A partial internal model for longevity risk

Thomas Møller

PFA Pension and University of Copenhagen

Life Colloquium, Lyon, June 2013

(Joint work with Søren Fiig Jarner, ATP & Univ. Copenhagen)

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Valuation and solvency in life insurance

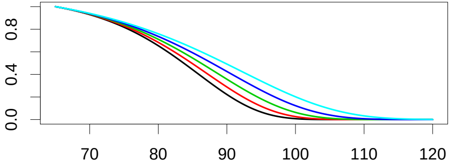
Longevity risk in Solvency 2:

- In standard model, 20 percent reduction in mortality rates
- **Possibility for partial or full internal models**

Modeling:

- Current mortality
- Expected future mortality development (**the trend**)

Survival probability (male, initial age 65)

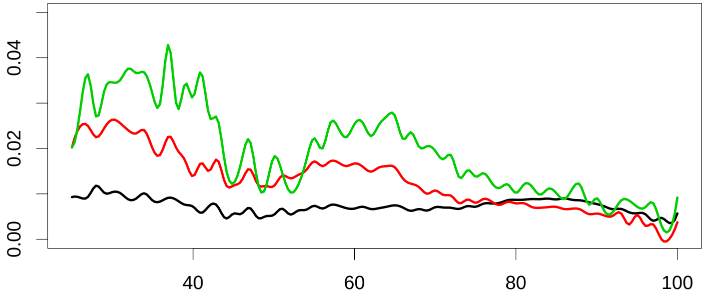


Trend	V	E	${}_{25}p_{65}$
0%	14.80	18.2	22.2%
1%	15.54	19.4	28.8%
2%	16.38	20.7	35.7%
3%	17.33	22.3	42.6%
4%	18.43	24.3	49.2%

(**Trend 0%, 1%, 2%, 3%, 4%**)

Improvement rates for different periods

Observed yearly relative decline, Denmark



(1956-2006, 1980-2006, 1990-2006)

Difficult to predict future changes:

- Yearly improvement rates depend on the period considered
- Largest improvement rates observed in the period 1990-2006

(PFA Pension and Human mortality database, www.mortality.org)

Approach to longevity modeling

- Danish FSA introduced in December 2010 a **mortality and longevity benchmark**
- Benchmark for **current mortality** $\mu^{FSA}(x, t_0)$ estimated from data for insured individuals with **5 years of data**
- Benchmark for **future trend** $R(x)$ estimated from total Danish population with **30 years of data**
- Pension funds perform yearly statistical tests whether they derive from benchmark.
  (Estimate $\beta = (\beta_1, \beta_2, \beta_3)$. Same level after age 100.)

Mortality model:

$$\mu^{model}(t, x) = e^{\sum_i \beta_i r_i(x)} \mu^{FSA}(x, t_0) \cdot (1 - R(x))^{t - t_0}$$

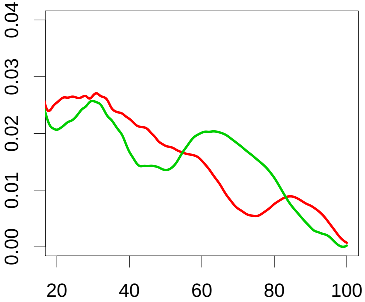
Two basic questions:

- Is portfolio-specific mortality equal to benchmark mortality?
- What is "best estimate" of portfolio specific mortality?

