A semi-Markov model to investigate the different transitions between states of dependency in elderly people

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Insuring Long Term Care for elderly people: a fast growing need of our society

Modeling the LTC risk

*Penalized actuarial tables and markov approach*

*semi-markov approach*

Estimating the parameters of a semi-markov model

*The data, the entry into dependency state and model dimension*

*Weibull - Linear model*

*Non-parametric approach and first results*

*Mixed Weibull – Linear model and final estimation results*

Application to a model-related insurance product design

*Mortality tables*

*Actuarial tables*

Perspectives
The number of people aged 75 and more is about to **triple** until 2050.

At the same time, number of elderly dependant people will **double**.

In France, public social insurance type of coverage is worth **20 G€ / year**, or **1% GDP**.

On average, the overall care costs for an elderly dependent person are **30 000 € / year**.

The APA (social scheme) is worth a **fifth of the need**

An other **2 to 3 fifth comes from the revenues of the househould**

The rest is not covered

Overall, Long term Care is an emerging risk with foreseeable extremely high costs…

...and only partially covered.
The insurance industry must provide solutions, whether social economy, provident societies or private sector.

At the same time and due to financial stakes, one needs to develop a refined knowledge of the insurable risk.

The objectives of this study are

- providing a robust and detailed modeling of the survival process into dependency state.
  - we will pay particular attention to states transitions and their length;
- building up annuities insurance products for LTC needs coverage.
  - the associated benefits will be adapted to the underlying LTC states.

The progressive french GIR grid is our reference for measuring LTC states and needs.
Modelling the risk: first approaches

The « old » penalized actuarial tables SCOR model

entry into total dependency state at age \( a > 60 \) yo:
\[
e_a = 0.0015 \times \exp (0.125 \times (a - 60))
\]
mortality of non-dépendants:
\[
q^{ND}_a = 0.8 \times q^{TD8890}_a
\]
mortality of dépendants:
\[
q^{ND}_a = 2 \times q^{TD8890}_a + 3.5 \%
\]

Markov model

Let \( E := \{ 0 ; 1 ; \ldots ; n \} \) be a set of states and \( X \) un process lying in \( E \).

For \( i \) and \( j \) in \( E \) and \( 0 < s < t \), the transition probability between \( i \) and \( j \) states, between \( s \) and \( t \) dates is
\[
p_{i,j}(s,t) = P(X(t) = j | X(s) = i).
\]

Let \( 0 < t_1 < \ldots < t_n \) be the moments where the process jumps into the \( i_1 , \ldots , i_n \) states, the transitions must only depend on the last previously visited state:
\[
P(X(t_{n+1}) = i_{n+1}|X(t_1) = i_1 , \ldots , X(t_n) = i_n) = P(X(t_{n+1}) = i_{n+1}|X(t_n) = i_n).
\]

Problem: the transition duration laws are then directly linked to the transition probabilities
The main goal of the semi-Markov approach is to have transition probabilities and duration of transition probability laws made independant.

Transition durations are as fundamental for the insurer as the transition probabilities themselves because
- dependency level is driven by transition probabilities, hence annuitiy level;
- transition durations drive the length of payment of the annuity on a given level.

Hence, the present value of annuities, which is a critical input for pricing, must be constructed from a model able to precisely determine those two components.

Definitions:
- \( J_n \) : variable aléatoire discrète à valeurs dans l’espace d’états \( E \) égale au \( n \)ème état visité par le processus;
- \( S_n \) : variable aléatoire continue positive égale à la date du \( n \)ème saut du processus;
- \( X_n \) : variable aléatoire continue positive égale à la durée de station dans l’état \( J_n \);
- \( N(s) \) : variable aléatoire discrète égale aux nombres de transitions intervenues avant la date \( s \).

