Modeling Mortality of Multiple Populations with Vector Error Correction Models: Applications to Solvency II

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Introduction

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Multi-Population Mortality Models

- Capture the dependence of mortality dynamics between different populations
- Useful for modeling mortality dynamics of:
  - Males and females
  - Different national populations
  - A population and its subpopulation
- Useful for estimating basis risk involved standardized longevity risk hedges

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Objectives

- A new approach for modeling mortality of multiple populations
  - Better capture the underlying correlations and long-term equilibrium
  - Does not require the user to assume which population is dominant
- An application to Solvency II capital requirements

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Three Approaches for Modeling the Time-Varying Factors

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The Framework of Cairns et al. (2011)

- A Lee-Carter parametric structure:

$$\ln(m_{x,t}^{(i)}) = \alpha_x^{(i)} + \beta_x^{(i)} \kappa_t^{(i)}, \qquad i = 1,2$$

- Necessary conditions for non-divergence:
  1. $\beta_x^{(1)} = \beta_x^{(2)}$
  2. $\kappa_t^{(1)} - \kappa_t^{(2)}$ is mean-reverting

- The resulting forecast depends heavily on how the time-varying factors $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ are modeled.

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The RWAR Approach

- Used in Cairns et al. (2011)
- A random walk (**RW**) with drift for $\kappa_t^{(1)}$:

$$\kappa_t^{(1)} = \kappa_{t-1}^{(1)} + \mu + \epsilon_t^{(1)}$$

- An autoregressive (**AR**) process for $\kappa_t^{(1)} - \kappa_t^{(2)}$:

$$\kappa_t^{(1)} - \kappa_t^{(2)} = \mu_\Delta + \phi(\kappa_{t-1}^{(1)} - \kappa_{t-1}^{(2)}) + \epsilon_t^{(2)}$$

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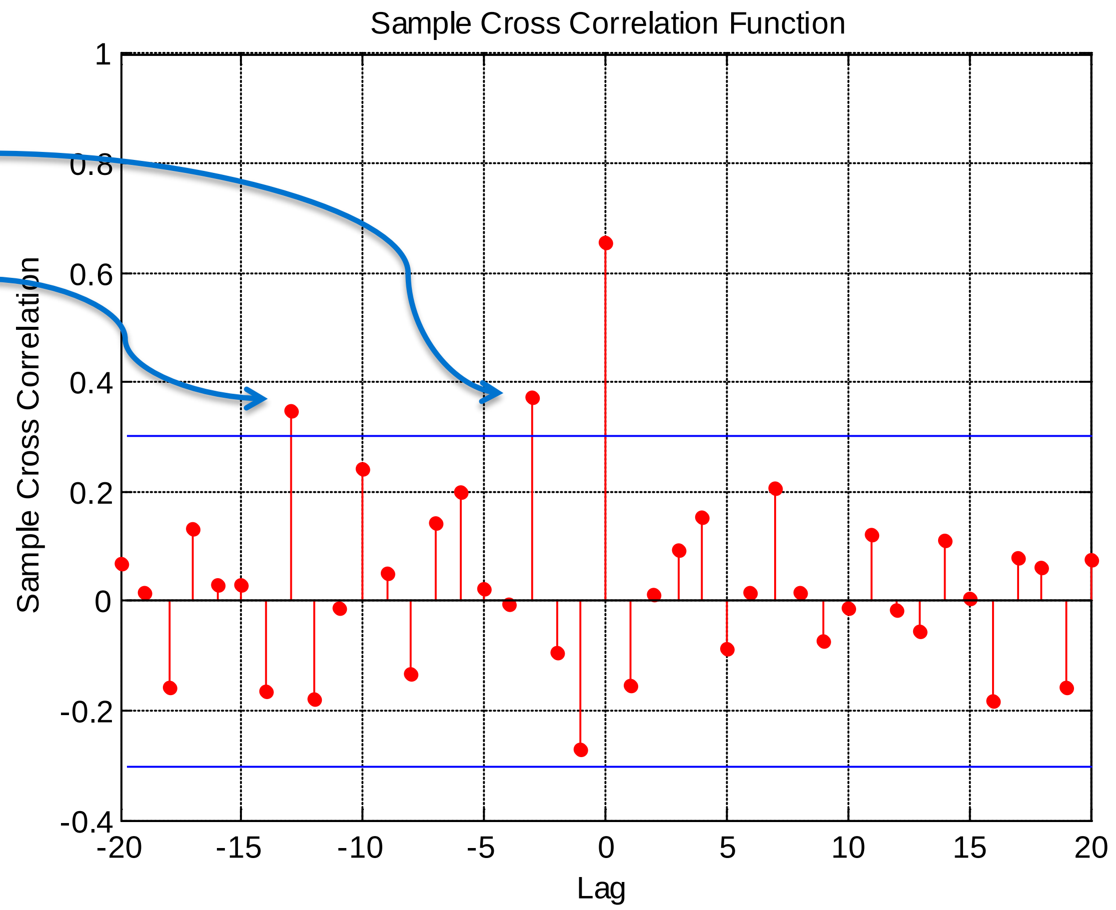
The Lead-Lag Problem

