MARKET VALUATION OF CASH BALANCE PENSION BENEFITS

Mary Hardy, David Saunders, Mike X Zhu
University of Waterloo

IAA/PBSS Symposium
Lyon, June 2013

Outline

1. Background
2. Framework, assumptions, notation
3. The valuation formulas
4. Some results for April 2013 interest rates
5. Results for past yield curves
6. Other valuation methods

Cash Balance Pensions

- Look like DC
  - contribution (% of salary) paid into participant’s account
  - account accumulates to retirement
  - lump sum retirement benefit
  - withdrawal benefit =account value (after vesting)
- Regulated like DB
- Participant accounts are nominal

Crediting rates

- Participant’s account accumulates at specified *crediting rate*.
- For example
  - Yield on 30-year government bonds
  - Yield on 10-year government bonds
  - Yield on 5-year government bonds + 25bp
  - Yield on 1-year government bonds + 100bp
  - Fixed rate, eg 5% p.y.
  - CPI rate

Some statistics...

- In 2010, 12 million CB participants in US
- Early popularity with sponsors, late 1990s
  - Simple transition from traditional DB to CB
    - Compared with DB to DC transition
  - Tax benefits
  - More transparent (apparently)
  - Less contribution volatility (apparently)
- With participants..
  - More portable, more transparent
  - But transition problems for older members

Framework, assumptions, notation

- Participant with n years service at valuation date.
- At valuation $t=0$.
- Retires at T with $n+T$ years
- Ignore exits, annuitization.
- Value future benefit arising from past contributions
- Use market valuation methods
  - Generates the cost of transferring the pension liability to capital markets

Framework, assumptions, notation

- F_t denotes the participant's fund at t
- $i^c(t)$, $r^c(t)$ denote the crediting rates at t
- $r_k(t)$ denotes the k-year spot rate at t
- $r(t)$ denotes the short rate at t
- $p(t, t+k)$ denotes the price at t of a \$1, k-year zero coupon bond.

Framework, assumptions, notation

- Recall that

$$p(t, t+k) = e^{-k\, r_k(t)}$$

- Using financial valuation principles, we also have

$$p(t, t+k) = \mathrm{E}_t^Q \left[\exp\left\{ -\int_t^{t+k} r(s)\,ds \right\} \right]$$

Framework, assumptions, notation

- Assume continuous crediting, given F_t

$$F_T = F_t \exp\left\{\int_t^T r^c(s)\,ds\right\}$$

- This is a random variable unless the crediting rate is constant.

The Valuation Formula

- The market value at t=0 of the benefit F_T is

$$
{}_0V = E_0^Q\left[F_T \, e^{-\int_0^T r(s)ds} \right]
$$

$$
= F_0 \, E_0^Q\left[\left(e^{\int_0^T r^c(s)ds} \right) \left(e^{-\int_0^T r(s)ds} \right) \right]
$$

$$
= F_0 \, E_0^Q\left[e^{\int_0^T \left(r^c(s) - r(s) \right) ds} \right]
$$

The Valuation Formula

- We let

$$V(t,T) = E_t^Q\left[\exp\left\{\int_t^T r^c(s) - r(s)\, ds\right\}\right]$$

That is

- $V(t,T)$ = market value at t of CB benefit at T
  - per $1 of nominal fund at *t*
  - No exits
  - No future contributions
  - With continuous compounding

Fixed crediting rate

- Suppose r^c(t) is constant, =r^c, say
- Then

$$V(0,T) = E_0^Q\left[\exp\left\{\int_0^T r^c(s) - r(s)\,ds\right\}\right]$$

$$= \exp(Tr^c)\,E_0^Q\left[\exp\left\{-\int_0^T r(s)\,ds\right\}\right]$$

$$= \exp(Tr^c)\,p(0,T)$$

- The T-year zcb price *p(0,T)*, is known at *t*=0

Fixed crediting rate

- For example, $r^c = \log(1.05)$
- Using US yield curve at 1/April/2013

$$V(0,5) = (1.05)^5 (0.96256) = 1.2285$$

$$V(0,10) = (1.05)^{10} (0.82250) = 1.3398$$

$$V(0,20) = (1.05)^{20} (0.58889) = 1.5626$$

- That is, with a 10-year horizon to retirement, every \$1 of fund costs \$1.4375
- Every \$1 of new contribution costs \$1.4375
- Model-free valuation result.

