Systematic Mortality Risk: An Analysis of Guaranteed Lifetime Withdrawal Benefits in Variable Annuities

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Background

- Variable annuity (VA): retirement savings with equity exposure and guarantees
- Guarantees embedded in VA can protect both downside investment risk, income and longevity risk (GMAB (accumulation), GMDB (death), GMIB (Income), GMWB (withdrawal))
- Guarantee lifetime withdrawal benefit (GLWB) is popular; provides guaranteed life time income in retirement
- Current literature focuses mainly on pricing and financial risks
- Our research considers longevity risk and its interaction with other risks

Market size

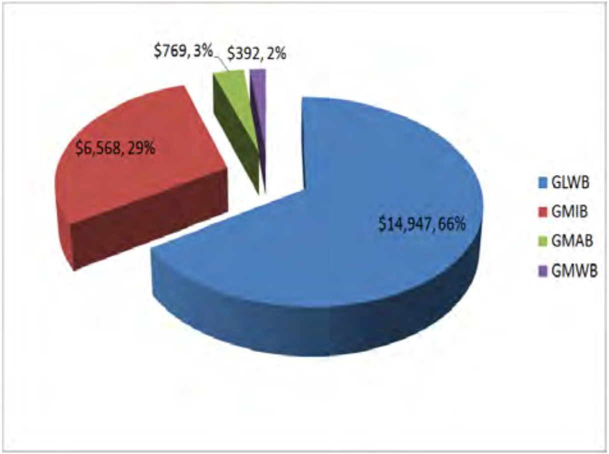

Figure : Market share (in $ in millions) in fourth quarter 2011 (US). Source: LIMRA

Presentation coverage

- Features of GLWB
- Investment and stochastic mortality model
- Pricing of GLWB
- Sensitivity analysis
- P&L analysis
- Static hedging of longevity risk
- Conclusions

Features of GLWB

- Attached to variable annuity, invested in mutual funds
- Policyholder can withdraw a capped amount of money from the account periodically (e.g. 5% at age 65)
- Insurer guarantees withdrawal for life, even if account drops to zero, for a guarantee fee as % of fund value
- Any money left in the account is returned to policyholder's beneficiary after death
- Providers subject to financial risk, demographic (longevity) risk and behaviorial risk

Simulated Fund Value GLWB

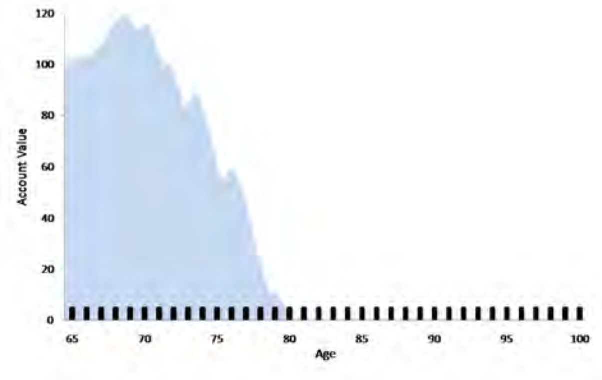

Figure : Simulated Fund Value: Liability for the insurer

Simulated Fund Value GLWB

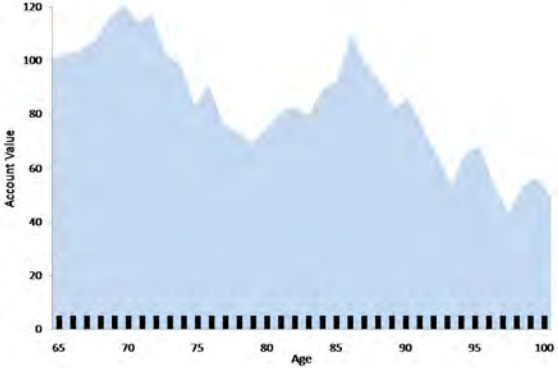

Figure : Simulated Fund Value: No liability for the insurer

Assumptions:

- Continuous time models
- geometric Brownian motion for equity
- Constant interest rates and volatility of equity
- Affine time homogeneous process for mortality rate
- Market prices of risk for equity and mortality
- “Static" withdrawal: policyholders withdraw exactly the guaranteed amount every period
- No other features such as roll-up and ratchet, though can be incorporated in our evaluation model

