Modeling Mortality of Multiple Populations with Vector Error Correction Models: Applications to Solvency II

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(Joint work with R. Zhou, Y. Wang, and K.S. Tan)
Introduction
Multi-Population Mortality Models

• Capture the dependence of mortality dynamics between different populations

• Useful for modeling mortality dynamics of:
  • Males and females
  • Different national populations
  • A population and its subpopulation

• Useful for estimating basis risk involved standardized longevity risk hedges
Objectives

• A new approach for modeling mortality of multiple populations
  – Better capture the underlying correlations and long-term equilibrium
  – Does not require the user to assume which population is dominant

• An application to Solvency II capital requirements
Three Approaches for Modeling the Time-Varying Factors
The Framework of Cairns et al. (2011)

- A Lee-Carter parametric structure:
  \[ \ln(m_x^{(i)}) = \alpha_x^{(i)} + \beta_x^{(i)} \kappa_t^{(i)}, \quad i = 1, 2 \]

- Necessary conditions for non-divergence:
  1. \( \beta_x^{(1)} = \beta_x^{(2)} \)
  2. \( \kappa_t^{(1)} - \kappa_t^{(2)} \) is mean-reverting

- The resulting forecast depends heavily on how the time-varying factors \( \kappa_t^{(1)} \) and \( \kappa_t^{(2)} \) are modeled.
The RWAR Approach

• Used in Cairns et al. (2011)

• A random walk (RW) with drift for $\kappa_t^{(1)}$:
  $$\kappa_t^{(1)} = \kappa_{t-1}^{(1)} + \mu + \epsilon_t^{(1)}$$

• An autoregressive (AR) process for $\kappa_t^{(1)} - \kappa_t^{(2)}$:
  $$\kappa_t^{(1)} - \kappa_t^{(2)} = \mu_\Delta + \phi(\kappa_{t-1}^{(1)} - \kappa_{t-1}^{(2)}) + \epsilon_t^{(2)}$$
The Lead-Lag Problem

Sample cross-correlations between \( \Delta \kappa_t^{(1)} \) and \( \Delta \kappa_t^{(2)} \)
Population 1: EW males
Population 2: CMI males

Indicates that it is the CMI population that leads the EW population
The VAR Approach

• A Vector Autoregressive (VAR) model:

\[ \Delta \kappa_t^{(1)} = \phi_0 + \phi_1 \Delta \kappa_{t-1}^{(1)} + \phi_2 \Delta \kappa_{t-1}^{(2)} + \epsilon_t^{(1)} \]

\[ \Delta \kappa_t^{(2)} = \theta_0 + \theta_1 \Delta \kappa_{t-1}^{(1)} + \theta_2 \Delta \kappa_{t-1}^{(2)} + \epsilon_t^{(2)} \]

• Non-divergence constraint:

\[ \frac{\phi_0}{1 - \phi_1 - \phi_2} = \frac{\theta_0}{1 - \theta_1 - \theta_2} \]

• Ensures that \( \Delta \kappa_t^{(1)} \) and \( \Delta \kappa_t^{(2)} \) converge to the same value in the long run
The VECM Approach

• A Vector Error Correction Model (VECM):

\[ \Delta \kappa_t^{(1)} = \rho^{(1)} \left( a \kappa_{t-1}^{(1)} + b \kappa_{t-1}^{(2)} + c \right) + \phi_0 + \phi_1 \Delta \kappa_{t-1}^{(1)} + \phi_2 \Delta \kappa_{t-1}^{(2)} + \epsilon_t^{(1)} \]

\[ \Delta \kappa_t^{(2)} = \rho^{(2)} \left( a \kappa_{t-1}^{(1)} + b \kappa_{t-1}^{(2)} + c \right) + \theta_0 + \theta_1 \Delta \kappa_{t-1}^{(1)} + \theta_2 \Delta \kappa_{t-1}^{(2)} + \epsilon_t^{(2)} \]