Approach to longevity modeling in Denmark

Benchmark

- Age-dependent yearly decline
- Different decline rates for males and females
- Trend and current mortality lead to best estimate
- Solvency II: 20% reduction of mortality rates (for longevity)



(**females**, **males**)

Expected life times

Age	Males	-20%	Females	-20%
40	84.4	1.8	86.6	1.9
50	83.9	1.8	86.3	1.9
60	84.1	1.7	86.7	1.8
70	85.3	1.5	87.7	1.5
80	88.2	1.1	90.1	1.2
90	93.9	0.7	94.7	0.8

Longevity stress in Solvency II – ongoing discussions

Background:

- Difficult to calibrate joint level for longevity risk for all EU countries (99.5% quantile with one-year time horizon)
- Until 2010: 25 percent
- Now: 20 percent stress, in addition to best estimate

News

Danish FSA has "substantial concerns" over longevity calibration for Solvency II

Published online only

Source: Insurance Risk
Author: Aaron Woolner, Laurie Carver
Source: Insurance Risk | 10 Jun 2010
Categories: Solvency II
Topics: Denmark, Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS), Longevity

Discussions in 2010:

...Parner said that with life expectancies increasing – on average – by three years in the last two decaded, Ceiops' approach of a uniform 25% stress for countries that updated their tables every 10 years was a good one, "but [it's] a bit too much for those that update on an annual basis"

Longevity stress in Solvency II

Standard model:

- Constant stress (not age-dependent)

Documentation for calibration of longevity stress

- Ceiops' "Solvency II Calibration Paper", 15.4.2010
- Background seems unclear
- Mortality tables not updated yearly in all countries?

> 3.283: "CEIOPS leaves the longevity stress unchanged because historic improvements in mortality rates observed in many countries are sometimes higher than 25% and, according to QIS4 report, the median stress in internal models equals 25%, with an interquartile range of 1% to 25%"

Paper by Lars Hougaard Nielsen, 2010:
Assessment of the VaR(99.5%) for longevity risk
Conclusion: Stress of 10–15% is conservative

Challenges with the longevity stress in Solvency II

Same longevity stress for all countries:

- Considerable differences in methods across the EU
- Stress seems to be based on updating every 5 or 8 years
- In DK, the longevity benchmark is being used
- Conclusion: 20% seems to be too big for Denmark

Possible adjustments:

- Full internal model, including longevity risk
- A partial internal model for longevity risk
- **Not possible** with country-specific parameters in the standard model

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The Danish Society of Actuaries – working group

Task: Develop suggestion for partial internal model for longevity risk ("Danish sector solution"?)

The working group:

- 10 members, one representative of FSA
- First meeting, June 1, 2012.
- Report, September 18, 2012

Wish list for the model:

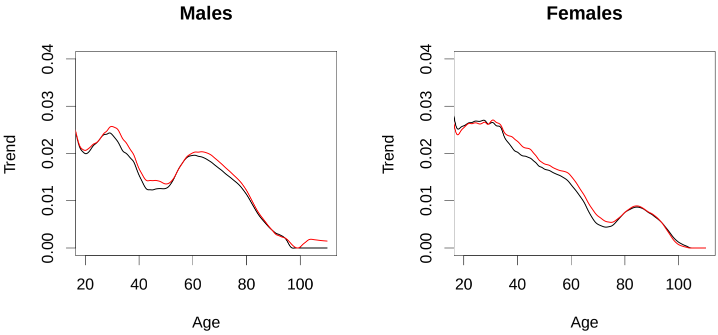
- Reflect risk in Danish longevity benchmark
- Address unsystematic risk (via company specific parameter)
- Simple and easy to understand
- Enter FSA pre-approval process for internal models

Further process:

- Danish CEA, FSA, Society of Actuaries, individual companies

2012-update of trend in FSA benchmark

In August 2012, the Danish FSA updated the benchmark

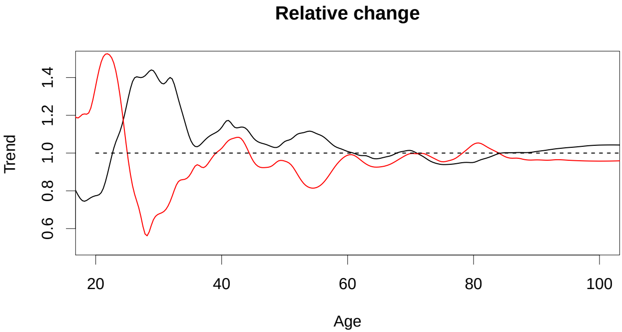


Trend increased from 2011- (black) to 2012-benchmark (red)

