Realistic Tables with Competing Risks

Application to Inception Rates Estimation for Long Term Care Insurance

Q. Guibert[1 2] F. Planchet[1 3]

[1]ISFA - Laboratoire SAF
Université de Lyon - Université Claude Bernard Lyon 1

[2]Optimind Winter

[3]Prim'Act

June 2013, AFIR-ERM/LIFE/PBSS Colloquium - Lyon

Agenda

Long Term Care Insurance Framework

- In France, LTC insurance provides benefits for elderly people suffering from a loss of mobility and autonomy in their activity of daily living.
- In addition to the social benefits.
- LTC insurance may be individual or collective.
- Payment of benefits depends to the level of dependency.

Long Term Care Insurance Framework

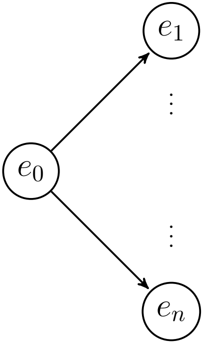
- Pricing, reserving and managing LTC risks strongly depend to the tables selected.
- LTC risks are tricky to estimate:
  - the first french products dates from the early of 80's,
  - only a few insurers have reliable data with sufficient amount (but for higher levels of dependency and not at older ages),
  - definitions and grids are not unique,
  - covariate effects (gender, place, pathology, etc.) are rarely taking into account.
- Forecasting LTC risks (longevity, disability and mortality) are also a very difficult exercice.
- Pratitioners often use empirical methods and expert opinions.

Motivation

- Constructing realistic tables for inception rates in dependency is an important challenge in Solvency II perspective.
- Distinguing entry by pathology is very usefull as:
  - waiting periods in contracts depend on the type of desease,
  - pathologies have a major role in the survival of LTC claimants.
- In presence of competing risks, practioners often use techniques based on latent failure times and arbitrary choices for modelling dependence between them.
- Aims of our approach:
  - use a more relevant multistate approach to estimate inception rates with right censored data,
  - measure the bias that the common approach used by prationers comprises.
- Limits of our approach:
  - a non-parametric approach requires a signicant amount of reliable data and does not permit forecasting to older ages,
  - longitudinal data are required.

Competing Risks Model

- A competing risks process models adequately both the entry in dependency according to different pathologies and other exit causes (e.g. death, cancellation).



Example of exit causes with 4 types of pathology.

	Exit causes
e_1	Neurologic pathologies
e_2	Various pathologies
e_3	Terminal cancers
e_4	Dementia
e_5	Death
e_6	Cancel

- Benefits and premiums are paid according to the papern of states of the policyholder.

Quantities of Interest

We introduce a Markov process $(X_t)_{t\geq 0}$ with finite state space $\mathcal{S} = \{e_0, e_1, \ldots, e_n\}$ where the state e_j represents the j-th exit cause from the initial state. T represents the survival time in initial state and C the right-censoring time.

- Transition probability

$$
\begin{aligned}
p_{0j}(s,t) &= \mathbb{P}\left(X_t = e_j \mid X_s = e_0\right) \\
&= \mathbb{P}\left(T \leq t, X_T = e_j \mid T > s\right).
\end{aligned}
$$

- Cause-specific hazard

$$
\mu_{0j}(t) = \lim_{\Delta t \to 0} \frac{p_{0j}(t, t+\Delta t)}{\Delta t}.
$$

- Latent failure time

$$
T_{0j} = \inf_{t\geq 0}(X_t = e_j).
$$

Technical provisions

- Technical provisions correspond to the expectation of future discounted cash-flows relating to the contract.
- With a policyholder aged x years at t_0, reserves at time $t \geq t_0$ are

$$\sum_{j \neq e_0} \int_{t-t_0}^{\infty} B\left(t, t_0+\tau\right) p_{00}\left(x, x+\tau\right) \mu_{0j}\left(x+\tau\right) c_j\left(x+\tau\right) d\tau$$
$$- \int_{t-t_0}^{\infty} B\left(t, t_0+\tau\right) p_{00}\left(x, x+\tau\right) b\left(x+\tau\right) d\tau.$$

- Inception rate $q_j\left(t\right)$ appears when we approximate the above formula

$$\approx \sum_{j \neq e_0} \sum_{k=t-t_0}^{\infty} B\left(t, t_0+k+1\right) p_{00}\left(x, x+k\right) q_j\left(x+k\right) c_j\left(x+k\right)$$
$$- \sum_{k=t-t_0}^{\infty} B\left(t, t_0+k\right) p_{00}\left(x, x+k\right) b\left(x+k\right).$$