Indicates that it is the CMI population that leads the EW population



Sample cross-correlations between $\Delta\kappa_t^{(1)}$ and $\Delta\kappa_t^{(2)}$

Population 1: EW males

Population 2: CMI males

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The VAR Approach

- A Vector Autoregressive (**VAR**) model:

$$\Delta\kappa_t^{(1)} = \phi_0 + \phi_1 \Delta\kappa_{t-1}^{(1)} + \phi_2 \Delta\kappa_{t-1}^{(2)} + \epsilon_t^{(1)}$$
$$\Delta\kappa_t^{(2)} = \theta_0 + \theta_1 \Delta\kappa_{t-1}^{(1)} + \theta_2 \Delta\kappa_{t-1}^{(2)} + \epsilon_t^{(2)}$$

- Non-divergence constraint:
  - $\frac{\phi_0}{1-\phi_1-\phi_2} = \frac{\theta_0}{1-\theta_1-\theta_2}$
  - Ensures that $\Delta\kappa_t^{(1)}$ and $\Delta\kappa_t^{(2)}$ converge to the same value in the long run

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The VECM Approach

- A Vector Error Correction Model (**VECM**):

$$\Delta\kappa_t^{(1)} = \rho^{(1)}\left(a\kappa_{t-1}^{(1)} + b\kappa_{t-1}^{(2)} + c\right) + \phi_0 + \phi_1\Delta\kappa_{t-1}^{(1)} + \phi_2\Delta\kappa_{t-1}^{(2)} + \epsilon_t^{(1)}$$

$$\Delta\kappa_t^{(2)} = \rho^{(2)}\left(a\kappa_{t-1}^{(1)} + b\kappa_{t-1}^{(2)} + c\right) + \theta_0 + \theta_1\Delta\kappa_{t-1}^{(1)} + \theta_2\Delta\kappa_{t-1}^{(2)} + \epsilon_t^{(2)}$$

- Intuitions:
  - $a\kappa_t^{(1)} + b\kappa_t^{(2)} + c = 0$ is the long-term equilibrium
  - $\rho^{(1)}$ and $\rho^{(2)}$ are the adjustment coefficients
- Non-divergence constraint: $a = -b$

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Evaluating the Three Approaches

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Evaluating the Three Approaches

- Evaluation criteria:
  - Goodness-of-fit
  - Forecasting performance
  - Robustness

- Data:
  - Population 1: EW males
  - Population 2: CMI males
  - Sample period: 1961 – 2005
  - Age range: 60 – 84

