis, with the capital requirements and with the requirements regarding technical provisions;*

- Introduces two highly quantitative issues inherent to the ORSA process
  - Definition and assessment of the Overall Solvency Needs, required capital amount to withstand a solvency constraint coherent with the Risk Appetite
  - Development a process to guarantee the continuous compliance with the regulatory requirements linked to Pillar I
Main contributions
- Practical formalization of the multi-year solvency concept
- Development of implementation tools for the Overall Solvency Needs assessment adapted to life insurance
- Implementation on a standardized life insurance product

Selected literature on the subject

**Regulatory notion of solvency**

Let $NAV_t$ be the Net Asset Value at date $t$, $SCR_t$ be the Solvency Capital Requirement at date $t$ and $SR_t$ be the Solvency Ratio at date $t$ (random variables as soon as $t > 0$)

- **Solvency constraint in a one-year framework (regulatory approach)**
  - **Constraint:** $RegulatorySolvency \iff P[NAV_1 \geq 0] \geq 99.5\%$
  - **Practical Issue:** assessment of the $NAV_{0.5\%}$ - empiric quantile at 1y (requires the use of Nested Simulations)

- Monte-Carlo approach, two levels of simulation required in the case of an Internal Model to assess $SCR_0 = NAV_0 + K / : K = -VaR_{0.5\%}(\delta_1 NAV_1)$
NOTION OF MULTI-YEAR SOLVENCY

ORSA framework – Various interpretation of the multi-year solvency

- Multi-year adaptation of the regulatory constraint (constraint on economic bankruptcy)
  - **Question**: level of own funds at \( t = 0 \) required to withstand economic bankruptcy on the whole horizon \([1, T]\) with a \( p \) threshold
  - **Constraint**: \((SC1) \iff \mathbb{P}[\cap_{0 < t \leq T} \{ \text{NAV}_t \geq 0 \}] \geq p \)
    
    \[
    \text{RequiredCapital}_{SC1} = \text{NAV}_0 + K / : K = \text{Argmin}_X \left( \mathbb{P} \left[ \cap_{0 < t \leq T} \left\{ \frac{\text{NAV}_t + \frac{X}{\delta_t}}{\text{SCR}_t} \geq \alpha \% \right\} \right] \right) \geq p
    \]

- Adjustment of the underlying risky variable (constraint on solvency shortfalls)
  - **Question**: level of own funds at \( t = 0 \) required to ensure the coverage of at least a level \( \alpha \) of the regulatory capital on the whole time horizon with a \( p \) threshold
  - **Constraint**: \((SC2) \iff \mathbb{P} \left[ \cap_{0 < t \leq T} \left\{ \frac{\text{NAV}_t}{\text{SCR}_t} \geq \alpha \% \right\} \right] \geq p \)
    
    \[
    \text{RequiredCapital}_{SC2} = \text{NAV}_0 + K / : K = \text{Argmin}_X \left( \mathbb{P} \left[ \cap_{0 < t \leq T} \left\{ \frac{\text{NAV}_t + \frac{X}{\delta_t}}{\text{SCR}_t} \geq \alpha \% \right\} \right] \right) \geq p
    \]
NOTION OF MULTI-YEAR SOLVENCY

Major practical issue: difficulties to assess the required capital

- In practice this assessment would require multi-year projections of NAV / SCR empirical outcomes
  - Three levels of Nested Simulations in an Internal Model framework (we add the time-dimensions)
  - If a Standard Formula is considered, only two levels of simulations

→ Unusable in practice, requires the development of proxy methodologies

In this presentation we address the (SC2) implementation issue and propose two proxy methodologies adapted to a life insurance
Elementary Risk Factors

Consider the simulations used to assess Monte-Carlo outcomes of NAV and SCR

- Primary simulations
  - Real World simulations of various economic drivers (stock index, ZCB on various maturities...)
  - Relevant to try and synthesize the information embedded in each drivers’ diffusion in a minimal number of simplified factors

  $\rightarrow$ Notion of Elementary Risk Factors (ERF)

