Realistic Tables with Competing Risks
Application to Inception Rates Estimation for Long Term Care Insurance

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## Agenda

1. **Introduction**
2. **Modelisation**
3. **Estimation methods for competing risks**
4. **Numerical Application**
Long Term Care Insurance Framework

- In France, LTC insurance provides benefits for elderly people suffering from a loss of mobility and autonomy in their activity of daily living.
- In addition to the social benefits.
- LTC insurance may be individual or collective.
- Payment of benefits depends to the level of dependency.
Long Term Care Insurance Framework

- Pricing, reserving and managing LTC risks strongly depend to the tables selected.
- LTC risks are tricky to estimate:
  - the first french products dates from the early of 80’s,
  - only a few insurers have reliable data with sufficient amount (but for higher levels of dependency and not at older ages),
  - definitions and grids are not unique,
  - covariate effects (gender, place, pathology, etc.) are rarely taking into account.
- Forecasting LTC risks (longevity, disability and mortality) are also a very difficult exercice.
- Pratitioners often use empirical methods and expert opinions.
Motivation

- Constructing realistic tables for **inception rates** in dependency is an important challenge in Solvency II perspective.
- Distinguishing entry by pathology is very useful as:
  - waiting periods in contracts depend on the type of disease,
  - pathologies have a major role in the survival of LTC claimants.
- In presence of competing risks, practitioners often use techniques based on latent failure times and arbitrary choices for modelling dependence between them.
- **Aims of our approach:**
  - use a more relevant multistate approach to estimate inception rates with right censored data,
  - measure the bias that the common approach used by practitioners comprises.
- **Limits of our approach:**
  - a non-parametric approach requires a significant amount of reliable data and does not permit forecasting to older ages,
  - longitudinal data are required.
Competing Risks Model

- A competing risks process models adequately both the entry in dependency according to different pathologies and other exit causes (e.g. death, cancellation).

Example of exit causes with 4 types of pathology.

<table>
<thead>
<tr>
<th>Exit causes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_1$</td>
<td>Neurologic pathologies</td>
</tr>
<tr>
<td>$e_2$</td>
<td>Various pathologies</td>
</tr>
<tr>
<td>$e_3$</td>
<td>Terminal cancers</td>
</tr>
<tr>
<td>$e_4$</td>
<td>Dementia</td>
</tr>
<tr>
<td>$e_5$</td>
<td>Death</td>
</tr>
<tr>
<td>$e_6$</td>
<td>Cancel</td>
</tr>
</tbody>
</table>

- Benefits and premiums are paid according to the paper of states of the policyholder.
Quantities of Interest

We introduce a Markov process \((X_t)_{t \geq 0}\) with finite state space \(S = \{e_0, e_1, \ldots, e_n\}\) where the state \(e_j\) represents the \(j\)-th exit cause from the initial state. \(T\) represents the survival time in initial state and \(C\) the right-censoring time.

- **Transition probability**

  \[
  p_{0j}(s, t) = \mathbb{P}(X_t = e_j \mid X_s = e_0) = \mathbb{P}(T \leq t, X_T = e_j \mid T > s).
  \]

- **Cause-specific hazard**

  \[
  \mu_{0j}(t) = \lim_{\Delta t \to 0} \frac{p_{0j}(t, t + \Delta t)}{\Delta t}.
  \]

- **Latent failure time**

  \[
  T_{0j} = \inf_{t \geq 0} (X_t = e_j).
  \]
Technical provisions

- Technical provisions correspond to the expectation of future discounted cash-flows relating to the contract.
- With a policyholder aged $x$ years at $t_0$, reserves at time $t \geq t_0$ are

$$\sum_{j \neq e_0} \int_{t-t_0}^{\infty} B(t, t_0 + \tau) p_{00}(x, x + \tau) \mu_{0j}(x + \tau) c_j(x + \tau) d\tau$$

$$- \int_{t-t_0}^{\infty} B(t, t_0 + \tau) p_{00}(x, x + \tau) b(x + \tau) d\tau.$$