Crediting with the short rate

- Suppose the crediting rate is the short rate plus a fixed margin m
  - That is $r^c(t) = r(t) + m$, then

$$V(0,T) = E_0^Q\left[\exp\left\{\int_0^T r^c(s) - r(s)\, ds\right\}\right]$$

$$= E_0^Q\left[\exp\left\{\int_0^T r(s) + m - r(s)\, ds\right\}\right]$$

$$= e^{mT}$$

Crediting with the short rate

- For example, $r^c(t) = r(t) + m$, with $m = 0.0175$
  - Then

    V(0,5) = e^{5m} = 1.09144

    V(0,10) = e^{10m} = 1.19125

    V(0,20) = e^{20m} = 1.41908

- This will be $\approx$ to the valuation for 3-month T-bill +175bp crediting rates.
- Model-free

Crediting with *k*-year spot rates

- Crediting with $r^c(t) = r_k(t) + m$
- We need a market model for $r_k(t)$
- We use one-factor Hull-White / ext Vasicek model

$$dr(t) = a\left(\theta(t) - r(t)\right)dt + \sigma dW_t$$

$$p(t,t+k) = E_t^Q\left[\exp\left\{-\int_t^{t+k} r(s)ds\right\}\right] = \exp\left\{A(t,t+k) - B(t,t+k)r(t)\right\}$$

$$B(t,t+k) = \frac{1-e^{-ak}}{a}$$

$$A(t,t+k) = \log\left(\frac{p(0,t+k)}{p(0,t)}\right) + r_t(0)B(t,t+k) - \frac{\sigma^2}{4a}B(t,t+k)^2\left(1-e^{-2at}\right)$$

Crediting with *k*-year spot rates

$$p(t,t+k) = e^{-kr_k(t)}$$

$$\Rightarrow \exp\{A(t,t+k) - B(t,t+k)r(t)\} = \exp\{-kr_k(t)\}$$

$$\Rightarrow r_k(t) = \frac{B(t,t+k)r(t) - A(t,t+k)}{k}$$

$$V(0,T) = \mathrm{E}_0^Q\left[\exp\left(\int_0^T r_k(t) + m - r(t)dt\right)\right]$$

$$= \mathrm{E}_0^Q\left[\exp\left(\int_0^T \frac{B(t,t+k)r(t) - A(t,t+k)}{k} + m - r(t)dt\right)\right]$$

Crediting with *k*-year spot rates

- Separate out terms in *r(t)*

$$V(0,T) = e^{mT} \exp\left(-\int_0^T \frac{A(t,t+k)}{k} dt \right) \mathrm{E}_0^Q \left[\exp\left(-\int_0^T \gamma r(t) dt \right) \right]$$

where $\gamma = 1 - \left(\dfrac{1-e^{-ak}}{ak} \right)$

- The second term is evaluated using numerical integration (partly).
- The third term can be solved analytically – similar to the case γ=1
- ...

Crediting with k-year spot rates

$$E_0^Q\left[\exp\left(-\int_0^T \gamma r(t)dt\right)\right]=$$

$$\exp\left\{\gamma\log p(0,T)+\frac{\sigma^2\gamma}{2a}\left(\frac{(1-e^{-aT})(1-2\gamma)}{a}\right)+\frac{(1-e^{-aT})^2+\gamma(1-e^{-2aT})}{2a}-T(1-\gamma)\right\}$$

- We use parameters a = 0.02, σ = 0.006
- For T=5, 10, 20 years
- $r^c(t)$= 30-yr spot rate, 20-yr spot rate, 10-yr spot rate, 5-yr + 25bp, 1-yr + 100bp, 0.5-yr+150bp
- Yield curve from 1/4/13 US treasuries.