Model quantifies risk associated with yearly updating procedure

2012-update of current mortality in FSA benchmark

Decrease in benchmark for the current mortality from 2011 to 2012



(Relative change, corrected for expected improvements, for females (red) and males (black))

Sensitive to changes in data (number of companies included). Parts of the changes due to change in population

Goals and results

Longevity risk in the Solvency II directive, article 105, 2(b):
"the risk of loss, or of adverse change in the value of insurance liabilities, resulting from **changes in the level, trend, or volatility of mortality rates**, where a decrease in the mortality rate leads to an increase in the value of insurance liabilities (longevity risk)"

The Danish FSA divides the longevity risk into two parts:

- **Systematic risk:** Risk associated with changes in the **level and trend** for the company's mortality
- **Unsystematic risk:** Uncertainty in the **estimation** of the company's mortality

The proposed model describes these risks!

- Model cannot handle (too) small companies
- Model risk is not included (FSA-model used "as it is")

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Main results

Stress for systematic risk:

$$\mu^{FSA}_{stress}(x,t) = \mu^{FSA}(x,T)(1 - S_{level})(1 - R(x)(1 + S_{trend}))^{t-T}$$

- Calibrated parameters: $S_{level} = 6.0\%$, $S_{trend} = 6.0\%$
- Quantifies risk associated with yearly updating procedure

Unsystematic risk:

$$\mu^{model}_{stress}(x,t) = e^{\beta_1 r_1(x) + \beta_2 r_2(x) + \beta_3 r_3(x)} \mu^{FSA}_{stress}(x,t)(1 - S_{rl})$$

- Calibrated parameter: $S_{rl} = 2.6/\sqrt{5H}$, where H is expected number of deaths in portfolio during last 5 years
- Quantifies uncertaincy associated with estimating the company's actual mortality

Systematic and unsystematic risk represent 99.5%-quantiles

Total stress - a comparison of life times

Remaining life times in 2012 with benchmark:

Gender	Age	Remaining lifetime	-20%	H=500	H=5000	Systematic
Males	20	66.3	1.7	1.1	0.8	0.7
	40	45.1	1.8	1.1	0.8	0.7
	60	24.7	1.7	0.9	0.7	0.5
	80	8.5	1.1	0.6	0.4	0.3
Females	20	68.6	1.8	1.2	0.9	0.7
	40	47.5	1.9	1.1	0.8	0.7
	60	27.4	1.8	1.0	0.7	0.6
	80	10.5	1.3	0.7	0.5	0.4

Remaining life times at various ages and increase associated with standard model stress, stress for $H = 500$ and $H = 5000$ and pure systematic risk

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Model framework

Consistent modeling of underlying mortalities and data

Framework: (gender-specific, but not included in notation)

National mortality:	$D^N(x,t) \sim \text{Poisson}(\mu^N(x,t)E^N(x,t))$
Sector-specific mortality:	$D^S(x,t) \sim \text{Poisson}(\mu^S(x,t)E^S(x,t))$
Company-specific mortality:	$D^C(x,t) \sim \text{Poisson}(\mu^C(x,t)E^C(x,t))$

Underlying mortalities:

- Systematic risk
- μ^N, μ^S, μ^C are stochastic with given dependence structure

Actual (realized) number of deaths:

- Unsystematic risk
- D^N, D^S, D^C are Poisson distributed and independent (given underlying mortality intensities)

Model for the development in the mortality

National mortality (gender specific)

- Poisson Lee-Carter model (Brouhns et al. (2002))

$$D^N(x,t) \sim \text{Poisson}(\mu^N(x,t)E^N(x,t))$$

  where $\log \mu^N(x,t) = a_x + b_x k_t$
- Age-dependent level a_x and "improvement rate" b_x
- Joint improvement index k_t
- All (systematic) improvements 100% correlated
- Realised number of deaths are independent (given μ^N)
- Handling of cells without deaths better than original Lee-Carter (1992) model

