Stochastic Simulation of Individual Retirement Accounts in Mexico

(Replacement Rates Comparison)

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   - Mexican Pension System
   - Uncertain Benefits

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Mexican Pension System

Individual Account System

Reforms in 1997 changed Mexican Social Security System from Collective Capitalization into Individual Retirement Accounts System.



Individual Account System

Administrators

Accounts are managed by Pension Fund Administrators AFORE (for its acronym in Spanish) and every worker may choose one of them.

Administrators	
Afirme Bajío	Metlife
Azteca	PENSIONISSSTE
Banamex	Principal
Bancomer	Profuturo GNP
Coopel	SURA
Inbursa	XXI
Invercap	

Administrators in Operation 2012, Source: Mexican Supervising Institution CONSAR.

SIEFOREs

SIEFOREs

According to Worker Age, Administrators invest Retirement Savings in Market Money by using Specialized Investment Societies denominated SIEFOREs. (Portfolios)

For Younger Persons, More Risk.

Benefits, Contributions and Requirements

Benefits

- Pension + Survival Insurance = Cumulated Fund
- Minimum Pension Guaranteed (MPG)

Contributions

- 6.5% of Salary+ Social Quota

Requirements

- 65 Years Old
- 1250 Contributions Weeks $\sim$ (24 Years)

Uncertain Benefits

- There are no Historical Replacement Rates to Compare Administrators.

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- Contribution Density affects directly the Pensions.
- So...

So, ¿Which Administrator Choose?

Answer

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- Which gives the **Highest Pension**

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- Which gives the **Highest Pension**
- Or the **Highest Replacement Rate**

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$$\Delta\% = \frac{Pension}{Last\ Salary}$$

In Probability Terms

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- Which has the **Smallest Probability of Exercising the (MPG) (Not Reaching the GOAL).**

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- Which has the **Smallest Probability of Exercising the (MPG) (Not Reaching the GOAL).**
- The Administrator (Portfolio) with the **Highest Probability of Being Better than the Others.**

Section 2

Stochastic Simulation of Individual Retirement Account

Individual Account Factors

- Contribution Density.
- Real Investment Returns.
- Salary History.

Contribution Density

Contribution Density

Percentage of Contributions Made

CONSAR (2010)

- 70% Males
- 66% Females

Modeling Contribution Density

- Non-Homogeneous Markov Chain Model
- Homogeneous Markov Chain Model
- Bayesian Model

Markov Chain Models

Let $C_n \in \{0, 1\}$ the Stochastic Process of Contributions at time n

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- **Non-Homogeneous Transition Matrix**

$$P_x = \begin{pmatrix} p_x & 1 - p_x \\ 1 - q_x & q_x \end{pmatrix}$$

Where $P_x(i, j) = \mathbb{P}[C_{x+1} = j | C_x = i]\ i, j \in \{0, 1\}$

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- **Homogeneous Transition Matrix**

$$\lim_{n \to \infty} \begin{pmatrix} p & 1 - p \\ 1 - q & q \end{pmatrix}^n = \begin{pmatrix} \delta & 1 - \delta \\ \delta & 1 - \delta \end{pmatrix}$$

Where: $\delta =$ Contribution Density

Bayesian Model

- Contribution C_n given δ is a $Bernoulli(\delta)$ Random Variable.

$f(C_n|\delta) \sim Bernoulli(\delta)$, *a priori distribution* $\pi_0(\delta) \sim Beta(\alpha, \beta)$ $\alpha, \beta > 0$

Then, the Predicted Distribution for C_{n+1} given $C_n, \ldots, C_1$ is:

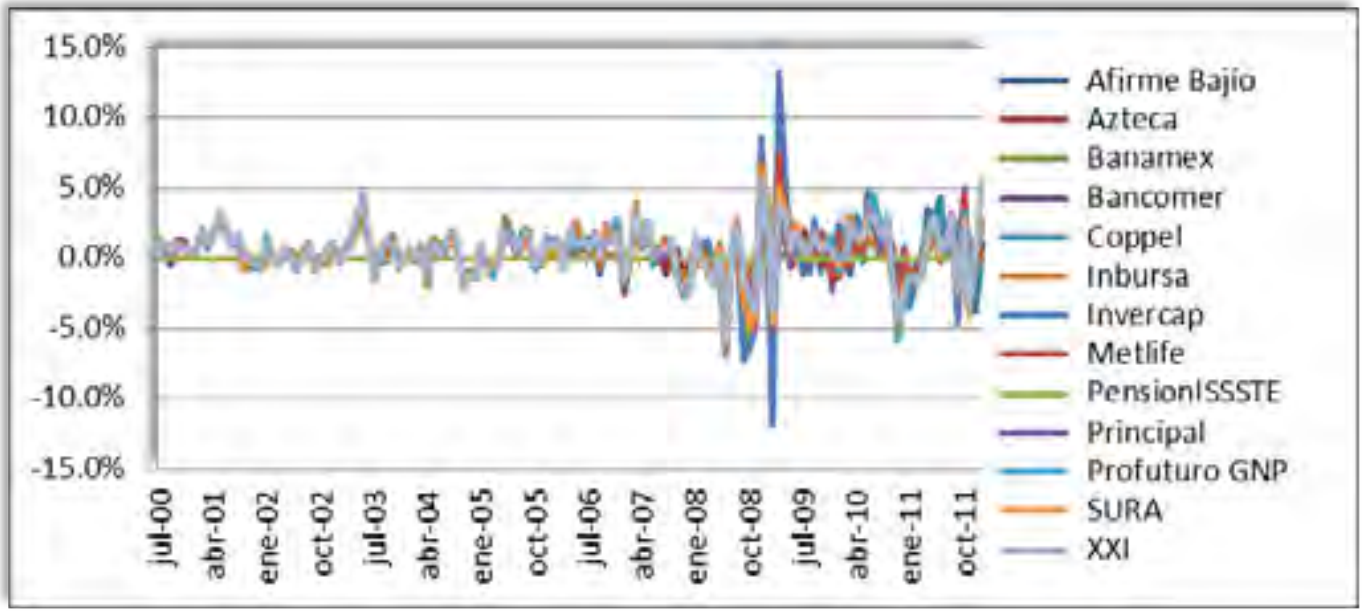
$$\hat{f}\left(C_{n+1}|C_n, \ldots, C_1\right) \sim Bernoulli(\delta_{n+1})$$

Where

$$\delta_{n+1} = E[\delta|C_n, \ldots, C_1] = \frac{\alpha + \sum_{j=1}^{n} C_j}{\alpha + \beta + n}$$

Real Investment Returns

Real Investment Returns are subject to Market Volatility.



Source: Own elaboration with Data published by Regulator Institution CONSAR and Mexican Central Bank

Modeling Real Investment Returns

- Bootstrapping Methods
- Heavy Tailed Distributions
- Mixed Distributions
- Multivariate Distributions
- Time Series Models (ARIMA, VARIMA, GARCH)
- Univariate Distributions
- So on...

Multivariate Normal Distribution

Let $R = (r_1, \ldots, r_n)$ Monthly Investment Returns for n Administrators. R is Normal Distributed if its Probability Density Function is:

$$\phi(R, \mu, \Sigma) = |2\pi\Sigma|^{-1/2} \exp\left(-\frac{1}{2}(R-\mu)' \Sigma^{-1} (R-\mu)\right)$$

Where
$\mu = (\mu_1, \ldots, \mu_n)$ Expected Monthly Real Investment Returns.
Σ Volatility and Correlation Structure.
Those Parameters are estimated by Maximum Likelihood Method.

Salary History

Salary History

Salary changes in response of Abilities, Productivity and other Characteristics. The Comparison among Administrators might be Independent of Salary History.

Salary Profiles

- Mexican Average Salary $\mu = 4.2$ Minimum Wages (MW)
- $\mu + \sigma = 8.8$ (MW)

Salary Growth

- 1% Real Annual Growth as Actuarial Valuations from Mexican Social Security Institute

Simulations

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- At time t is determined:

$$SIEFORE_j = \begin{cases} 1, & x_t \leqslant 22 \\ 2, & 26 < x_t \leqslant 37 \\ 3, & 37 < x_t \leqslant 45 \\ 4, & 45 < x_t \leqslant 59 \\ 5, & 59 < x_t \end{cases}$$

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- Given $SIEFORE_j$ it is generated a Normal Vector of Investment Returns $R_j = (r_1, ..., r_n)$ for n Administrators.

