Systematic Mortality Risk: An Analysis of Guaranteed Lifetime Withdrawal Benefits in Variable Annuities

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Background

- Variable annuity (VA): retirement savings with equity exposure and guarantees
- Guarantees embedded in VA can protect both downside investment risk, income and longevity risk (GMAB (accumulation), GMDB (death), GMIB (Income), GMWB (withdrawal))
- Guarantee lifetime withdrawal benefit (GLWB) is popular; provides guaranteed life time income in retirement
- Current literature focuses mainly on pricing and financial risks
- Our research considers longevity risk and its interaction with other risks
Market size

Figure: Market share (in $ in millions) in fourth quarter 2011 (US). Source: LIMRA
Presentation coverage

- Features of GLWB
- Investment and stochastic mortality model
- Pricing of GLWB
- Sensitivity analysis
- P&L analysis
- Static hedging of longevity risk
- Conclusions
Features of GLWB

- Attached to variable annuity, invested in mutual funds
- Policyholder can withdraw a capped amount of money from the account periodically (e.g. 5% at age 65)
- Insurer guarantees withdrawal for life, even if account drops to zero, for a guarantee fee as % of fund value
- Any money left in the account is returned to policyholder’s beneficiary after death
- Providers subject to financial risk, demographic (longevity) risk and behavioral risk
Simulated Fund Value GLWB

Figure: Simulated Fund Value: Liability for the insurer
Simulated Fund Value GLWB

Figure: Simulated Fund Value: No liability for the insurer
Assumptions:

- Continuous time models
- geometric Brownian motion for equity
- Constant interest rates and volatility of equity
- Affine time homogeneous process for mortality rate
- Market prices of risk for equity and mortality
- “Static” withdrawal: policyholders withdraw exactly the guaranteed amount every period
- No other features such as roll-up and ratchet, though can be incorporated in our evaluation model
Models

Equity fund & savings account:

\[ dS(t) = \mu S(t)dt + \sigma S(t)dW_1(t), \quad dB(t) = r B(t)dt \]

Mixed fund (\( \pi \): equity exposure):

\[ dV(t) = (\mu \pi + r(1 - \pi))V(t)dt + \sigma \pi V(t)dW_1(t) \]

Investment account:

\[ dA(t) = (\mu \pi + r(1 - \pi) - \alpha_g)A(t)dt - g A(0) dt + \sigma \pi A(t)dW_1(t) \]

where \( \alpha_g \): guaranteed fee rate and \( g \): guaranteed withdrawal rate
Stochastic mortality model:

\[ d\mu_{x+t}(t) = (a + b \mu_{x+t}(t))dt + \sigma_\mu \sqrt{\mu_{x+t}(t)}dW_2(t), \quad \mu_x(0) > 0 \]

where \( \mu_{x+t}(t) \) is mortality intensity

- Non mean reverting when \( b > 0 \)
- Closed form expressions for survival probability
- Similar to Gompertz model when \( \sigma_\mu = 0 \); allows comparison between deterministic (no long.) and stochastic mortality (with long.)
- Under \( \mathbb{Q} \): \( dW^Q_2(t) = \lambda \sqrt{\mu_{x+t}(t)}dt + dW_2(t) \)
Pricing

Policyholder’s perspective

Value of GLWB

\[ V^P(t) = V^P_1(t) + V^P_2(t) - 1\{\hat{\tau} > t\}A(t) \]

where

\[ V^P_1(t) = 1\{\hat{\tau} > t\} g A_0 \int_0^{\omega-x-t} sP_{x+t} e^{-rs} \, ds \]

and

\[ V^P_2(t) = 1\{\hat{\tau} > t\} \int_0^{\omega-x-t} f_{x+t}(s) E_t^Q \left( e^{-rs} (\tilde{A}(t+s))^+ \right) \, ds \]

Fair guaranteed fee rate \( \alpha^*_g \): solve

\[ V^P(0) = 0 \Rightarrow V^P_1(0) + V^P_2(0) = A(0) \]
Insurer's perspective

Value of GLWB

\[ V^l(t) = V_1^l(t) - V_2^l(t) \]

where

\[
V_1^l(t) = 1\{\hat{\tau}>t\} \int_0^{\omega-x-t} f_{x+t}(s) E_t^Q \left( \int_{t+\hat{u}}^{t+s} g A_0 e^{-r(v-t)} 1\{s>\hat{u}\} dv \right) ds
\]

and

\[
V_2^l(t) = 1\{\hat{\tau}>t\} \int_0^{\omega-x-t} f_{x+t}(s) E_t^Q \left( \int_{t}^{t+(\hat{u} \wedge s)} e^{-r(v-t)} \alpha_g A(v) dv \right) ds
\]

Fair guaranteed fee rate \(\alpha_g^*:\) solve

\[ V^l(0) = 0 \Rightarrow V_1^l(0) = V_2^l(0) \]
Sensitivity Analysis

Figure: (Left) Sensitivity of vol. of mortality $\sigma_\mu$ (Right) Sensitivity of market price of longevity risk represented by $\lambda$