Improvement index modeled via random walk with drift

$$k_{t+1} = k_t + \varepsilon_{t+1}$$

where ε_t are independent $\text{N}(\xi, \sigma^2)$
Model estimated for each gender separately, period 1980-2009

Model for the development in the mortality (continued)

Sector-specific mortality (gender-dependent)

- Starting point, most recent benchmark mortality
- Development (for logμ) fully correlated with national mortality
- Simulate data one year ahead

$$\begin{aligned} \mu^S(x, 2012) &= \mu^{FSA}(x, 2011)\mu^N(x, 2010)/\mu^N(x, 2009) \\ &= \mu^{FSA}(x, 2011)\exp(b_x \varepsilon_{2010}) \end{aligned}$$

- Shift in time periods represents "worst case"

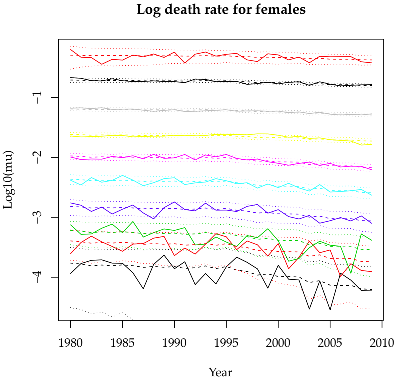
Company-specific mortality (joint α for males and females)

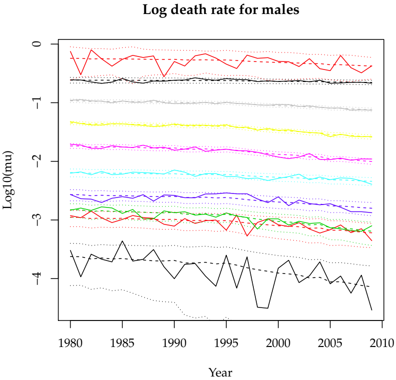
- Model for assessment of unsystematic risk in addition to benchmark

$$\mu^C(x, t) = \alpha\mu^S(x, t)$$

Poisson Lee-Carter model fitted to Danish data

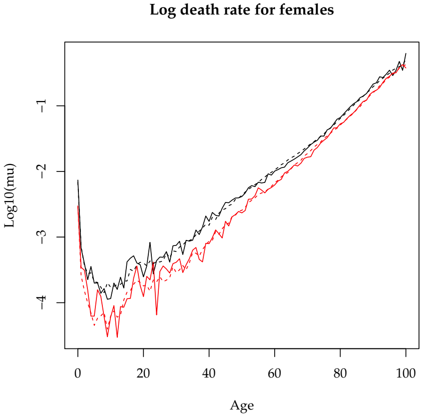
- Poisson Lee-Carter model fitted to 1980-2009 and age 0-105
- Data (solid line), fit (dashed line), 95% confidence interval (dotted) for age 0, 10, 20, ..., 100
- Unsystematic vol in data in accordance with Poisson variation
- Model reproduces observed drift and volatility in the data

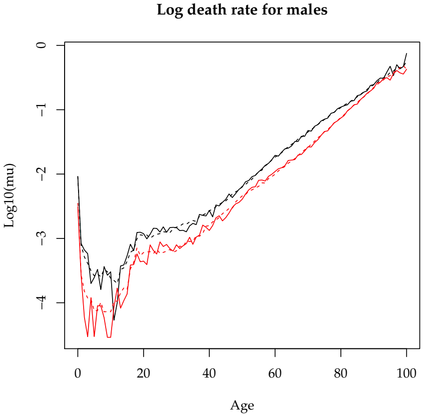




Poisson Lee-Carter model - continued

- Fit i 1980 (black) and 2009 (red)
- In the Lee-Carter model, fit is often adjusted in the final year, such that expected number of deaths in the model is equal to observed number
  - Automatically satisfied in this variant of the Lee-Carter model