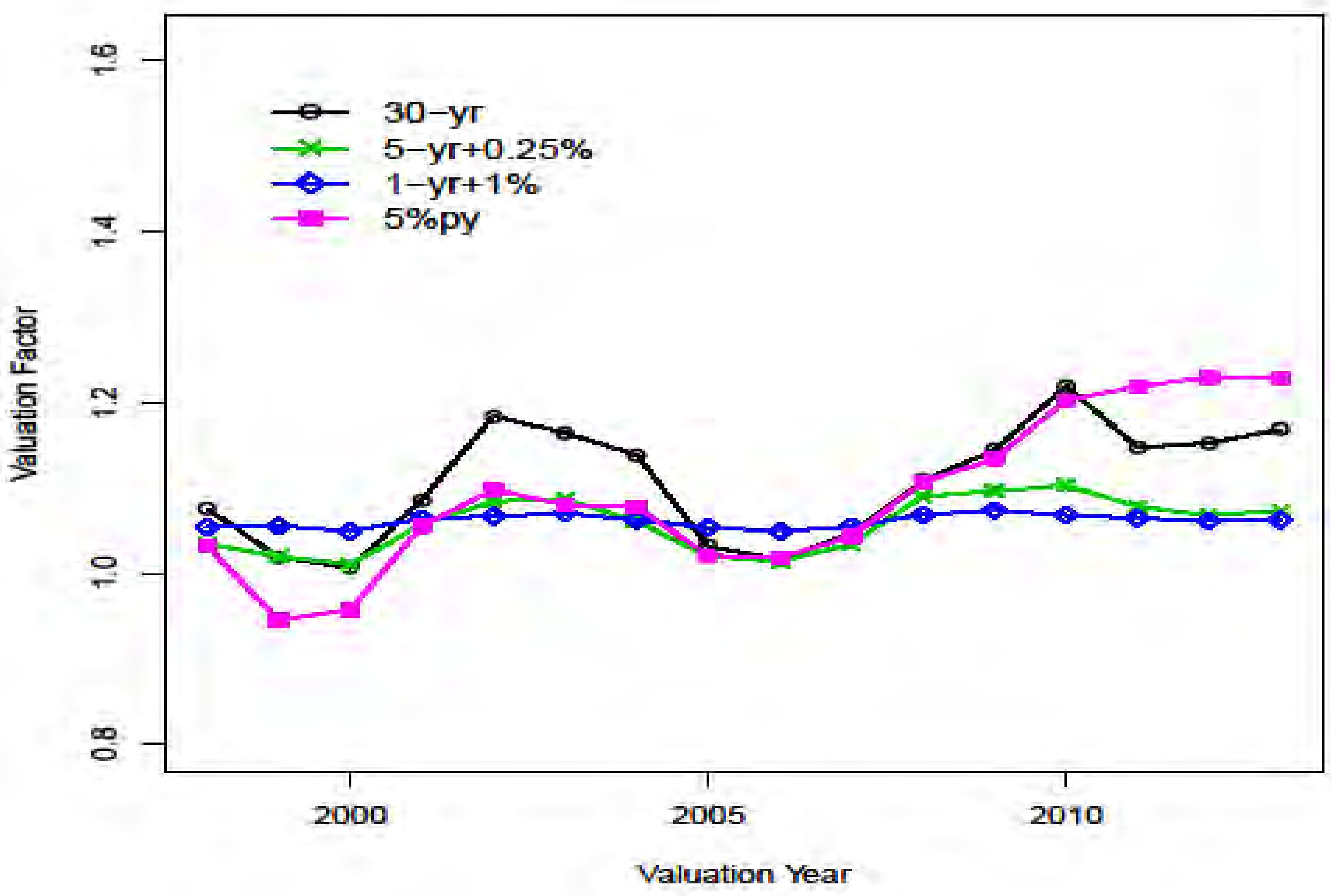
Crediting with k-year spot rates

	V(0,T); 2013 YC		
Crediting Rate	T=5	T=10	T=20
30-yr	1.168	1.235	1.380
20-yr	1.130	1.189	1.361
10-yr	1.095	1.106	1.230
5-yr+0.25%	1.073	1.091	1.177
1-yr+1.0%	1.062	1.120	1.250
½-yr+1.5%	1.083	1.170	1.366
short+1.75%	1.091	1.191	1.419
5% fixed	1.229	1.340	1.562