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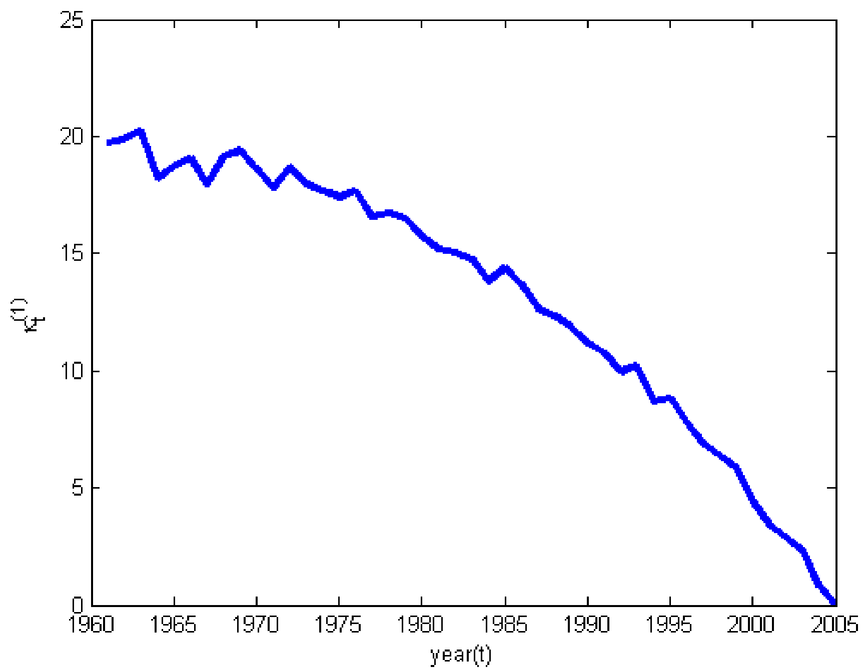
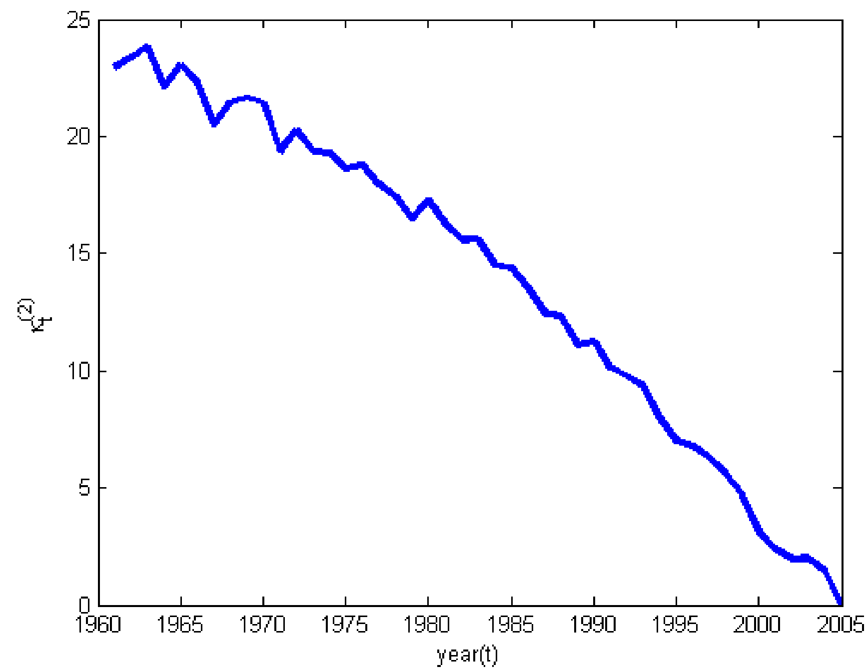
First Stage Estimates of $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$