For \( i \) and \( j \) in \( E \) and \( 0 < s < x \), one defines the semi-markov kernel \( Q \) between \( i \) and \( j \) and between \( s \) and \( x \) by

\[
Q_{i,j}(s, x) = P(J_{N(s)+1} = j, X_{N(s)+1} \leq x | J_{N(s)} = i, S_{N(s)} = s)
\]
Characteristics of the semi-Markov model:

It is a generalization of the standard Markov model:

$$p_{i,j}(s) = \lim_{x \to \infty} Q_{i,j}(s, x) = P(J_{N(s)+1} = j | J_{N(s)} = i, S_{N(s)} = s).$$

The law of transition length from state $i$, entered in state $s$ and state $j$, for any $x > 0$ is

$$F_{i,j}(s, x) = P(X_{N(s)+1} \leq x | J_{N(s)+1} = j, J_{N(s)} = i, S_{N(s)} = s).$$

Fundamental assumption and property:

Under the assumption that the time spent in a given state and the entry date into it are independent

$$Q_{i,j}(s, x) = F_{i,j}(x) \times p_{i,j}(s).$$

Hence, to build-up of the $Q$ kernel, one only needs
- the transition durations laws between states;
- the functional forms of the probability transitions between states.

Practically speaking, the underlying models for the two can be parametric or not.
Estimation: the data

A fundamental source: the APA data

The DREES granted us access to compulsory APA data for 4 French départements:
- 04: Alpes de Haute Provence;
- 71: Saône et Loire;
- 76: Seine Maritime;
- 95: Val d’Oise.

They consist in the complete trajectories of dependant 52,000 dependent people between 1st Jan 2002 and 31st Dec 2005.

More than 27,000 GIR transitions are observed.

A secondary source: the CPRPSNCF data

The CPRPSNCF built up a dependency social coverage since 1st Oct 2005.

Data regarding 6,500 elderly dependent people are observed over 15 months.

But too few GIR transitions are actually observed.
Entry into dependency by GIR state is estimated by the measured empirical frequency

An exponential smoothing of these frequencies, all GIR levels compounded, allows us to compare measured frequencies to the genuine SCOR model.

For a given age $a > 60$ ans, one gets an entry into dependency rate $e_a$:

- **SCOR**: $e_a = 0.0015 \times \exp(0.125 \times (a - 60))$;
- **APA**: $e_a = 0.0020 \times \exp(0.126 \times (a - 60))$;
- **SNCF**: $e_a = 0.0003 \times \exp(0.173 \times (a - 60))$;
Estimation: reduction of model dimension

Observed frequencies of state transitions in the APA data:

<table>
<thead>
<tr>
<th></th>
<th>GIR 4</th>
<th>GIR 3</th>
<th>GIR 2</th>
<th>GIR 1</th>
<th>GIR 0</th>
<th>Censures</th>
</tr>
</thead>
<tbody>
<tr>
<td>GIR 4</td>
<td>1,0%</td>
<td>8,5%</td>
<td>9,3%</td>
<td>1,1%</td>
<td>6,9%</td>
<td>74,2%</td>
</tr>
<tr>
<td>GIR 3</td>
<td>2,3%</td>
<td>2,6%</td>
<td>16,4%</td>
<td>2,8%</td>
<td>10,8%</td>
<td>67,7%</td>
</tr>
<tr>
<td>GIR 2</td>
<td>1,8%</td>
<td>2,6%</td>
<td>4,2%</td>
<td>4,2%</td>
<td>18,3%</td>
<td>73,1%</td>
</tr>
<tr>
<td>GIR 1</td>
<td>0,5%</td>
<td>0,5%</td>
<td>1,8%</td>
<td>25,9%</td>
<td>71,3%</td>
<td></td>
</tr>
</tbody>
</table>

Final transition model:
Observed data show a **heavy right hand side censorship**.

Likelihood function must take this censorship into account.

Likelihood contribution of an observed transition from state $i$ (entry in date $s$) to state $j$ with duration $x$:

$$c_{i,j}(s, x) = p_{i,j}(s) \times f_{i,j}(x),$$

likelihood contribution of a censored trajectory since state $i$ (entry in date $s$) after a waiting time $x$:

$$c_i(s, x) = S_i(s, x) = \sum_{j \neq i} p_{i,j}(s) \times S_{i,j}(x) \quad \text{avec} \quad S_{i,j}(x) = 1 - F_{i,j}(x).$$

For $K$ individuals following, for $k = 1, \ldots, K$ a series of

- $n_k$ visited states;
- at dates $s_{n0}, \ldots, s_{nk-1}$;
- with durations $x_{n1}, \ldots, x_{nk}$.