Classical Methods

- A extensive literature is dedicated to infer competing risks model:
  - methods based on latent failure times (e.g. Prentice *et al.* (1978)),
  - proportional hazards models (e.g. Fine and Gray (1999)),
  - multistate approaches (e.g. Andersen *et al.* (2002)).
- Practioners often use latent failure times approachs (e.g. Deléglise *et al.* (2009)) but:
  - latent failure time variable T_{0j} is artificial,
  - dependence structure between latent failure times is unknow.
- These last estimators may overestimate the inception rates (see Gooley *et al.* (1999)).

Multistate Approach

We aim to estimate inception rates with a non-parametric multistate approach (see Andersen *et al.* (1993)).

- We observe continuously M independent policyholders $\left(\bar{T}^m, V^m\right)_{m=1,\dots,M}$ where $\bar{T}^m = \min\left(T^m, C^m\right)$ and $V = X_T$ the failure cause.
- The cumulative cause-specific hazards are estimated with Nelson-Aalen estimator.
- The inception rates are consequently obtained with the Aalen-Johansen estimator
$$\hat{q}_j\left(t\right) = \sum_{\left\{m, t < \bar{T}^{(m)} \leq t+1\right\}} \hat{S}\left(\bar{T}^{(m)}-\right) \frac{\mathbb{1}_{\left\{V^{(m)}=j\right\}}}{Y_0\left(\bar{T}^{(m)}\right)}.$$
- These estimators are asymptotically normally distributed.

Latent Failure Times Approach

- Praticioners usually assume latent failure times are independent and then we have

$$\mathbb{P}\left(T_{0j} > t\right) = \exp\left(-\int_0^t \mu_{0j}\left(\tau\right) d\tau\right).$$

- Consequently, the inception rates per event are estimated with Kaplan-Meier estimator and noted $q_j^*\left(t\right)$.
- Then, arbitrary dependence rule are applied on $q_j^*\left(t\right)$ to verify the egality

$$1 - \sum_{j=1}^{n} q_j\left(t\right) = p_{00}\left(t, t+1\right).$$

Latent Failure Times Approach

- Inception rates adjustment algorithm for some order $(j_1, \ldots, j_n)$ is:

Latent Failure Times Approach

- Inception rates adjustment algorithm for some order $(j_1, \ldots, j_n)$ is:
  - $\breve{q}_{j_1}(t) = \hat{q}^*_{j_1}(t)$

Latent Failure Times Approach

- Inception rates adjustment algorithm for some order $(j_1, \ldots, j_n)$ is:
  - $\breve{q}_{j_1}(t) = \hat{q}^*_{j_1}(t)$
  - $\breve{q}_{j_2}(t) = \hat{q}^*_{j_2}(t)\left(1 - \hat{q}^*_{j_1}(t)\right),$

Latent Failure Times Approach

- Inception rates adjustment algorithm for some order $(j_1, \ldots, j_n)$ is:
  - $\breve{q}_{j_1}(t) = \hat{q}^*_{j_1}(t)$
  - $\breve{q}_{j_2}(t) = \hat{q}^*_{j_2}(t)\left(1 - \hat{q}^*_{j_1}(t)\right),$
  - $\ldots$
  - $\breve{q}_{j_n}(t) = \hat{q}^*_{j_n}(t)\prod_{k=1}^{n-1}\left(1 - \hat{q}^*_{j_k}(t)\right).$

Latent Failure Times Approach

- Inception rates adjustment algorithm for some order $(j_1, \dots, j_n)$ is:
  - $\breve{q}_{j_1}(t) = \hat{q}^*_{j_1}(t)$
  - $\breve{q}_{j_2}(t) = \hat{q}^*_{j_2}(t)\left(1 - \hat{q}^*_{j_1}(t)\right)$,
  - $\dots$
  - $\breve{q}_{j_n}(t) = \hat{q}^*_{j_n}(t)\prod_{k=1}^{n-1}\left(1 - \hat{q}^*_{j_k}(t)\right)$.