Models

Equity fund & savings account:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW_1(t), \quad dB(t) = r\,B(t)dt$$

Mixed fund (π: equity exposure):

$$dV(t) = (\mu\pi + r(1-\pi))V(t)dt + \sigma\pi V(t)dW_1(t)$$

Investment account:

$$dA(t) = (\mu\pi + r(1-\pi) - \alpha_g)A(t)dt - g\,A(0)\,dt + \sigma\pi A(t)dW_1(t)$$

where α_g: guaranteed fee rate and g: guaranteed withdrawal rate

Stochastic mortality model:

$$d\mu_{x+t}(t) = (a + b\,\mu_{x+t}(t))dt + \sigma_\mu \sqrt{\mu_{x+t}(t)}dW_2(t), \quad \mu_x(0) > 0$$

where $\mu_{x+t}(t)$ is mortality intensity

- Non mean reverting when $b > 0$
- Closed form expressions for survival probability
- Similar to Gompertz model when $\sigma_\mu = 0$; allows comparison between deterministic (no long.) and stochastic mortality (with long.)
- Under $\mathbb{Q}$: $dW_2^{\mathbb{Q}}(t) = \lambda\sqrt{\mu_{x+t}(t)}dt + dW_2(t)$

Pricing

Policyholder's perspective
Value of GLWB

$$V^P(t) = V_1^P(t) + V_2^P(t) - 1_{\{\hat{\tau}>t\}} A(t)$$

where

$$V_1^P(t) = 1_{\{\hat{\tau}>t\}}\, g\, A_0 \int_0^{\omega-x-t} {}_sP_{x+t}\, e^{-rs}\, ds$$

and

$$V_2^P(t) = 1_{\{\hat{\tau}>t\}} \int_0^{\omega-x-t} f_{x+t}(s) E_t^{\mathbb{Q}} \left(e^{-rs} \, (\tilde{A}(t+s))^+ \right) ds$$

Fair guaranteed fee rate α_g^*: solve

$$V^P(0) = 0 \Rightarrow V_1^P(0) + V_2^P(0) = A(0)$$

Insurer's perspective

Value of GLWB

$$V^I(t) = V_1^I(t) - V_2^I(t)$$

where

$$V_1^I(t) = 1_{\{\hat{\tau}>t\}} \int_0^{\omega-x-t} f_{x+t}(s)\, E_t^{\mathbb{Q}} \left(\int_{t+\hat{u}}^{t+s} g\, A_0 e^{-r(v-t)} 1_{\{s>\hat{u}\}}\, dv \right) ds$$

and

$$V_2^I(t) = 1_{\{\hat{\tau}>t\}} \int_0^{\omega-x-t} f_{x+t}(s)\, E_t^{\mathbb{Q}} \left(\int_t^{t+(\hat{u}\wedge s)} e^{-r(v-t)} \alpha_g\, A(v)\, dv \right) ds$$

Fair guaranteed fee rate α_g^*: solve

$$V^I(0) = 0 \Rightarrow V_1^I(0) = V_2^I(0)$$

Sensitivity Analysis

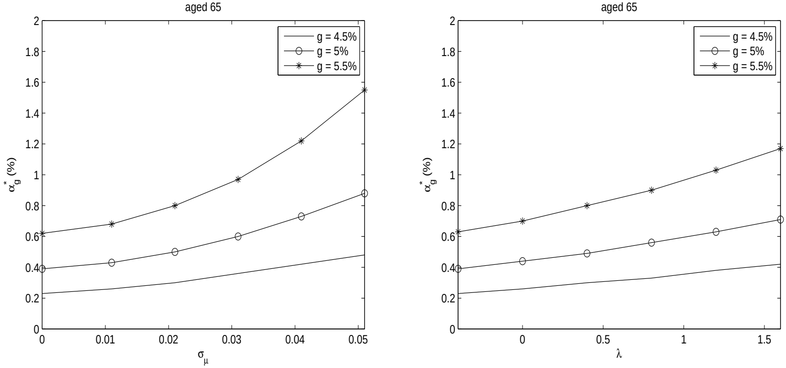

Figure : (Left) Sensitivity of vol. of mortality σ_μ (Right) Sensitivity of market price of longevity risk represented by λ