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Goodness-of-Fit

	RWAR	VAR	VECM
BIC	179.19	176.45	174.63

$$BIC = k\ln(n) - 2\hat{l}$$

$\hat{l}$ maximized log likelihood

k number of parameters

n sample size

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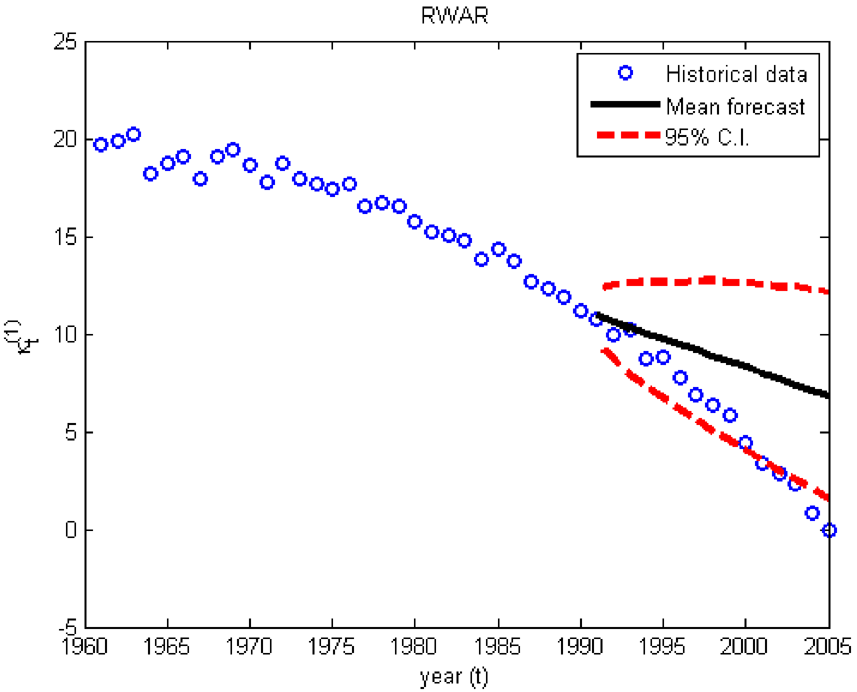
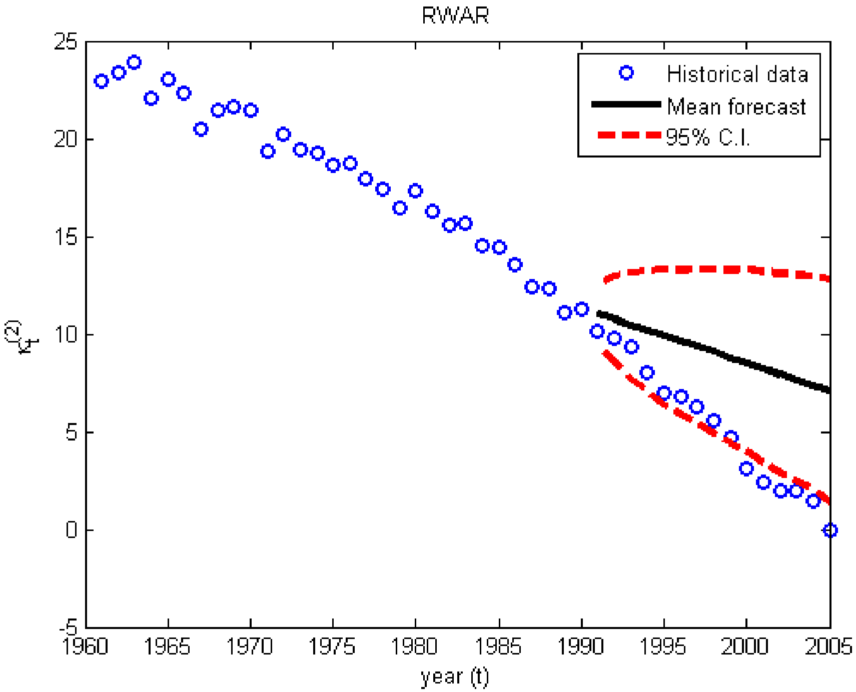
In-Sample Forecast RWAR