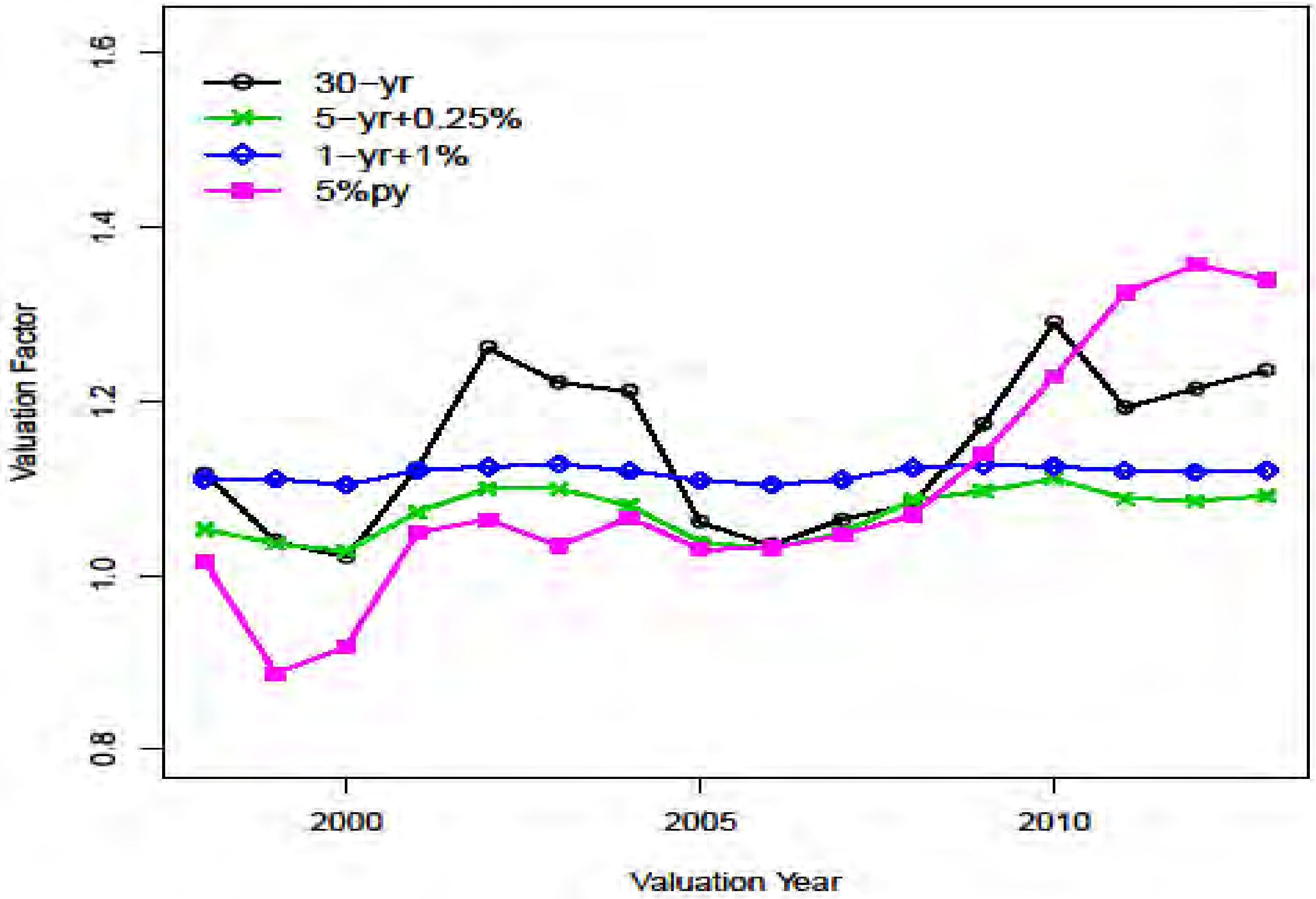
Impact of the starting YC

- Repeat the valuation for yield curves
  - 1998 →2013
- Plot V(0,T) over time for different $r^c(t)$ definitions and for T=5, 10, 20
- Same scale

T=5-years

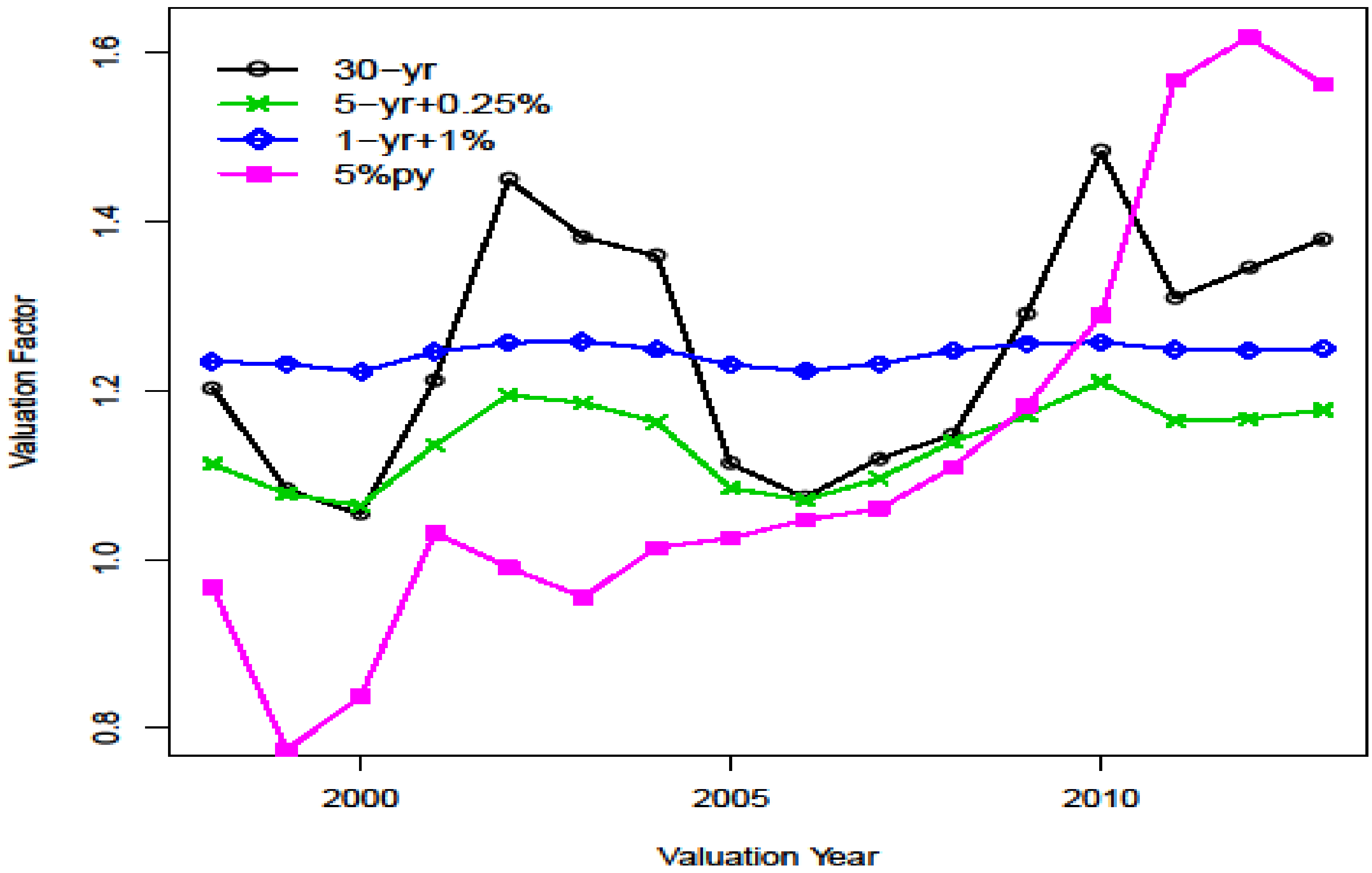

Valuation Factor
0.8
1.0
1.2
1.4
1.6
30−yr
5−yr+0.25%
1−yr+1%
5%py
2000
2005
2010
Valuation Year