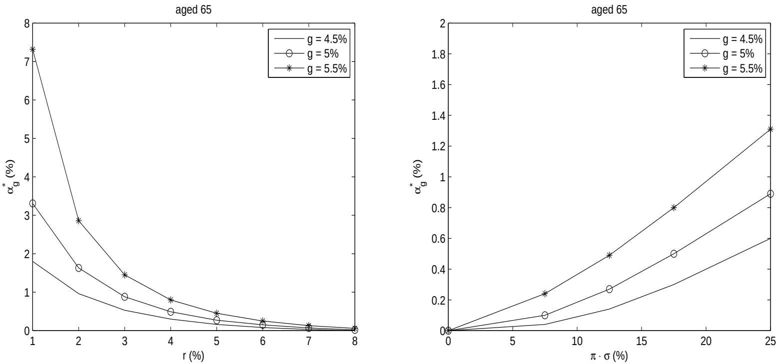

Figure : (Left) Sensitivity of interest rate *r* (Right) Sensitivity of investment account's volatility $\pi \cdot \sigma$

Table : Longevity risk; Discounted P&L per dollar received; 1000 policyholders

Case	Mean	Std	$VaR_{0.995}$	$ES_{0.995}$
		$\pi = 0$		
no long.	0.0011	0.0008	-0.0010	-0.0013
with long.	0.0062	0.0123	-0.0470	-0.0556
		$\pi = 0.5$		
no long.	0.0590	0.0624	-0.1595	-0.1863
with long.	0.0807	0.0838	-0.1988	-0.2369
		$\pi = 1$		
no long.	0.2136	0.3579	-0.3236	-0.3532
with long.	0.2520	0.4599	-0.3628	-0.4047

Table : Parameter risk: Discounted P&L distribution

Case	Mean	Std	$\mathrm{VaR}_{0.995}$	$\mathrm{ES}_{0.995}$
		$r = 4\%$		
5%	0.0731	0.1214	-0.2855	-0.3264
6%	0.0326	0.0901	-0.2872	-0.3280
7%	0.0055	0.0718	-0.2882	-0.3299
		$\sigma = 25\%$		
10%	0.0446	0.0995	-0.2882	-0.3292
15%	0.0731	0.1214	-0.2855	-0.3264
20%	0.1058	0.1475	-0.2764	-0.3175
		$\sigma_\mu = 0.021$		
0.005	0.1079	0.1487	-0.2816	-0.3231
0.01	0.1175	0.1564	-0.2733	-0.3152
0.015	0.1250	0.1633	-0.2751	-0.3212
		$\lambda = 0.4$		
0.1	0.1244	0.1619	-0.2743	-0.3186
0.2	0.1263	0.1637	-0.2772	-0.3162
0.3	0.1321	0.1684	-0.2759	-0.3188

Static Hedging with S-forwards

Table : Discounted P&L per dollar received; 1000 policyholders

Case	Mean	Std	$VaR_{0.995}$	$ES_{0.995}$
		$\pi = 0$		
no hedge	0.0062	0.0123	-0.0470	-0.0556
static hedge	0.0033	0.0119	-0.0266	-0.0303
		$\pi = 0.5$		
no hedge	0.0807	0.0838	-0.1988	-0.2369
static hedge	0.0779	0.0847	-0.1983	-0.2343
		$\pi = 1$		
no hedge	0.2520	0.4599	-0.3628	-0.4047
static hedge	0.2492	0.4604	-0.3629	-0.4030

Conclusions

- Evaluation of GLWB with longevity risk in continuous time framework considering both equity and longevity risk
- Sensitivity analysis for different financial and mortality variables including equity volatility, interest rates and price of mortality risk -- financial risk is important as is longevity risk
- Longevity risk and parameter risk using P&L distribution - present valuing shows pricing risks are less dependent on stochastic mortality assumption
- Static hedging using S-forwards - confirms effectiveness of these approaches

Thank you for your attention

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http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2279274