Penalized Least Squares Smoothing of Two-Dimensional Mortality Tables with Imposed Smoothness

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We present a method to impose a desired amount of smoothness when smoothing two-dimensional mortality tables via Penalized Least Squares. We can decide at the outset a desired percentage of smoothness in the dimension of age, the dimension of year or both, in order to obtain comparable smoothed mortality trends, for different datasets. Since the smoothing method is mainly determined by two smoothing parameters, we propose to employ some indices that relate those parameters to desired percentages of smoothness for each dimension. We first define a smoothness index for the one-dimensional case and generalize it to the two-dimensional one; we then establish some theoretical results that lend support to this approach. For illustrative purposes, we apply the proposed method to data coming from the Continuous Mortality Investigation Bureau of the UK. The numerical examples are useful to appreciate that different log-mortality patterns can be obtained when each marginal smoothness changes, but the joint percentage of smoothness stays fixed. Thus, emphasizing that we should care about marginal as well as joint (two-dimensional) percentages of smoothness that can be achieved in practice.

Keywords: comparability, index of smoothness, Generalized Least Squares, mean squared error, smoothness parameter