T=10-years



30−yr
5−yr+0.25%
1−yr+1%
5%py
Valuation Factor
0.8
1.0
1.2
1.4
1.6
2000
2005
2010
Valuation Year

T=20-years



30−yr
5−yr+0.25%
1−yr+1%
5%py
Valuation Factor
Valuation Year
0.8
1.0
1.2
1.4
1.6
2000
2005
2010

Comments

What is the most stable choice for r^c?

- Long rates are more stable than short rates
- Constant rates are even more stable
- But long rates and constant rates produce more volatility than short rates.

Comments

Has the cost risen since the early transitions in 1998?

- For fixed rates – yes
- For market based rates – it's more complicated
  - Interest rates were high in 1999, $r_{30} \approx 6.3\%$
  - But the cost is low because short rates were also high.
  - The risk is from the spread, $r_k(t) - r(t)$ not from the absolute values

Crediting with k-year par yields

- More realistic for k >1
- But requires simulation, no analytic results
- For 1/4/2013 valuation:

	V(0,T); 2013 YC		
Crediting Rate	T=5	T=10	T=20
30-yr par	1.150	1.218	1.366
20-yr par	1.123	1.172	1.340
10-yr par	1.093	1.105	1.227
5-yr+0.25%	1.073	1.091	1.177

Actuarial valuations

- Review traditional approaches
- Consider three CB methods
- Principles and notation:
  - AL_t = actuarial liability = target asset requirement
  - NC_t = Normal Contribution = contribution needed to fund the expected increase in *AL*, *t* to *t+1*
- Under valuation assumptions, ignoring exits

$$(AL_t + NC_t)(1 + i_t) = AL_{t+1}$$

Actuarial valuation for final-salary DB

- Accruals based $\Rightarrow$ past service earned benefits are included in the valuation
- Accruals methods are PUC and CUC(=TUC)
  - Projected accrued $\Rightarrow$ benefits from past service indexed to retirement by salary scale.
  - Current accrued $\Rightarrow$ benefits from past service valued assuming no further increases.

CB Valuation 1: Past service, projected credited interest

- Past service $\Rightarrow$ no allowance for future contributions to participant's fund
- This is the method used above, with market rates and models

$$\boxed{\begin{aligned} AL_t &= F_t\, V(t,T) \\ NC_t &= cS_t\, V(t,T) \end{aligned}}$$

CB Valuation 2: Past service, current credited interest

- Past service $\Rightarrow$ no allowance for future contributions to participant's fund
- Current credited interest $\Rightarrow$ no allowance for future credited interest
- $v_i(s)$ denotes the valuation discount factor for s-yrs ahead

$$AL_t = F_t$$

$$NC_t = cS_t + \left(F_t + cS_t\right)\Big((1+i^c(t))v_i(1) - 1\Big)$$

CB Valuation 3: Full service, projected credited interest, pro-rata accrual

- Let $\widetilde{B_t}(T)$ denote the projected final benefit, and let n denote service at the valuation date
- Deterministic salary growth and crediting rate assumptions

$$AL_t = \left(\widetilde{B}_t(T)\, v_i(T-t)\right)\frac{n}{n+T-t}$$

$$NC_t = \frac{AL_t}{n}$$

Example

- Olivia
  - 19 years service
  - 1 year to retirement
  - S=$75 000, F=100 000
- Harriet
  - 10 years service
  - 10 years to retirement
  - S=60 000, F=55 000
- Beatrice
  - 1 year service
  - 19 years to retirement
  - S=50 000, F=4 000