- The new estimator $\breve{q}_j(t)$ has upper and lower bounds

$$b_j^-(t) = \hat{q}^*_j(t)\prod_{k\neq j}(1 - \hat{q}^*_k(t)) \leq \breve{q}_j(t) \leq \hat{q}^*_j(t) = b_j^+(t).$$

Estimation with Multistate Approach

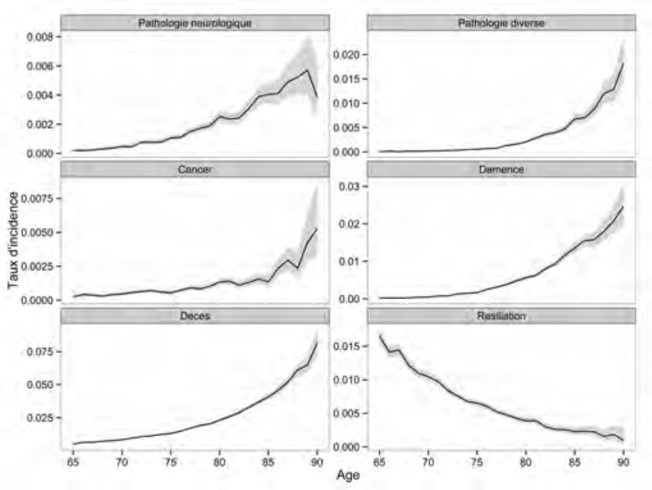


Figure: Inception rates estimates with approximate pointwise 95% confidence intervals

Smoothed Rates

- We smooth the crude inception rates with non-parametric Whittaker-Henderson model.
- Smoothing parameters are selected on the basis of residuals analysis and by regarding the following fitting criteria:
  - cross validation and generalized cross validation,
  - AIC and AICC.

Smoothed Rates

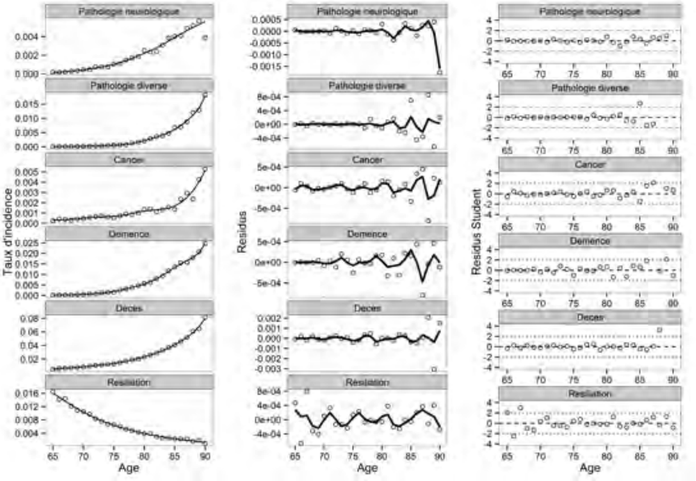


Figure: Smoothed rates, residuals and Student residuals

Measuring Estimation Risk

- We consider systematic risks induced by the construction of such tables and resulting from:
  - estimation of crude rates (due to sampling variation),
  - parameters estimation.
- These risks are taken into account with non-parametric bootstrap.

Measuring Estimation Risk

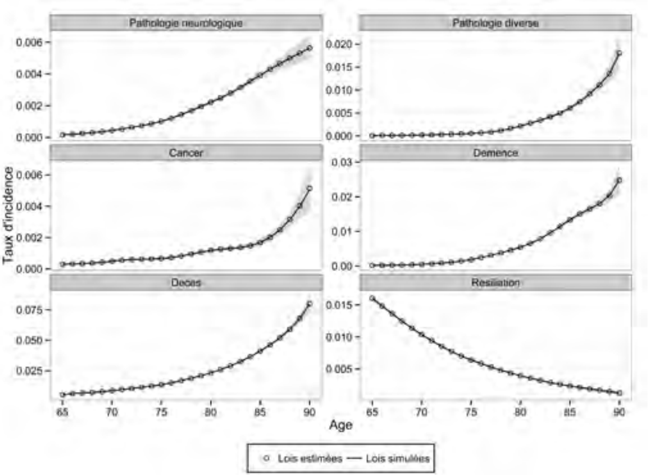


Figure: Simulated rates ($K = 1000$) with 95% simulated confidence intervals

Measuring Estimation Risk

- We compute dispersion coefficients

$$c\left(\psi_{jx}\right) = \frac{\sqrt{\sum_{k=1}^{K}\left(\tilde{q}_j^k\left(x\right) - \tilde{q}_j\left(x\right)\right)^2}}{\tilde{q}_j\left(x\right)}.$$

- The risk is relatively important and should be considered carefully for technical provisions valuation.