- Elementary risk factor := simple tools that enable one to trace the evolution of the modeled risks on each one-year period $[t - 1, t]$

  - Consider a stochastic stock index denoted $S^i_t$ at date $t$ and for scenario $i$, and a ZCB denoted $ZCB^i(t, m)$ at date $t$, for scenario $i$ and maturity $m$
**Parametric proxies**

Basic idea: The NAV can be approximated by its conditional expectation given the \( \sigma \)-field generated by the ERF

**H0:** This conditional expectation is a polynomial form of the ERF

- Two distinct methodologies can be intuited, the Curve Fitting (CF) and Least Squares Monte-Carlo (LSMC)
- **Same principle, calibration of a polynomial function of the ERF that approximates the NAV at each date** \( t \).
- Concept opposition between the Net Asset Value and the Net Present Value of margins (NPV)

The CF (resp. LSMC) methodology consists in the implementation of a multiple linear regression on a small (resp. large) number of \( \hat{NAV}_t \) (resp. \( NPV_t \)) outcomes
PARAMETRIC PROXIES: 1-YEAR FRAMEWORK

Implementation sequence - case of two risk factors (stocks and IR - level risk)

Standard multilinear regression: Calibration of the optimal set of regressors $X_t = (I, X^n_t, \ldots, X^n_k)$, with

$$i X_t = s \varepsilon^i_t . Z C \varepsilon^t_y,$$

then determination of $\hat{\beta}_t = (\hat{\beta}_t, \hat{\beta}_t, \ldots, \hat{\beta}_t)'$, the OLS parameters’ estimator of the multiple regression $Y_t = X_t . \beta_t + u_t$ where $Y_t = \hat{NAV}_t / NPV_t$.

The underlying assumption of both the CF and LSMC methodologies at each date $t$ can be written

$$\exists \beta_t, E [Y_t | X_t] = X_t . \beta_t$$

Illustration at 1 year to replicate the central Net Asset Value

- CF calibration on $N_1 \hat{NAV}_1$ outcomes
- LSMC calibration on $N_2 NPV_1$ outcomes ($N_1 << N_2$)
**Parametric proxies: multi-year framework**

- Adaptation at $T$ years to obtain empirical distributions of $NAV$ and $SR$
  
  - Calibration of one polynomial proxy per considered Standard Formula shock (Stock, IR, spread,...) and per projection year
  
  - Standard Formula aggregation of the approximated $NAV$ (central and marginally shocked) to assess the $SCR$ and then the $SR$
THEORETICAL COMPARISON

EQUIVALENCE OF THE OPTIMAL PARAMETERS

Remark: The NAV outcomes considered in the CF framework are Monte-Carlo approximations of the real NAV outcomes (unknown random variable in practice). One can consider three distinct multi-linear regressions:

- **CF regression** - $\hat{NAV}_t = \beta_1.X_t + u_t$, assumption $\mathbb{E}[\hat{NAV}_t | X_t] = X_t.\beta_1$
- **LSMC regression** - $NPV_t = \beta_2.X_t + v_t$, assumption $\mathbb{E}[NPV_t | X_t] = X_t.\beta^2$
- **Underlying regression** - $NAV_t = \beta.X_t + v_t$, assumption $\mathbb{E}[NAV_t | X_t] = X_t.\beta$

Under these assumptions

Result 1: $\beta_1 = \beta_2 = \beta$
THEORETICAL COMPARISON

COMPARISON UNDER THE ASYMPTOTIC OLS ASSUMPTIONS

**Curve Fitting:**
*N* primary simulations
Secondary tables of *P* simu.
Algorithmic complexity *N*.

<table>
<thead>
<tr>
<th>Question?</th>
</tr>
</thead>
<tbody>
<tr>
<td>For a similar asymptotic efficiency: <em>N</em> = ?</td>
</tr>
</tbody>
</table>

**Answer:**

For a similar asymptotic efficiency: *N*.