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GLWB: Risk Analysis
Figure: (Left) Sensitivity of interest rate $r$ (Right) Sensitivity of investment account’s volatility $\pi \cdot \sigma$
<table>
<thead>
<tr>
<th>Case</th>
<th>Mean</th>
<th>Std</th>
<th>VaR(_{0.995})</th>
<th>ES(_{0.995})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi = 0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no long.</td>
<td>0.0011</td>
<td>0.0008</td>
<td>-0.0010</td>
<td>-0.0013</td>
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<tr>
<td>with long.</td>
<td>0.0062</td>
<td>0.0123</td>
<td>-0.0470</td>
<td>-0.0556</td>
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<tr>
<td>(\pi = 0.5)</td>
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<tr>
<td>no long.</td>
<td>0.0590</td>
<td>0.0624</td>
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<td>-0.1863</td>
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<td>with long.</td>
<td>0.0807</td>
<td>0.0838</td>
<td>-0.1988</td>
<td>-0.2369</td>
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<tr>
<td>(\pi = 1)</td>
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<tr>
<td>no long.</td>
<td>0.2136</td>
<td>0.3579</td>
<td>-0.3236</td>
<td>-0.3532</td>
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<tr>
<td>with long.</td>
<td>0.2520</td>
<td>0.4599</td>
<td>-0.3628</td>
<td>-0.4047</td>
</tr>
</tbody>
</table>
Table: Parameter risk: Discounted P&L distribution

<table>
<thead>
<tr>
<th>Case</th>
<th>Mean</th>
<th>Std</th>
<th>$\text{VaR}_{0.995}$</th>
<th>$\text{ES}_{0.995}$</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$r = 4%$</td>
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<tr>
<td>5%</td>
<td>0.0731</td>
<td>0.1214</td>
<td>-0.2855</td>
<td>-0.3264</td>
</tr>
<tr>
<td>6%</td>
<td>0.0326</td>
<td>0.0901</td>
<td>-0.2872</td>
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<tr>
<td>7%</td>
<td>0.0055</td>
<td>0.0718</td>
<td>-0.2882</td>
<td>-0.3299</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sigma = 25%$</td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>0.0446</td>
<td>0.0995</td>
<td>-0.2882</td>
<td>-0.3292</td>
</tr>
<tr>
<td>15%</td>
<td>0.0731</td>
<td>0.1214</td>
<td>-0.2855</td>
<td>-0.3264</td>
</tr>
<tr>
<td>20%</td>
<td>0.1058</td>
<td>0.1475</td>
<td>-0.2764</td>
<td>-0.3175</td>
</tr>
<tr>
<td></td>
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<td>$\sigma_{\mu} = 0.021$</td>
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</tr>
<tr>
<td>0.005</td>
<td>0.1079</td>
<td>0.1487</td>
<td>-0.2816</td>
<td>-0.3231</td>
</tr>
<tr>
<td>0.01</td>
<td>0.1175</td>
<td>0.1564</td>
<td>-0.2733</td>
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<tr>
<td>0.015</td>
<td>0.1250</td>
<td>0.1633</td>
<td>-0.2751</td>
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<td></td>
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<td>$\lambda = 0.4$</td>
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<tr>
<td>0.1</td>
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<td>0.1619</td>
<td>-0.2743</td>
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<tr>
<td>0.2</td>
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<tr>
<td>0.3</td>
<td>0.1321</td>
<td>0.1684</td>
<td>-0.2759</td>
<td>-0.3188</td>
</tr>
</tbody>
</table>
### Table: Discounted P&L per dollar received; 1000 policyholders

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<th>ES$_{0.995}$</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\pi = 0$</td>
<td></td>
</tr>
<tr>
<td>no hedge</td>
<td>0.0062</td>
<td>0.0123</td>
<td>-0.0470</td>
<td>-0.0556</td>
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<tr>
<td>static hedge</td>
<td>0.0033</td>
<td>0.0119</td>
<td>-0.0266</td>
<td>-0.0303</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\pi = 0.5$</td>
<td></td>
</tr>
<tr>
<td>no hedge</td>
<td>0.0807</td>
<td>0.0838</td>
<td>-0.1988</td>
<td>-0.2369</td>
</tr>
<tr>
<td>static hedge</td>
<td>0.0779</td>
<td>0.0847</td>
<td>-0.1983</td>
<td>-0.2343</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\pi = 1$</td>
<td></td>
</tr>
<tr>
<td>no hedge</td>
<td>0.2520</td>
<td>0.4599</td>
<td>-0.3628</td>
<td>-0.4047</td>
</tr>
<tr>
<td>static hedge</td>
<td>0.2492</td>
<td>0.4604</td>
<td>-0.3629</td>
<td>-0.4030</td>
</tr>
</tbody>
</table>
Conclusions

- Evaluation of GLWB with longevity risk in continuous time framework considering both equity and longevity risk
- Sensitivity analysis for different financial and mortality variables including equity volatility, interest rates and price of mortality risk - financial risk is important as is longevity risk
- Longevity risk and parameter risk using P&L distribution - present valuing shows pricing risks are less dependent on stochastic mortality assumption
- Static hedging using S-forwards - confirms effectiveness of these approaches
Thank you for your attention

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