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- After a year, Salary is upgrade one percent and the Process Continue until Retirement Age.

Replacement Rate Simulation

- For Administrator i the Pension is:

$$P_i = \Delta\% S_n = \max\left(\frac{F_{i,r}}{\frac{13}{12}(\ddot{a}_x^{(12)} + \ddot{a}_y^{(12)} - \ddot{a}_{xy}^{(12)})}, MPG\right)$$

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- It is Constructed N Replacement Rates Scenarios.

Replacement Rate Simulation

- Probability of Exercising MPG is estimated for Administrator i as follows:

$$\hat{p}_i = \sum_{i=1}^{N} \frac{I\{P_i = MPG\}}{N}$$

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- Probability of Administrator i is worse than Administrator j

$$\hat{p}_{i,j} = \sum_{i=1}^{N} \frac{I\{\delta_i < \delta_j\}}{N}$$

Salary Profiles

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- 20, 30 Years Old

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- Contribution Density .70 and .66 for Males and Females

Section 3

Results and Conclusions

Simulation Results

Simulations

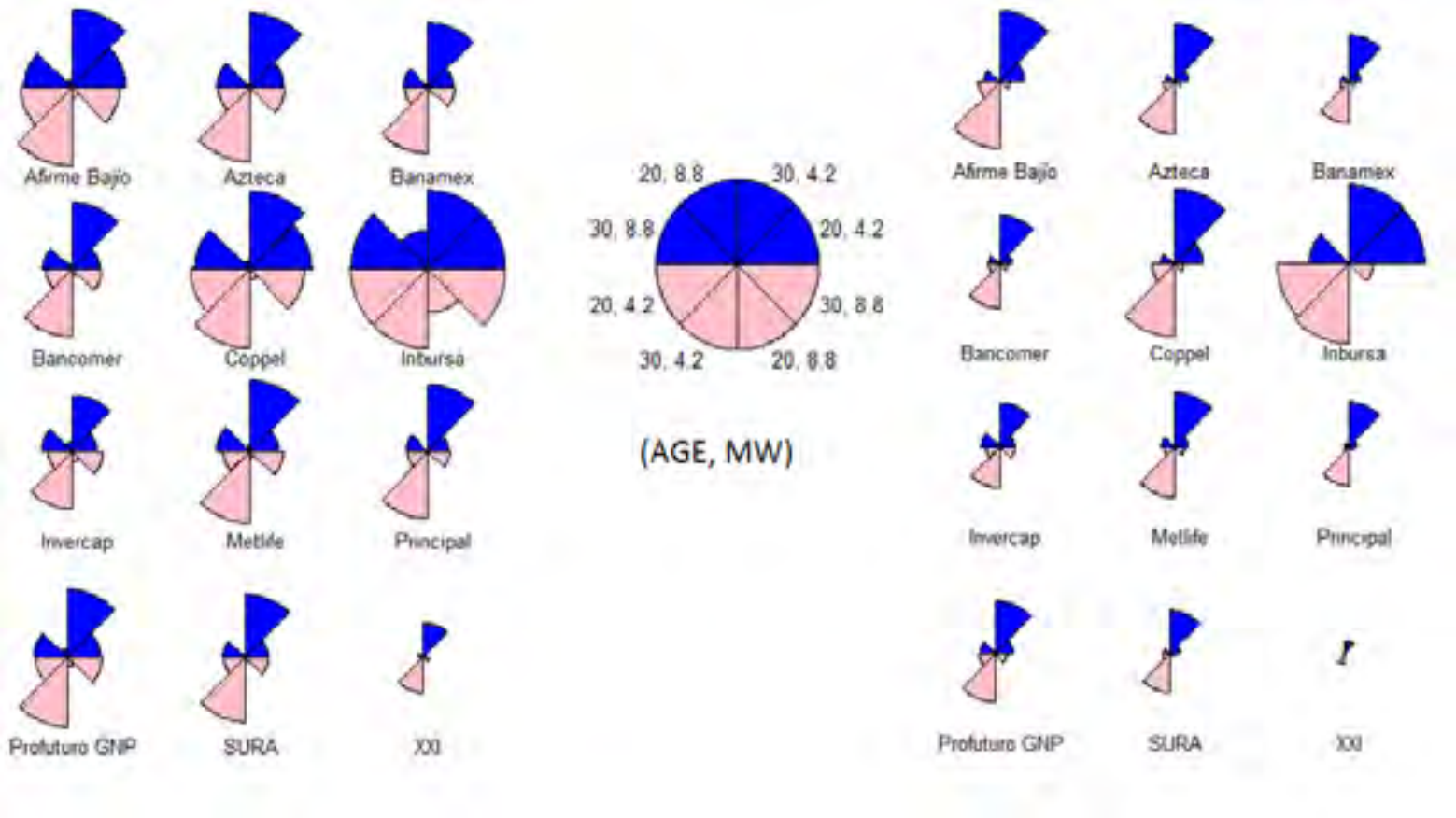
- Simulations were elaborated with the statistical package R.
- Data was obtained from the Regulator Institution CONSAR and the Mexican Central Bank webpages at February 2012.