• Intuitions:
  • \( a \kappa_t^{(1)} + b \kappa_t^{(2)} + c = 0 \) is the long-term equilibrium
  • \( \rho^{(1)} \) and \( \rho^{(2)} \) are the adjustment coefficients

• Non-divergence constraint: \( a = -b \)
Evaluating the Three Approaches
Evaluating the Three Approaches

• Evaluation criteria:
  – Goodness-of-fit
  – Forecasting performance
  – Robustness

• Data:
  – Population 1: EW males
  – Population 2: CMI males
  – Sample period: 1961 – 2005
  – Age range: 60 – 84
First Stage Estimates of $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$
Goodness-of-Fit

<table>
<thead>
<tr>
<th></th>
<th>RWAR</th>
<th>VAR</th>
<th>VECM</th>
</tr>
</thead>
<tbody>
<tr>
<td>BIC</td>
<td>179.19</td>
<td>176.45</td>
<td>174.63</td>
</tr>
</tbody>
</table>

\[ BIC = k \ln(n) - 2\hat{l} \]

- \( \hat{l} \): maximized log likelihood
- \( k \): number of parameters
- \( n \): sample size
In-Sample Forecast

RWAR
In-Sample Forecast

VAR
In-Sample forecast

VECM
Testing Robustness

• **Test 1**
  Fit the models to two different lookback periods: 1961-2005 (45 years) and 1976-2005 (30 years)

• **Test 2**

**Conclusion of Tests 1 and 2:**
VECM is the most robust among the three approaches

• **Test 3**
  Examines how forecasts will change if we swap populations 1 and 2

**Conclusion of Test 3:**
VAR and VECM: No change to the resulting forecasts
RWAR: The forecasts change significantly if we swap the two populations
Multi-Population Models Applied to Longevity Swap
Real-World Applications

Project survivorship of a portfolio of lives

- Pensioners, insured lives, healthy / disabled / dependent

• Current mortality: well understood
  - Experience analysis, survival models

• Future mortality trend: data?

• Use general population as a proxy

→ Introduces Longevity Basis Risk
Longevity Basis Risk

Our case: Annuitants

• Insured Lives vs. General Population
  – Misestimation of Best Estimate Liabilities
  – Misestimation of Capital Requirements

→ Underprice the Risk
Longevity Swap Reinsurance

- UK: Reinsurance market
  ~ £10 billion transferred per year
- Also transactions in the USA, Netherlands
- Trade floating pensions/annuity benefits for fixed stream of reinsurance premiums
- What is the fair value?
Solvency 2

- Used in Cairns et al. (2011)

- A random walk (RW) with drift for $\kappa_t^{(1)}$:
  \[ \kappa_t^{(1)} = \kappa_{t-1}^{(1)} + \mu + \varepsilon_t^{(1)} \]

- An autoregressive (AR) process for $\kappa_t^{(1)} - \kappa_t^{(2)}$:
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Longevity Swap Pricing

- Solvency Capital creates frictional cost
- Risk Margin \(\sim\) Cost of Capital

\[\text{Risk Margin} = 6\% \times PV(SCR_t)\]

\[PV(\text{Reinsurance Fees}) = \text{Risk Margin}\]
# Longevity Swap Prices

100 mn annual benefit, 99.5%-ile SCR

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<tr>
<td>Liabilities [bn]</td>
<td>1.679</td>
<td>1.697</td>
<td>1.701</td>
<td>1.728</td>
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<tr>
<td>increase</td>
<td>1.1%</td>
<td>1.3%</td>
<td>2.9%</td>
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<td>Risk Margin [mn]</td>
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<td>Risk Margin [% annual benefits]</td>
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<td>3.1%</td>
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# Longevity Swap Prices

**[Standard Formula]**

100 mn annual benefit, -20% Stress SCR

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<td>Risk Margin [mn]</td>
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