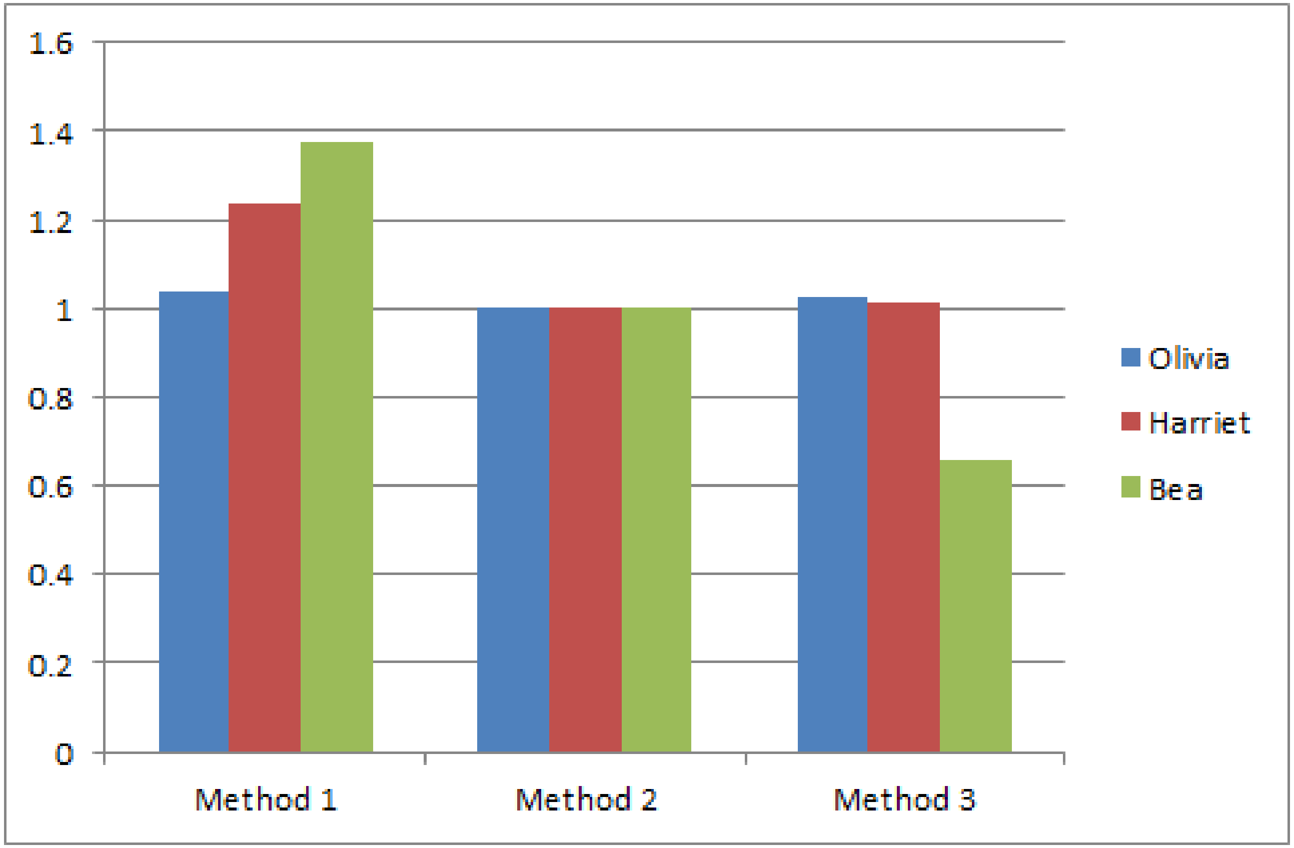
Example

- Assume 1/4/2103 market rates for $v_i(s)$
- Crediting rate = 0.036 (30-year rate)
- Future crediting rate assumption (for method 3)