Probabilities of Exercicing the MPG

Probability of exercising MPG		System			
		Salary			
		4.2 MW		8.8 MW	
		AGES			
Sex	Density	20	30	20	30
M	70%	0.446	0.844	0.103	0.446
	100%	0.253	0.661	0.014	0.172
F	66%	0.427	0.853	0.117	0.454
	100%	0.222	0.613	0.010	0.119

Probabilities grows up as age as well. Not taking into account Contribution Density Underestimate Actuarial Valuations of MPG Benefits

Probabilities of Exercising the MPG by Administrators, Contribution Density, Salary and Sex



Density Average (Left), Density of 100% (Right)

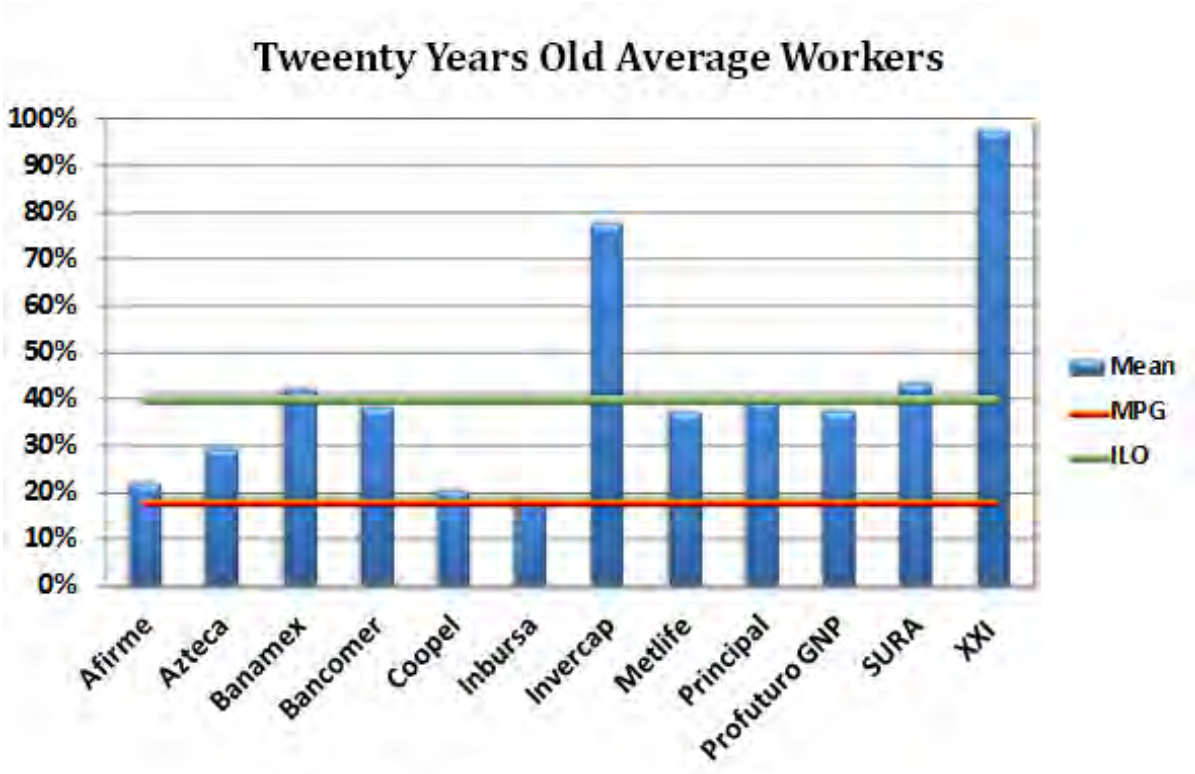
System Replacement Rates

Replacements Rates (μ, σ)		System							
		Salary							
		4.2 MW				8.8 MW			
		Ages							
		20		30		20		30	
Sex	Density	μ	σ	μ	σ	μ	σ	μ	σ
M	70%	42%	38%	23%	8%	39%	39%	16%	7%
	100%	59%	60%	27%	13%	54%	51%	22%	11%
F	66%	43%	39%	22%	6%	36%	31%	16%	9%
	100%	63%	64%	28%	14%	57%	54%	23%	12%

Replacement Rates are higher for Average Workers due Social Quota.
Only Young Workers could reach Replacement Rates above 40%.

Differences due Contribution Density go from 4% to 20%

Administators Expected Replacement Rate



Only two Administrators can give Pensions above the International Labour Organization recommendation of 40%