The partial likelihood function to be maximized with respect to model parameters is:

$$L(s, x) = \prod_{k=1}^{K} S_{j_{nk}^k}(s_{nk-1}^k, x_{nk}^k) \prod_{n=1}^{n_{k-1}} c_{j_{n-1}^k, j_{nk}^k}(s_{n-1}^k, x_{nk}^k)$$
Estimation : Weibull – linear model (1 / 2)

First parametrisation choice :
- linear transition probabilities \( p_{i,j}(s) = a_{i,j} \times s + b_{i,j} \);
- Weibull-type transition duration laws \( f(x) = \nu \sigma^\nu x^{\nu-1} \exp\left(-\sigma x^\nu\right) \);
- Hence: 24 paramètres ;
- a number of individuals \( K = 52 \, 000 \);
- a total number of visited states \( n_k = 1 \) to 4 per individual.

Parameters estimation results :

<table>
<thead>
<tr>
<th>Transition vers GIR</th>
<th>Transition vers le décès</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{4,3} = W(1,66;0,36) )</td>
<td>( f_{4,0} = W(1,24;0,13) )</td>
</tr>
<tr>
<td>( f_{4,2} = W(1,81;0,39) )</td>
<td>( f_{3,0} = W(1,18;0,20) )</td>
</tr>
<tr>
<td>( f_{3,2} = W(1,58;0,40) )</td>
<td>( f_{2,0} = W(1,35;0,25) )</td>
</tr>
<tr>
<td>( f_{2,1} = W(1,46;0,27) )</td>
<td>( f_{1,0} = W(1,17;0,22) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GIR 3</th>
<th>GIR 2</th>
<th>GIR 1</th>
<th>GIR 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>GIR 4</td>
<td>0,003 s + 0,182</td>
<td>0,007 s + 0,091</td>
<td>−0,010 s + 0,727</td>
</tr>
<tr>
<td>GIR 3</td>
<td>0,002 s + 0,426</td>
<td>−0,006 s + 0,331</td>
<td>−0,002 s + 0,574</td>
</tr>
<tr>
<td>GIR 2</td>
<td></td>
<td></td>
<td>0,006 s + 0,669</td>
</tr>
<tr>
<td>GIR 1</td>
<td></td>
<td></td>
<td>0,000 s + 1,000</td>
</tr>
</tbody>
</table>
Example of transition probabilities behavior (from GIR 4 to other states):

Average transition lengths between states:

<table>
<thead>
<tr>
<th>Transition</th>
<th>GIR 3</th>
<th>GIR 2</th>
<th>GIR 1</th>
<th>GIR 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>GIR 4</td>
<td>2, 4</td>
<td>2, 2</td>
<td>3, 4</td>
<td>7, 1</td>
</tr>
<tr>
<td>GIR 3</td>
<td></td>
<td>2, 2</td>
<td>4, 7</td>
<td>4, 2</td>
</tr>
<tr>
<td>GIR 2</td>
<td></td>
<td></td>
<td>3, 7</td>
<td></td>
</tr>
<tr>
<td>GIR 1</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Model to be fine tuned...
Our goal: get hints about the shapes of the laws to be estimated.

To do so, we estimate a model where:

- transition probabilities are assumed to be **piecewise constant**;
- transition durations laws are also assumed to **piecewise constant**.

**Estimation Results:**

- Transition probabilities linearity validated
- Inadequacy of Weibull for transitions GIR → death
Final choice of modeling:

- linear transition probabilities \( p_{i,j}(s) = a_{i,j} \times s + b_{i,j} \);
- Weibull type laws for GIR ➔ GIR transitions \( f(x) = \nu \sigma^\nu x^{\nu-1} \exp \left(-\left(\sigma x\right)^\nu\right) \);
- Convex combination of two Weibull laws GIR ➔ Death transitions, with unknown weight
  \[
  f = \alpha \times w(\nu_1, \sigma_1) + (1 - \alpha) \times w(\nu_2, \sigma_2)
  \]
- hence 36 parameters to estimate, with still \( K = 27\,000 \) individuals and \( n_k = 1 \) to \( 4 \).