- Inception rate $q_j(t)$ appears when we approximate the above formula

$$\approx \sum_{j \neq e_0} \sum_{k=t-t_0}^{\infty} B(t, t_0 + k + 1) p_{00}(x, x + k) q_j(x + k) c_j(x + k)$$

$$- \sum_{k=t-t_0}^{\infty} B(t, t_0 + k) p_{00}(x, x + k) b(x + k).$$
Classical Methods

- A extensive literature is dedicated to infer competing risks model:
  - methods based on latent failure times (e.g. Prentice et al. (1978)),
  - proportional hazards models (e.g. Fine and Gray (1999)),
  - multistate approaches (e.g. Andersen et al. (2002)).

- Practioners often use latent failure times approaches (e.g. Deléglise et al. (2009)) but:
  - latent failure time variable $T_{0j}$ is artificial,
  - dependence structure between latent failure times is unknow.

- These last estimators may overestimate the inception rates (see Gooley et al. (1999)).
Multistate Approach

We aim to estimate inception rates with a non-parametric multistate approach (see Andersen et al. (1993)).

- We observe continuously $M$ independent policyholders $(\bar{T}_m, V_m)^{m=1,\ldots,M}$ where $\bar{T}_m = \min(T_m, C_m)$ and $V = X_T$ the failure cause.
- The cumulative cause-specific hazards are estimated with Nelson-Aalen estimator.
- The inception rates are consequently obtained with the Aalen-Johansen estimator
  \[
  \hat{q}_j(t) = \sum_{\{m,t<\bar{T}_m \leq t+1\}} \hat{S}(\bar{T}_m -) \frac{1}{Y_0(\bar{T}_m)} \frac{1}{Y_0(\bar{T}_m)} \mathbb{1}\{V_m=j\}.
  \]
- These estimators are asymptotically normally distributed.
Latent Failure Times Approach

- Practitioners usually assume latent failure times are independent and then we have

\[ P(T_{0j} > t) = \exp \left( - \int_0^t \mu_{0j}(\tau) \, d\tau \right). \]

- Consequently, the inception rates per event are estimated with Kaplan-Meier estimator and noted \( q_j^* (t) \).

- Then, arbitrary dependence rule are applied on \( q_j^* (t) \) to verify the equality

\[ 1 - \sum_{j=1}^{n} q_j (t) = p_{00} (t, t + 1). \]
Latent Failure Times Approach

- Inception rates adjustment algorithm for some order \((j_1, \ldots, j_n)\) is:

\[
\hat{q}_{j_1}(t) = \hat{q}^*_{j_1}(t) \prod_{k \neq j} \left(1 - \hat{q}^*_k(t)\right),
\]

\[
\hat{q}_{j_2}(t) = \hat{q}^*_{j_2}(t) \left(1 - \hat{q}^*_{j_1}(t)\right),
\]

\[
\vdots
\]

\[
\hat{q}_{j_n}(t) = \hat{q}^*_{j_n}(t) \prod_{k=1}^{n-1} \left(1 - \hat{q}^*_k(t)\right).
\]
Latent Failure Times Approach

- Inception rates adjustment algorithm for some order \((j_1, \ldots, j_n)\) is:
  - \(\hat{q}_{j_1}(t) = \hat{q}_{j_1}^*(t)\)
Latent Failure Times Approach

- Inception rates adjustment algorithm for some order \((j_1, \ldots, j_n)\) is:
  - \(\tilde{q}_{j_1}(t) = \hat{q}_{j_1}^*(t)\)
  - \(\tilde{q}_{j_2}(t) = \hat{q}_{j_2}^*(t)(1 - \hat{q}_{j_1}^*(t))\)
Inception rates adjustment algorithm for some order \((j_1, \ldots, j_n)\) is:

- \(\tilde{q}_{j_1}(t) = \hat{q}_{j_1}(t)\)
- \(\tilde{q}_{j_2}(t) = \hat{q}_{j_2}(t) \left( 1 - \hat{q}_{j_1}(t) \right)\)
- \(\ldots\)
- \(\tilde{q}_{j_n}(t) = \hat{q}_{j_n}(t) \prod_{k=1}^{n-1} \left( 1 - \hat{q}_{j_k}(t) \right)\).
Inception rates adjustment algorithm for some order \((j_1, \ldots, j_n)\) is:

- \(\tilde{q}_{j_1}(t) = \hat{q}_{j_1}^*(t)\)
- \(\tilde{q}_{j_2}(t) = \hat{q}_{j_2}^*(t) \left(1 - \hat{q}_{j_1}^*(t)\right)\),
- \(\ldots\)
- \(\tilde{q}_{j_n}(t) = \hat{q}_{j_n}^*(t) \prod_{k=1}^{n-1} \left(1 - \hat{q}_{j_k}^*(t)\right)\).

The new estimator \(\tilde{q}_j(t)\) has upper and lower bounds

\[
b_j^- (t) = \hat{q}_j^* (t) \prod_{k \neq j} (1 - \hat{q}_k^* (t)) \leq \tilde{q}_j (t) \leq \hat{q}_j^* (t) = b_j^+ (t) .
\]
Estimation with Multistate Approach

Figure: Inception rates estimates with approximate pointwise 95% confidence intervals
Smoothed Rates

- We smooth the crude inception rates with non-parametric Whittaker-Henderson model.
- Smoothing parameters are selected on the basis of residuals analysis and by regarding the following fitting criteria:
  - cross validation and generalized cross validation,
  - AIC and AICC.
Smoothed Rates

Figure: Smoothed rates, residuals and Student residuals
Measuring Estimation Risk

- We consider systematic risks induced by the construction of such tables and resulting from:
  - estimation of crude rates (due to sampling variation),
  - parameters estimation.
- These risks are taken into account with non-parametric bootstrap.
Measuring Estimation Risk

Figure: Simulated rates ($K = 1000$) with 95% simulated confidence intervals
We compute dispersion coefficients

\[ c(\psi_{jx}) = \sqrt{\sum_{k=1}^{K} \left( \tilde{q}_j^k(x) - \tilde{q}_j(x) \right)^2 / \tilde{q}_j(x)} \].

The risk is relatively important and should be considered carefully for technical provisions valuation.

<table>
<thead>
<tr>
<th>Exit causes</th>
<th>Average ( c(\psi_{jx}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neurologic pathologies</td>
<td>6.01%</td>
</tr>
<tr>
<td>Various pathologies</td>
<td>12.12%</td>
</tr>
<tr>
<td>Terminal cancers</td>
<td>7.94%</td>
</tr>
<tr>
<td>Dementia</td>
<td>6.53%</td>
</tr>
<tr>
<td>Death</td>
<td>2.05%</td>
</tr>
<tr>
<td>Cancel</td>
<td>3.76%</td>
</tr>
</tbody>
</table>
Comparing with the latent failure times approach

Figure: Comparing multistate and latent failure times approaches
Comparing with the latent failure times approach

- We compare 4 different priority orders.
- At 65 years old, the larger gap measured on technical provisions is around 2.5%.
- Priority orders should be selected soundly!

**Figure**: Gap on technical provisions with the both approaches
Summary

- Multistate approaches are rarely used by practitioners which prefer Kaplan-Meier estimators and marginal approaches for crude rates estimation.
- Multistate approaches are a little more complex but theoretically more appropriate for competing risks as the joint distribution of latent failure times are unobserved.
- A non-parametric approach may be difficult to implement due to the lack of available data but:
  - it provides a realistic fit that one can expect in Solvency II perspective.
  - it can be used to perform goodness-of-fit tests.
- Bias observed with latent failure times approach depends on the treatment of priorities applied on each cause. In our application, a sound choice may reduce significantly this bias and justify the use of latent failure times approach.

Outlook
- Extrapolating inception rates to older ages.
- Taking into account other sources of heterogeneity.
Thank you for your kind attention.
Some References I