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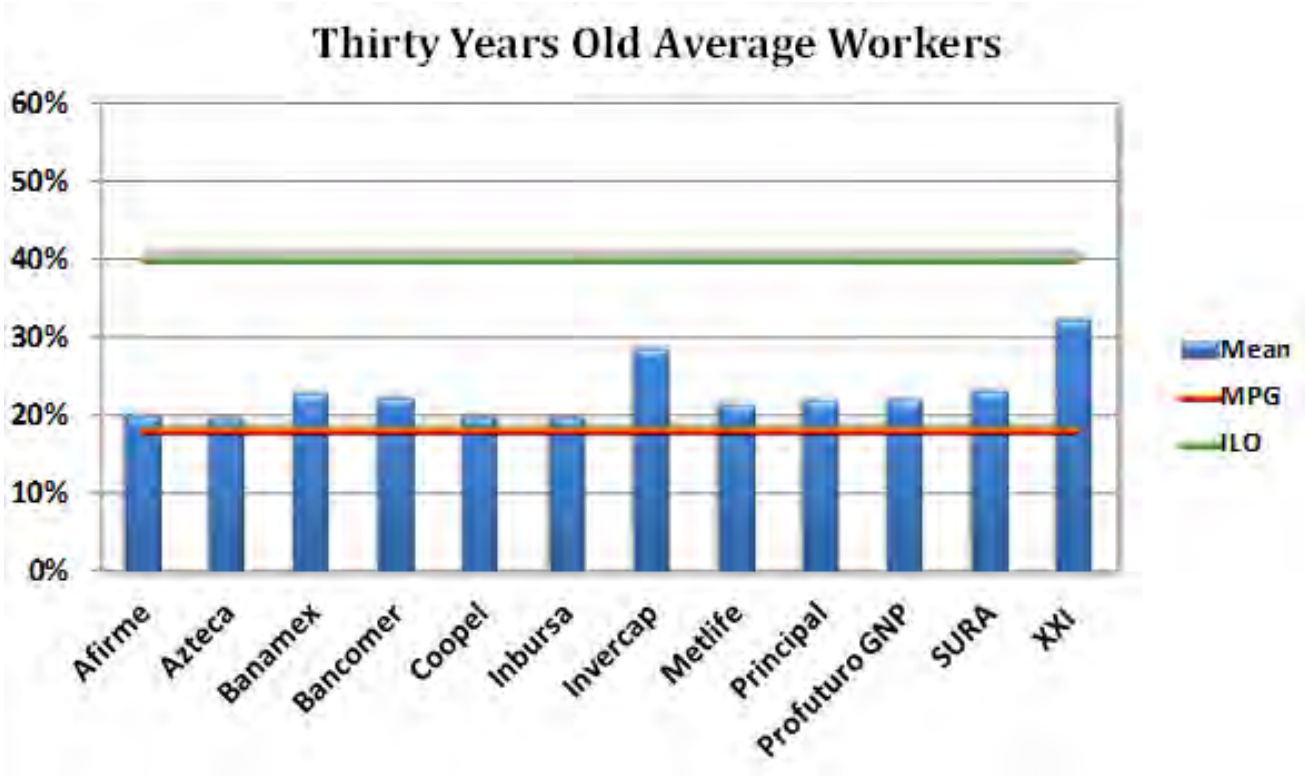
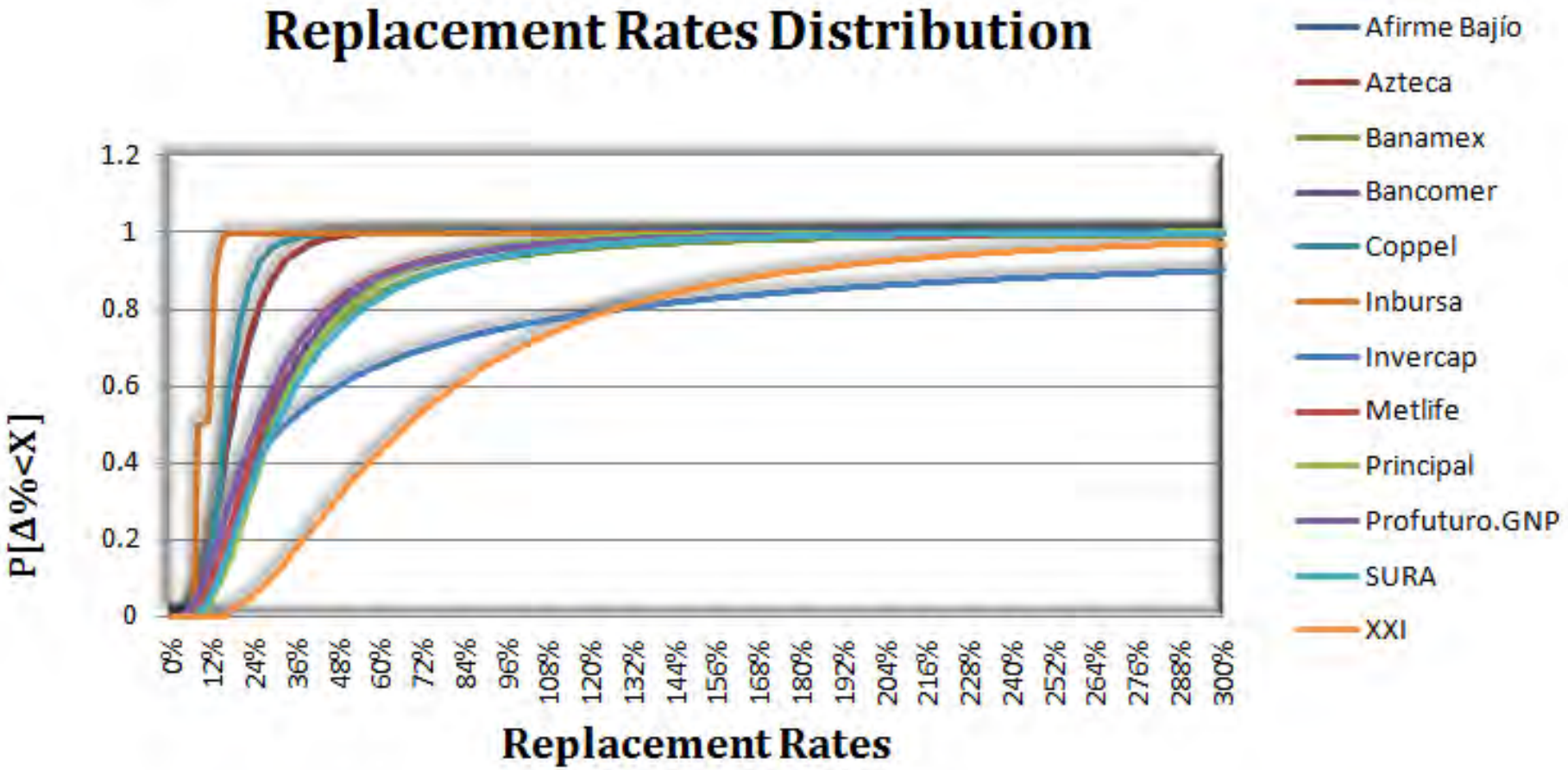