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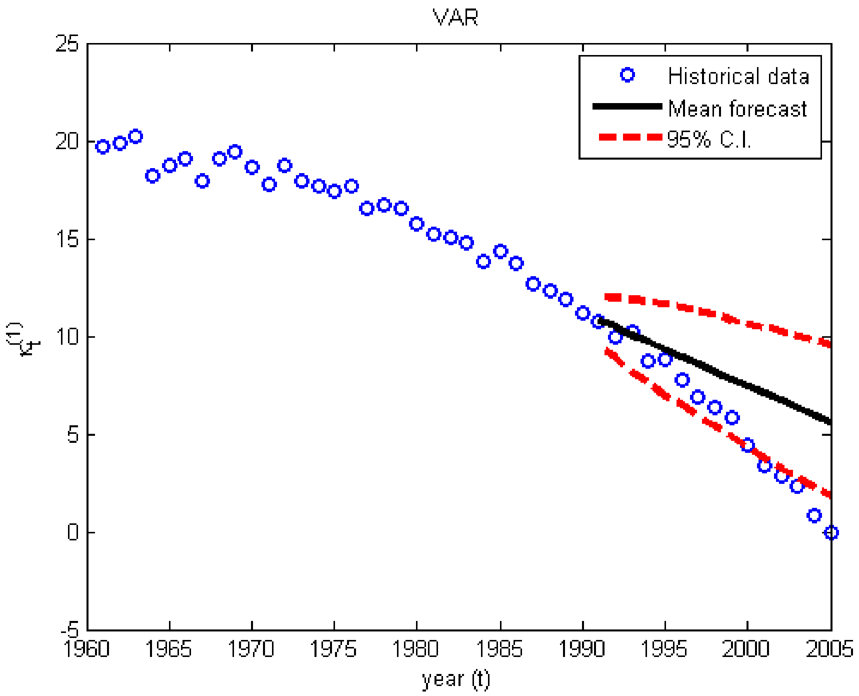
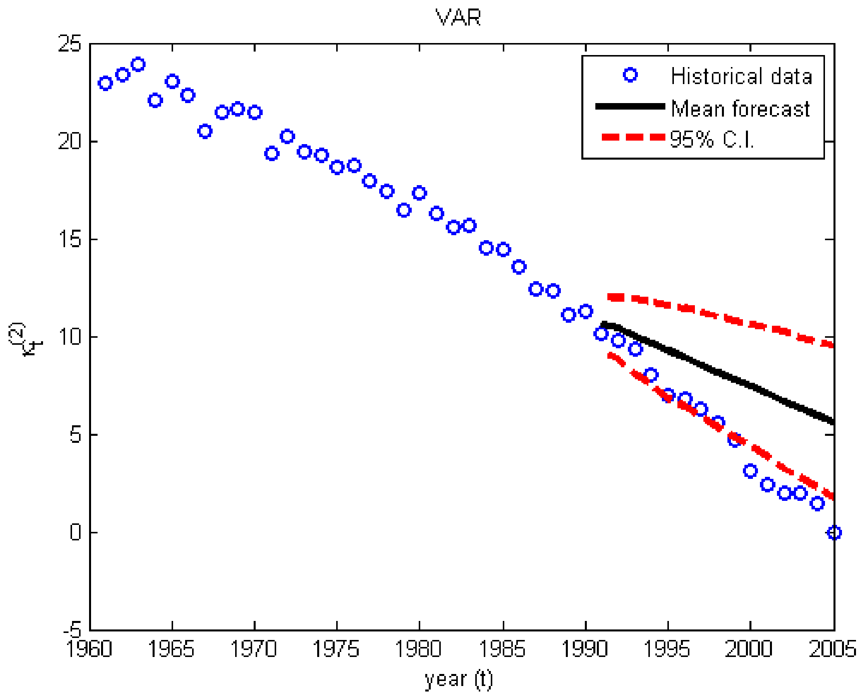
In-Sample Forecast
VAR





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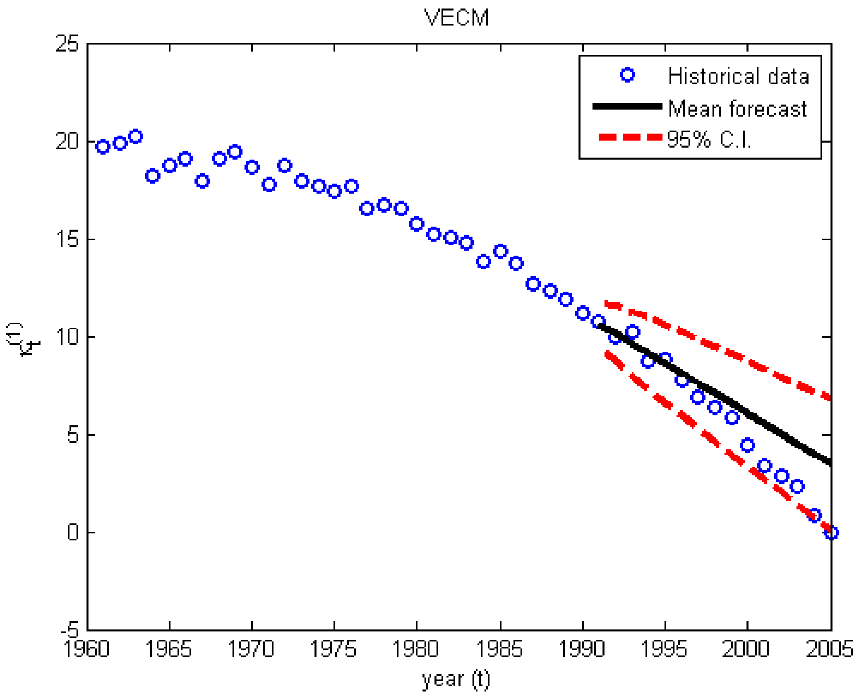
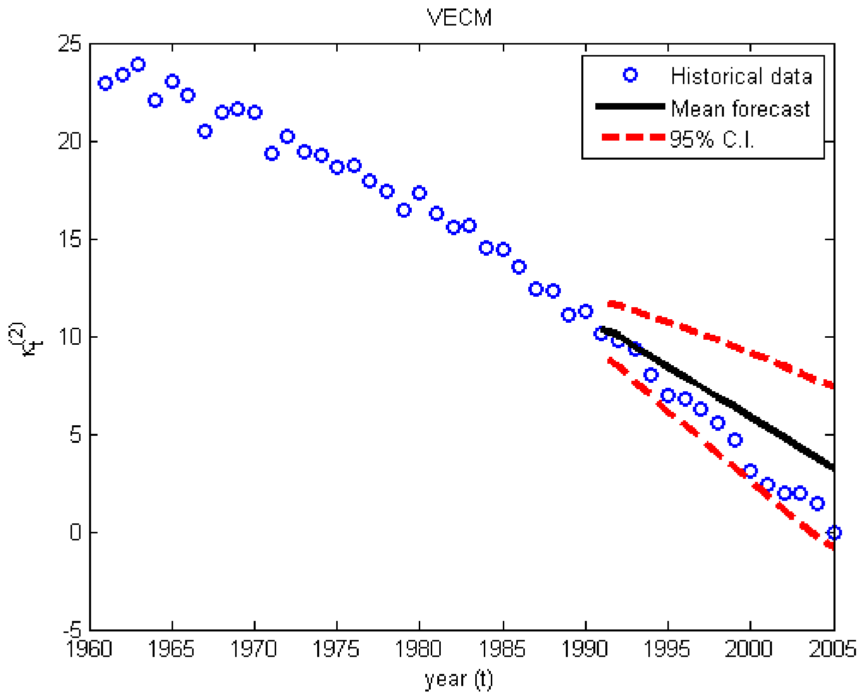
In-Sample forecast VECM