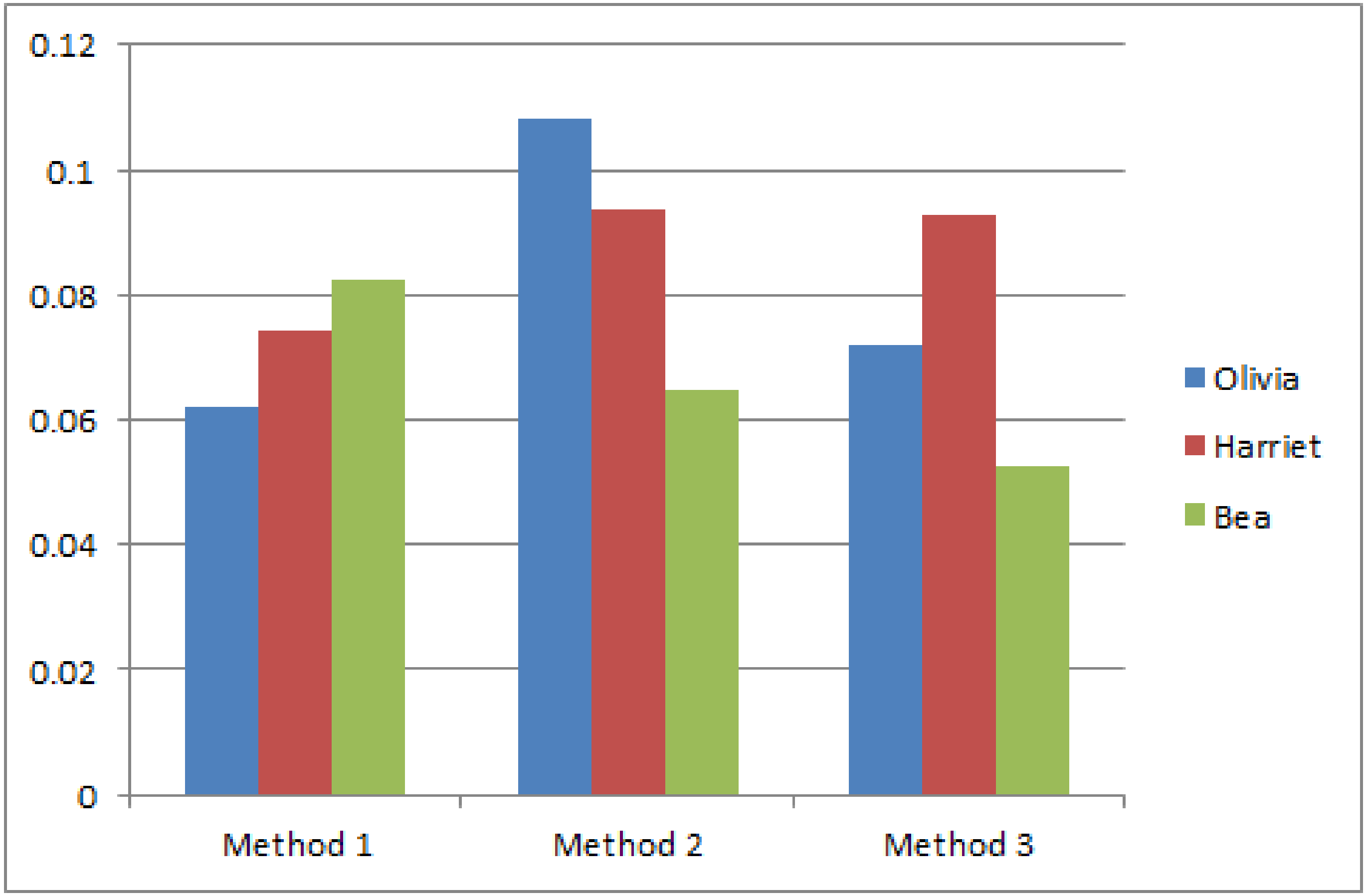
  $i^c(s) = 0.036$

- Future salary growth assumption 2% p.y. (method 3)