All Administrators among the MPG and ILO Recommendation

Replacement Rates Distribution



Administrators except Invercap and XXI with Probability of .8 will no give Replacement Rates higher than 56% for 8.8 MW Average Workers

Correlation Matrix

ρ_{xy}	Afirme Bajío	Azteca	Banamex	Bancomer	Coppel	Inbursa	Invercap	Metlife	Principal	Profuturo GNP	SURA	XXI
Afirme Bajío	1	0.92	0.84	0.88	0.95	0.51	0.63	0.89	0.91	0.88	0.85	0.81
Azteca		1	0.89	0.93	0.9	0.47	0.74	0.91	0.94	0.91	0.91	0.84
Banamex			1	0.98	0.82	0.46	0.88	0.92	0.94	0.97	0.98	0.91
Bancomer				1	0.86	0.48	0.87	0.94	0.96	0.98	0.98	0.91
Coppel					1	0.53	0.63	0.87	0.9	0.84	0.84	0.78
Inbursa						1	0.28	0.47	0.55	0.47	0.47	0.46
Invercap							1	0.8	0.77	0.83	0.87	0.79
Metlife								1	0.96	0.95	0.94	0.9
Principal									1	0.96	0.97	0.9
GNP										1	0.97	0.91
SURA											1	0.93
XXI												1

High Correlation due Investment Returns.

Probability of Being Better than other Administrator

$P[\Delta_i < \Delta_j]$	Afirme Bajío	Azteca	Banamex	Bancomer	Coppel	Inbursa	Invercap	Metlife	Principal	Profuturo GNP	SURA	XXI
Afirme Bajío		0.9	0.91	0.9	0.22	0.02	0.83	0.89	0.97	0.81	0.94	1
Azteca	0.1		0.8	0.75	0.03	0	0.75	0.69	0.93	0.59	0.87	1
Banamex	0.09	0.2		0.26	0.02	0	0.64	0.31	0.54	0.17	0.63	1
Bancomer	0.1	0.25	0.74		0.02	0	0.71	0.41	0.74	0.26	0.83	1
Coppel	0.78	0.97	0.98	0.98		0.02	0.84	0.91	0.97	0.83	0.96	1
Inbursa	0.98	1	1	1	0.98		0.88	0.93	0.97	0.89	0.96	1
Invercap	0.17	0.25	0.36	0.29	0.16	0.12		0.23	0.36	0.18	0.35	0.84
Metlife	0.11	0.31	0.69	0.59	0.09	0.07	0.77		0.73	0.4	0.75	1
Principal	0.03	0.07	0.46	0.26	0.03	0.03	0.64	0.27		0.19	0.53	1
GNP	0.19	0.41	0.83	0.74	0.17	0.11	0.82	0.6	0.81		0.86	1
SURA	0.06	0.13	0.37	0.17	0.04	0.04	0.65	0.25	0.47	0.14		1
XXI	0	0	0	0	0	0	0.16	0	0	0	0	

XXI And Invercap are more probable to be better than the others Administrator

Conclusions

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- Replacement Rates System are between MPG and ILO recommendation.
- XXI and Invercap are more probable to give Higher Replacement Rates.
- System could be Over valuated due not taking into account Contribution Density effects.
- Methodology can be also used for Comparing Investment Strategies, Portfolios, Competitors in terms in final Benefits and Probabilities.

We need to think about our future before this:

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Merci!!