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Testing Robustness

- **Test 1**

  Fit the models to two different lookback periods: 1961-2005 (45 years) and 1976-2005 (30 years)

- **Test 2**

  Fit the models to data over three sample periods: 1961-1990, 1966-1995, and 1971-2000

Conclusion of Tests 1 and 2:

VECM is the most robust among the three approaches

- **Test 3**

  Examines how forecasts will change if we swap populations 1 and 2

Conclusion of Test 3:

VAR and VECM: No change to the resulting forecasts

RWAR: The forecasts change significantly if we swap the two populations

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Multi-Population Models Applied to Longevity Swap

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Real-World Applications

Project survivorship of a portfolio of lives

- Pensioners, insured lives,
  healthy / disabled / dependent

- Current mortality: well understood
  - Experience analysis, survival models
- Future mortality trend: data?
- Use general population as a proxy

→Introduces Longevity Basis Risk

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Longevity Basis Risk

Our case: Annuitants

- Insured Lives vs. General Population
  - Misestimation of Best Estimate Liabilities
  - Misestimation of Capital Requirements

→ Underprice the Risk

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Longevity Swap Reinsurance

- UK: Reinsurance market
  ~ £10 billion transferred per year
- Also transactions in the USA, Netherlands
- Trade floating pensions/annuity benefits for fixed stream of reinsurance premiums
- What is the fair value?

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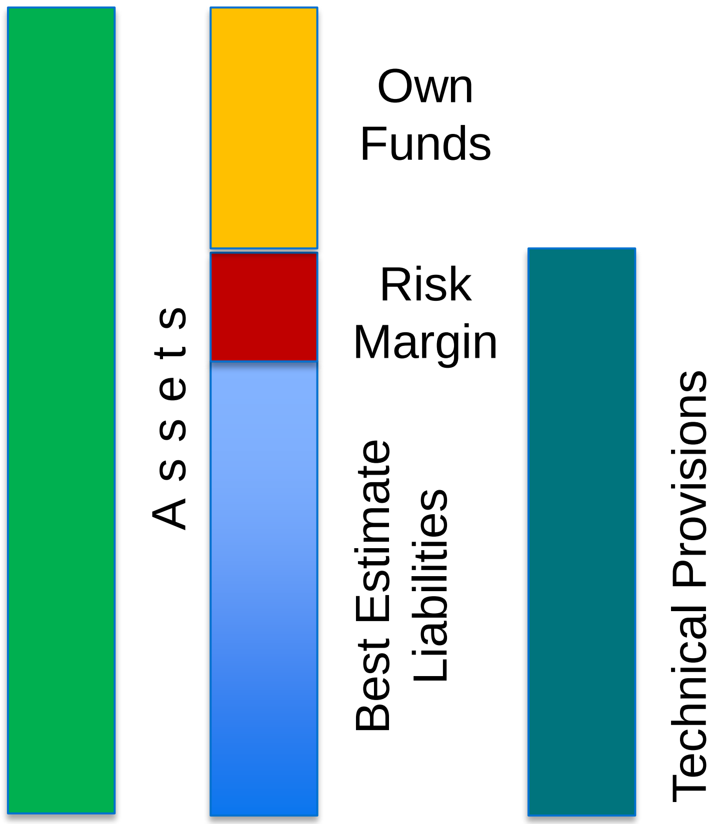
Solvency 2



