Further developments in the Erlang($n$) risk model

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For actuarial applications we consider the Sparre–Andersen risk model when the interclaim times are Erlang($n$) distributed. We first address the problem of solving an integro–differential equation that is satisfied by $\chi(u, b)$, the probability that the surplus reaches a given level $b$ from $u$ without first falling below zero, and show an alternative and improved method to solve such equation to that presented by Li (2008).

This is done by first considering the roots with positive real parts of the generalized Lundberg’s equation, and then establishing a one-to-one relation between them and the solutions of the integro–differential equation mentioned above.

Afterwards, we apply our findings above in the computation of the distribution of the maximum severity of ruin.

Finally, we introduce interest force to the model, consider the calculation of the expected discounted dividends prior to ruin, by finding and solving an integro–differential equation. Numerical examples are also provided for illustration.

Keywords: Sparre-Andersen risk model; Erlang($n$) interclaim times; generalized Lundberg’s equation; probability of reaching an upper barrier; maximum severity of ruin; discounted dividends.