Mean-Variance Optimal Portfolios under State-dependent Preferences

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Summary

It is common in the academic literature to study optimal investment strategies using an objective function that balances “return” and “risk”. This idea was introduced by Markowitz (1952), who together with Roy (1952) first proposed a quantitative approach to determine the optimal trade-off between mean (return) and variance (risk). The mean-variance framework has become very influential due to the fact that it combines simplicity with practical applicability. Markowitz’s (1952) paper initiated a tremendous amount of further academic research on optimal portfolio selection problems. Most of the existing approaches have in common that the objective function being optimized is “law-invariant” or “state-independent”. In other words the objective is solely driven by the distribution of returns and does not depend on the states of the economy in which they are generated. Such preferences include a wide range of behavioral theories, among them expected utility theory, cumulative prospect theory and rank dependent utility theory.

In practice, agents may face state-dependent constraints when making investment decisions. Purchasing a put option may mean that the agent would like to protect an investment in the underlying asset. In this paper we show how it is possible to derive optimal mean-variance efficient strategies under state-dependent constraints, where each such constraint is interpreted as information on the relationship between the outcomes of the strategy and the states of the financial market. Using the theory of copulas, we develop a constructive method to obtain the cheapest investment strategy which preserves, under state dependent preferences, a given mean and variance.

The results in the present paper require that the market is frictionless, arbitrage-free

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and that the agents agree on the same pricing operator. The paper makes the following contributions:

Traditional optimal mean-variance optimization consists in finding the best allocation of assets assuming a buy-and-hold strategy or a constant mix strategy. The question then is how mean-variance efficient portfolios can be derived when there is no restriction on trading; in other words, when all trading strategies are allowed and available. Of course, allowing for more trading strategies and thus more degrees of freedom will further enhance optimality. The first contribution in this paper is to derive optimal strategies explicitly when there are no restrictions on trading. We show that the optimal portfolio is a short position in the stochastic discount factor used for pricing derivatives and a long position in cash. To derive this result we use recent results on cost-efficient strategies, initiated by Cox and Leland (1982) and Dybvig (1988), and recently extended in Bernard, Boyle and Vanduffel (2011).

In the absence of additional constraints, we are then able to compute the maximum Sharpe ratio when all trading strategies are allowed. This could be useful for example to detect fraud and to check whether the reported performance of a strategy is feasible or not. Recall for example that the Sharpe ratio of Madoff’s strategy lied far above the maximum Sharpe ratio for feasible strategies (Bernard and Boyle (2009)). In the second part of the paper we extend our study to the case when there is additional information on the strategy, for example on the way it interacts with the financial market or any other benchmark asset. Our second contribution is then to derive tighter bounds on the Sharpe ratio. This could be useful to regulators to improve fraud detection or abnormal performance reporting.

Finally, we also observe that the mean-variance efficient portfolio exhibits its worst outcomes exactly in high states for the stochastic discount factor. This is also very intuitive in the sense that the states of a declining market are the most expensive states to insure, which corresponds to the states where the highest values for the discount factor are observed. To summarize, mean-variance efficient portfolios do not offer protection against bear markets but rather reinforce this. In practice, many investors reward strategies that show resilience against declining markets or, more generally, that exhibit some desired dependence with any other source of background risk (which we generically call a benchmark). Our third contribution is to derive mean-variance optimal allocation schemes for investors who exhibit state-dependent preferences in the sense that they care about the first two moments of the strategy’s distribution and additionally aim at obtaining a desired dependence with a benchmark asset.

References


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1A sufficient condition for this to occur is when the market is complete.
Short bios of the authors

• **Carole Bernard** joined the University of Waterloo in 2007 as an Assistant Professor in the Department of Statistics and Actuarial Science where she had previously been a postdoctoral fellow. In 2005, she obtained her PhD in Finance from the University of Lyon in France on the subject of "Valuation of Guarantees in Insurance and in Finance using the Option Theory". It received the award for the best PhD in Finance (2005) in France. Since then, Dr. Bernard has published in many journals in actuarial science, mathematics, economics and finance. She was recently awarded the EGRIE Young Economist Best Paper Award for the paper "Financial bounds for Insurance Claims" with Steven Vanduffel.

• **Steven Vanduffel** is an Associate Professor at the Vrije Universiteit Brussel (VUB). His current research is in the field of actuarial and financial mathematics with a current emphasis on the valuation of insurance claims and the optimal design of derivatives. He has published extensively in leading (financial) mathematics and actuarial journals and he has also written articles for, or has been quoted by, business oriented journals and newspapers. He is holding the BNP Paribas Fortis Chair in Banking at the VUB. By training he has MSc degrees in mathematics (KULeuven) and a PhD from the University of Amsterdam (2005).