Calibrating stress for trend and level

Simulate 10.000 underlying mortalities from the stochastic model for year $T + 1 = 2010$ (trend) and $T + 1 = 2012$ (level)

- In each scenario, generate Poisson number of deaths for national data and insurance sector data with exposure from 2009 and 2011
- Trend estimated from national data for 1981–2010 (FSA approach)
- Level estimated from sector data for 2008–2012 (FSA approach)
- Assume development in underlying mortalities are fully correlated

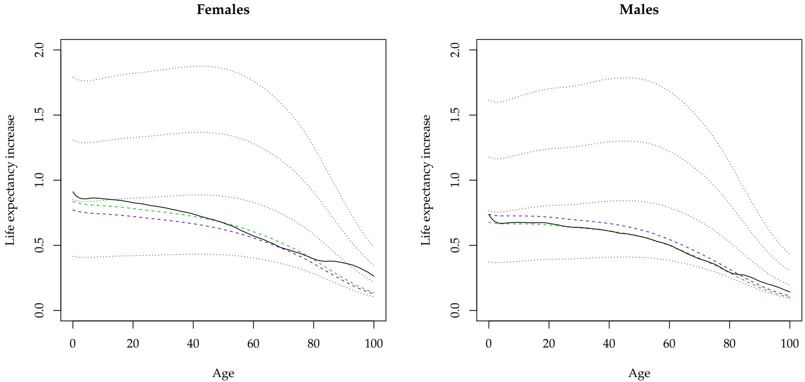
For each set of improvement rates and benchmark for observed mortality, calculate cohort life times in 2012 for females and males

- For each gender and age, calculate 99.5%-quantile
- Quantiles are approximated by uniform stress for trend and current mortality
- Alternative: Pointwise quantiles

Calibration of trend and level – life times

Increase in cohort life times relative to benchmark in 2012

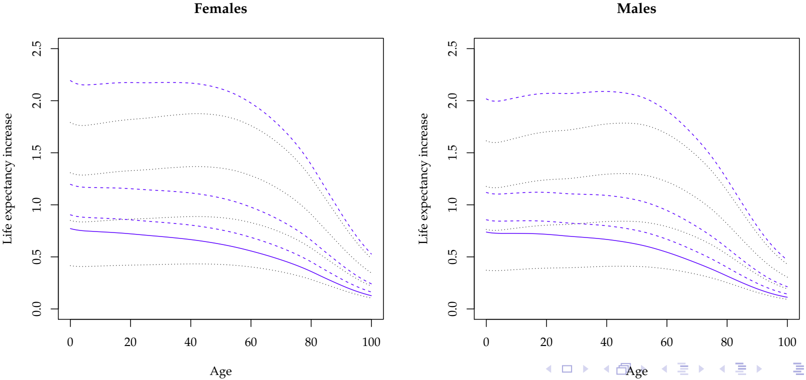
- Solid line: 99.5%-quantile in life time distribution (black), gender specific stress in trend and level (green), unisex stress for trend (S_{stress}=6%) and level (S_{level}=6,0%) (blue)
- Dotted lines: Benchmark surface, reduced by 5%, 10%, 15%, 20%



Total longevity stress - life times

Increase in cohort life times relative to benchmark in 2012

- Solid line: Unisex stress for trend (S_{stress}=6%) and level (S_{level}=6,0%)
- Dashed lines: Stress including unsystematic risk for H = 1,000, 10,000 and 100,000
- Dotted lines: Benchmark surface, reduced by 5%, 10%, 15%, 20%



Main references

Talk based on:

- Jarner and Møller (2013). A partial internal model for longevity risk, *Preprint*

Related work:

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- Richards, Currie and Ritchie (2012). A value-at-risk framework for longevity trend risk, *Discussion paper presented to The Institute and Faculty of Actuaries*