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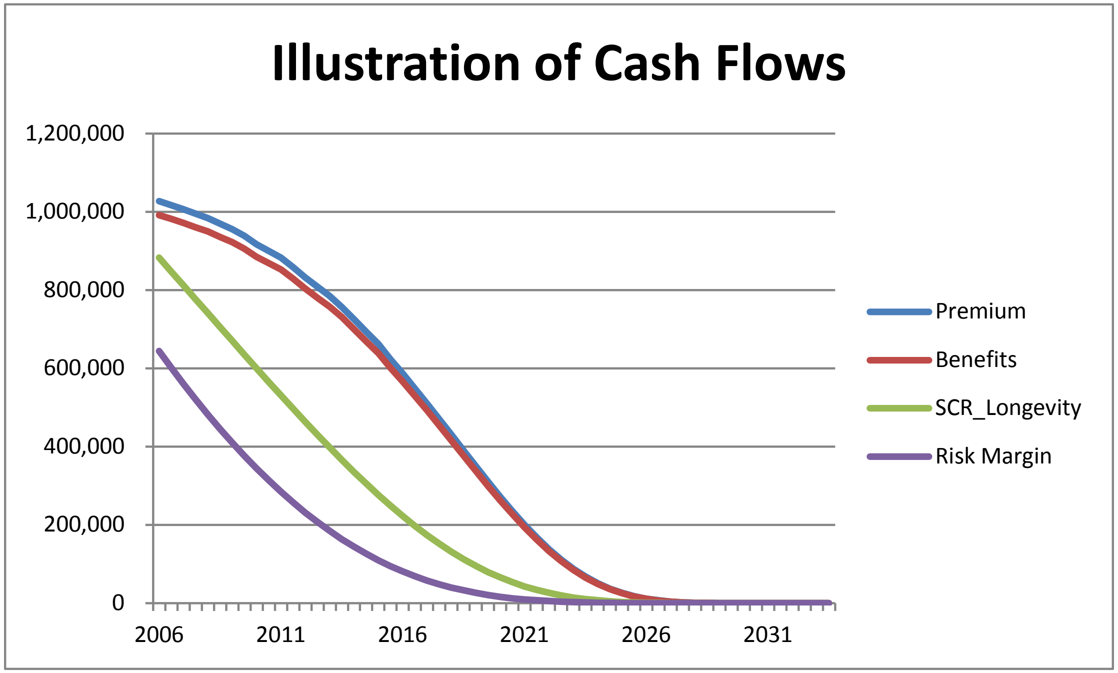
Longevity Swap Pricing

- Solvency Capital creates frictional cost
- Risk Margin
  ~ Cost of Capital



$$Risk\ Margin = 6\% \times PV(SCR_t)$$

$$PV(Reinsurance\ Fees) = Risk\ Margin$$

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Longevity Swap Prices

100 mn annual benefit, 99.5%-ile SCR

	Single	RWAR	VAR	VECM
Liabilities [bn]	1.679	1.697	1.701	1.728
increase		1.1%	1.3%	2.9%
Risk Margin [mn]	54.2	84.8	61.5	80.0
Risk Margin [% annual benefits]	2.0%	3.1%	2.2%	5.1%

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Longevity Swap Prices
[Standard Formula]

100 mn annual benefit, -20% Stress SCR

	Single	RWAR	VAR	VECM
Liabilities [bn]	1.679	1.697	1.701	1.728
increase		1.1%	1.3%	2.9%
Risk Margin [mn]	86.9	86.1	89.3	91.5
Risk Margin [% annual benefits]	3.2%	3.2%	3.3%	3.3%

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