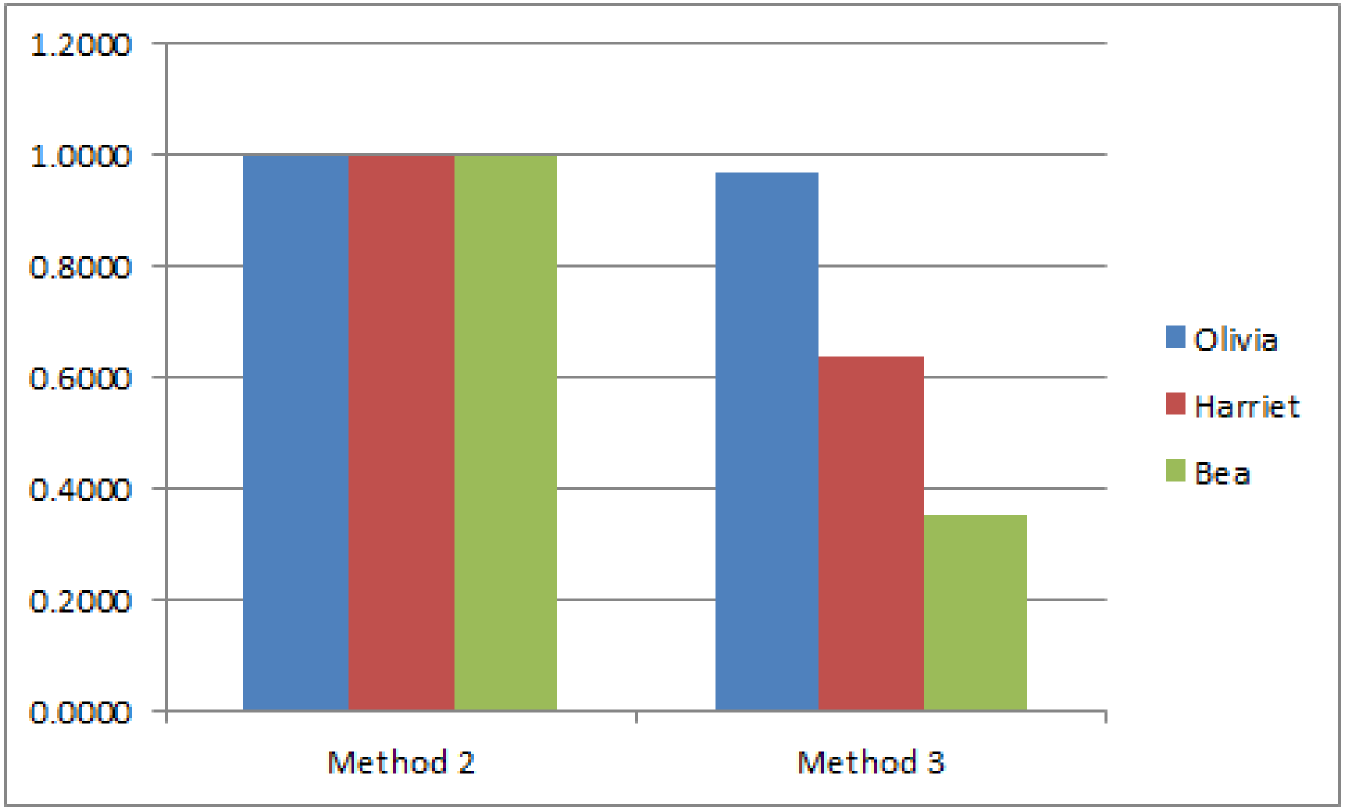
Example: AL_t/F_t , market rates



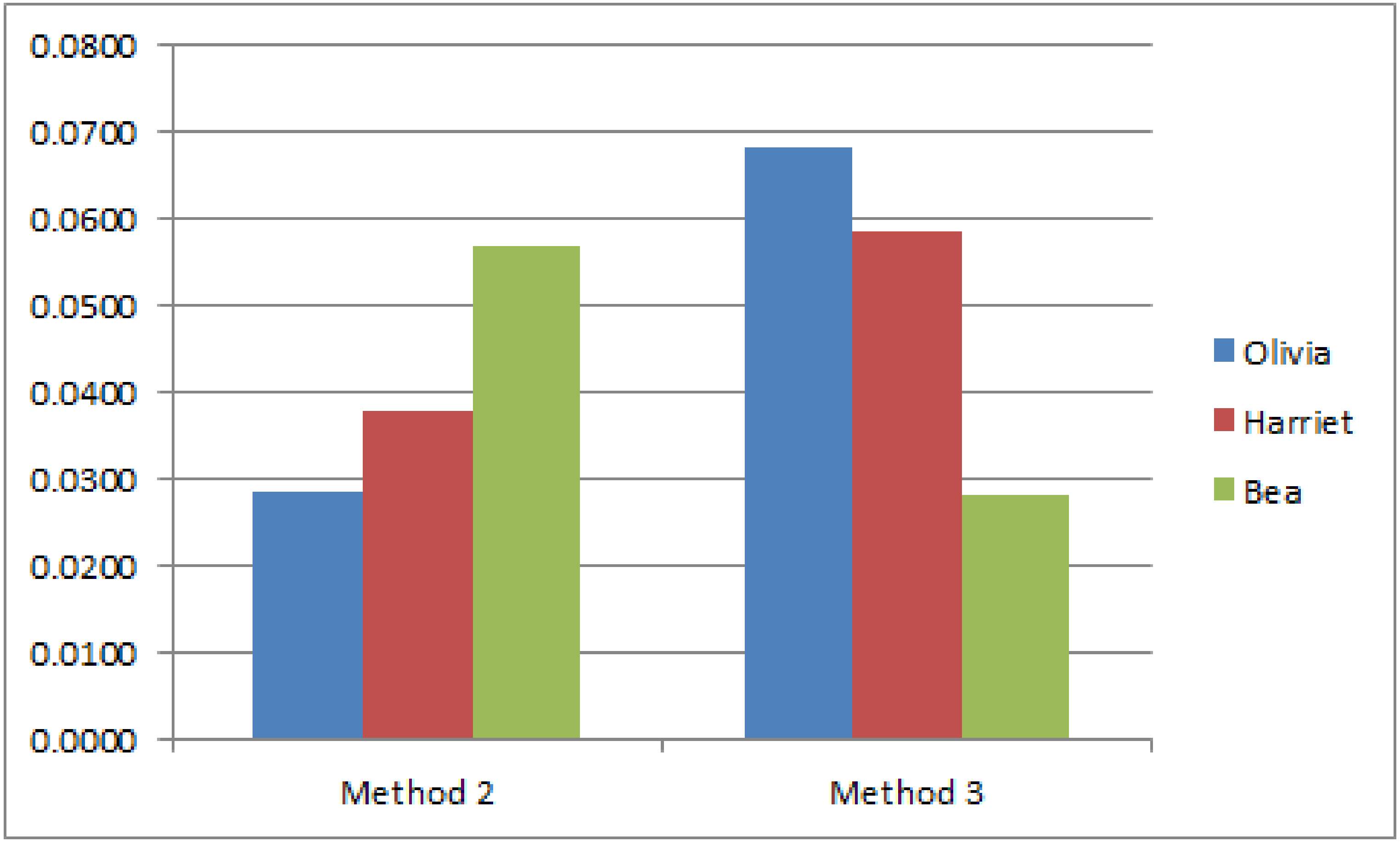
Example: NC_t/S_t , market rates



Example: AL_t/F_t , i=6%



Example: NC_t/S_t , i=6%



Comments

- Valuation by projecting/discounting/pro-rata is more like TUC than PUC
- Assuming a non-market interest rate generates AL considerably less than fund values
  - Potential for spectacular losses and reputational risk
- Assuming credited interest for long term may be overly conservative
  - This is the true PUC analogy
- But not projecting leads to high contribution rates
  - Similar to TUC method

Conclusions

- This benefit isn’t as simple as we thought
- This benefit isn’t as cheap as we thought/think
- Underfunding relative to F_t should not be permitted
- CB beginning to gain popularity outside US
  - Is this really desirable?
- Risk management is for future work
  - Managing the 30-year rate guarantee is not easy