**LSMC:**
*N* primary simulations
1 secondary scenarios
Algorithmic complexity *N*

Recall the three multi-linear regressions

- **CF regression** - \( \hat{NAV}_t = \beta^1.X_t + u_t \), assumption \( \mathbb{E}[\hat{NAV}_t | X_t] = X_t.\beta^1 \)
- **LSMC regression** - \( NPV_t = \beta^2.X_t + \nu_t \), assumption \( \mathbb{E}[NPV_t | X_t] = X_t.\beta^2 \)
- **Underlying regression** - \( NAV_t = \beta.X_t + \nu_t \), assumption \( \mathbb{E}[NAV_t | X_t] = X_t.\beta \)

Equalizing the asymptotic speeds of convergence of the OLS estimators \( \beta^1 \) and \( \beta^2 \) and denoting \( \mathbb{E}[\mathbb{V}[NPV_t | F_t]] = \sigma_{NPV_t}^2 \), one obtains

**Result 2:** \( N_2 = N_1.P. \left( 1 + \frac{\sigma_w^2}{\sigma_{NPV_t}^2} \right) \leq N_1.P \)
IMPLEMENTATION FRAMEWORK

Test on a realistic but simplified framework

- Two major financial risks
  - Stock (level)
  - IR (level)

Three implementations

- Reference implementation
  - Projection through 5y and MC calculation of 5000 joint outcomes (central / shocked NAV)
  - Secondary tables of 500 scenarios

- Curve Fitting calibration
  - 100 primary scenarios × 500 secondary scenarios

- LSMC calibration
  - 50 000 primary scenarios (× 1 secondary scenario)
Results Curve Fitting

QQ plots $NAV_t$ vs. $c_t NAV_t$

QQ plots $SR_t$ vs. $c_t SR_t$
**Results LSMC**

Example of proxy obtained for the central $NAV$ at $t = 5$

$$LSMC_{NAV_5} = I + \alpha_1^{LSMC} NAV_4 + \alpha_2^{LSMC} NAV_4 \cdot s^5 + \alpha_3^{s} e_5 + \alpha_4^{zc} e_3 + \alpha_5^{s^2} e_5^2 + \alpha_6^{zc} e_5^2 + \alpha_7^{zc} e_5 \cdot zc e_4 + \alpha_8^{zc} e_5 \cdot zc e_3 + \alpha_9^{zc} e_5 \cdot zc e_2 + \alpha_{10}^{zc} e_5 \cdot zc e_1 + \alpha_{11}^{zc} e_1$$
Overall Solvency Needs assessment

Relative differences between the assessed required capitals

<table>
<thead>
<tr>
<th>Multi-year solvency constraint</th>
<th>Curve Fitting</th>
<th>LSMC</th>
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</thead>
<tbody>
<tr>
<td>((SC2) : \mathbb{P}[ \cap_{0 &lt; t \leq T} \left{ \frac{NAV_t}{SCR_t} \geq 110% \right}] \geq 85%)</td>
<td>11.9%</td>
<td>9.3%</td>
</tr>
</tbody>
</table>

One can also consider a constraint on solvency shortfalls that does not take path-dependence into account

<table>
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<th>Curve Fitting</th>
<th>LSMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>((SC2bis) : \forall 0 &lt; t \leq T, \mathbb{P}\left[ \frac{NAV_t}{SCR_t} \geq 110% \right] \geq 85%)</td>
<td>7.9%</td>
<td>6.7%</td>
</tr>
</tbody>
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Remark: During the application we have observed that the relative differences between the NAV and the approximated NAV (central / shocked) can partially be explained by the sampling bias introduced by the MC calculations.
CONCLUSION

- Development of proxy methodologies with theoretical limits but great practical interest to assess future values of the central and shocked NAV
- The use of CF or LSMC allows to consider the most complex multi-year solvency metrics with fast and satisfactory empirical results

Future developments
- Test of the efficiency comparison formula on simple financial instruments
- Proxy recalibration frequency issue
- Study of the sample bias impact
- Assessment of theoretical results in an heteroskedastic framework