Exit causes	**Average** $c\left(\psi_{jx}\right)$
Neurologic pathologies	6.01%
Various pathologies	12.12%
Terminal cancers	7.94%
Dementia	6.53%
Death	2.05%
Cancel	3.76%

Comparing with the latent failure times approach

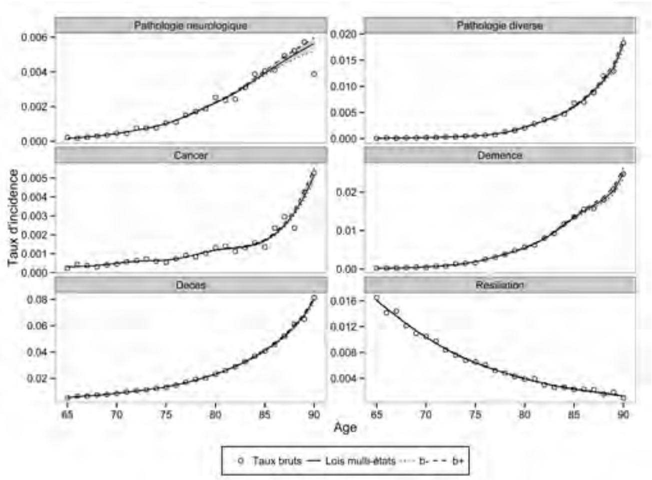


Figure: Comparing multistate and latent failure times approachs

Comparing with the latent failure times approach

- We compare 4 differents priority orders.
- At 65 years old, the larger gap measured on technical provisions is around 2.5%.
- Priority orders should be selected soundly!

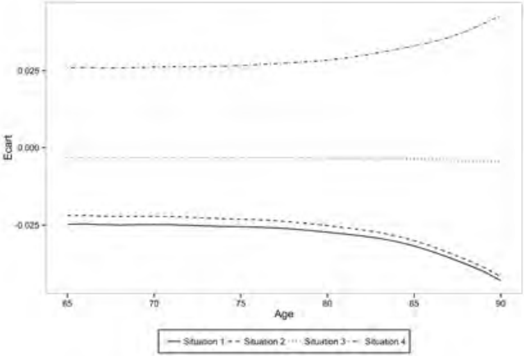


Figure: Gap on technical provisions with the both approachs

Summary

- Multistate approaches are rarely used by pratictioners which prefer Kaplan-Meier estimators and marginal approaches for crude rates estimation.
- Multistate approaches are a little more complex but theorically more appropriate for competing risks as the joint distribution of latent failure times are unobserved.
- A non-parametric approach may difficult to implement due to the lack of available data but:
  - it provides an realistic fit that one can expected in Solvency II perspective.
  - it can be used to perform goodness-of-fit tests.
- Bias observed with latent failure times approach depends on the treatment of priorities applied on each cause. In our application, a sound choice may reduce significatively this bias and justify the use of latent failure times approach.

- Outlook
  - Extrapoling inception rates to older ages.
  - Taking into account other sources of heterogeneity.

Thank you for your kind attention.

Some References I

Andersen, P. K., Borgan, r., Gill, R. D., and Keiding, N. (1993). *Statistical Models Based on Counting Processes*. Springer Series in Statistics. Springer-Verlag New York Inc.

Andersen, P. K., Abildstrom, S. Z., and Rosthøj, S. (2002). Competing risks as a multi-state model. *Statistical Methods in Medical Research*, **11**(2), 203–215.

Deléglise, M. P., Hess, C., and Nouet, S. (2009). Tarification, provisionnement et pilotage d'un portefeuille dépendance. *Bulletin Français d'Actuariat*, **9**(17), 70–108.

Fine, J. P. and Gray, R. J. (1999). A proportional hazards model for the subdistribution of a competing risk. *Journal of the American Statistical Association*, **94**(446), 496–509.

Gooley, T. A., Leisenring, W., Crowley, J., and Storer, B. E. (1999). Estimation of failure probabilities in the presence of competing risks: new representations of old estimators. *Statistics in medicine*, **18**(6), 695–706.

Guibert, Q. and Planchet, F. (2013). Construction de lois d'expérience en présence d'évènements concurrents – application à l'estimation des lois d'incidence d'un contrat dépendance. Les cahiers de recherche de l'ISFA WP 2013.6.

Prentice, R. L., Kalbfleisch, J. D., Peterson, A. V., Flournoy, N., Farewell, V. T., and Breslow, N. E. (1978). The analysis of failure times in the presence of competing risks. *Biometrics*, **34**(4), 541–554.