Parameters estimation:

<table>
<thead>
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<th>Transition vers GIR</th>
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<tbody>
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<td>( f_{4,3} = W(1,40;0,22) )</td>
<td>( f_{4,0} = 0,42 \times W(1,35;0,69) + 0,59 \times W(5,08;0,28) )</td>
</tr>
<tr>
<td>( f_{4,2} = W(1,69;0,40) )</td>
<td>( f_{3,0} = 0,73 \times W(1,08;0,31) + 0,27 \times W(5,90;0,27) )</td>
</tr>
<tr>
<td>( f_{3,2} = W(1,47;0,30) )</td>
<td>( f_{2,0} = 0,51 \times W(1,17;0,51) + 0,49 \times W(5,98;0,28) )</td>
</tr>
<tr>
<td>( f_{2,1} = W(1,47;0,20) )</td>
<td>( f_{1,0} = 0,26 \times W(1,16;0,95) + 0,74 \times W(4,14;0,24) )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GIR 3</th>
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<th>GIR 1</th>
<th>GIR 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>GIR 4</td>
<td>(-0,008,s + 0,708)</td>
<td>(0,006,s + 0,139)</td>
<td>(0,002,s + 0,153)</td>
</tr>
<tr>
<td>GIR 3</td>
<td>(-0,001,s + 0,638)</td>
<td>(-0,011,s + 0,652)</td>
<td>(0,001,s + 0,362)</td>
</tr>
<tr>
<td>GIR 2</td>
<td>(-0,011,s + 0,652)</td>
<td>(-0,011,s + 0,348)</td>
<td>(0,000,s + 1,000)</td>
</tr>
<tr>
<td>GIR 1</td>
<td>(-0,001,s + 0,638)</td>
<td>(-0,011,s + 0,652)</td>
<td>(-0,001,s + 0,362)</td>
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</table>
Mixed Weibull vs Weibull...
Example of transition probabilities behavior (from GIR 4 to other states):

Average transition lengths between states:

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<td>GIR 4</td>
<td>4, 1</td>
<td>2, 2</td>
<td>2, 3</td>
<td>2, 4</td>
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<tr>
<td>GIR 3</td>
<td>2, 2</td>
<td>2, 3</td>
<td>4, 6</td>
<td>3, 2</td>
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<tr>
<td>GIR 2</td>
<td>2, 3</td>
<td>4, 6</td>
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<td>GIR 1</td>
<td>4, 6</td>
<td>2, 6</td>
<td>3, 1</td>
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</table>
Application: mortality tables

The model allows heavy simulation of trajectories into dependency states for an entry at given age $a$, according to the following protocol:

1. Generate initial state $i_1$ according to entry tables;
2. Generate next state $i_2$ according to estimated probabilities $p_{i_1,i_2}$ estimated for age $a$;
3. Generate transition length between states $i_1$ and $i_2$ according to estimated Weibulls if $i_2$ is not death state and estimated Mixed-Weibull otherwise;
4. If last visited state is not Death then repeat stages (2) and (3).

The table is built out of 100 000 simulated trajectories for each starting age, corrected by TPG93 table is death rates prove to be too low. Here are the derived life expectancies at entry:

![Graph showing life expectancies at entry](image)
Application : costs tables

Following the same idea as for mortality table, Tables of Costs per GIR measure the Probable Present Value of a 100€ monthly annuity paid by the insurer to an individual that became dependent at age $a$, according to current GIR state.

For a portfolio of insured, the entry table per GIR and the Costs tables per GIR allow to Derive the Probable Present Value of an annuity product with benefits depending on the various GIR states.

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<th>GIR 4</th>
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</tbody>
</table>
Still to be addressed…

On the insurance side:
- evaluate all kind of provisions thanks to the model;
- account for waiting periods and other product features;
- get access to longer time series of data.

About modeling:
- Evaluate robustness of central modeling assumption;
- Assess volatility of parameters estimation;
- Take into account covariables or explanatory variables (diseases) through Cox-type modeling;
- Challenge Markov hypothesis ?

On the socio-political debate :
- Eventually create a fifth branch of Social Security scheme in France !
Thanks a lot for your attention