Which Banana Piece Could be Your Retirement Nest Egg?

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Abstract
In a defined contribution (DC) pension system, the interest rate risk is borne by the affiliate. In Colombia, the DC system is in charge of private companies and the capital is managed through a multifund scheme, a similar system exists in México since 1997. These systems have a responsibility on capital management; they focus on means, not on results, fact that could be not crystal clear for the affiliate. Retirement planning should take into account this financial risk, fact that is not always adequately communicated.

In this paper, a capital accrual process in a DC system is discussed, the model takes into account the age at which contributions begin, the contributions made as time passes by and the capital accrual. The model allows illustrating how the affiliate could perceive the interest rate risk and how this risk affects the final capital that he could have at the end of his accumulation phase. Some statistical conclusions about the process are presented.

Some considerations are made in order to have a model suitable for Colombia, another model is obtained for Mexico and a comparison between these two capitalization systems is made. Finally, taking into account the obtained results, some retirement planning considerations are discussed.

Keywords
Contribution defined pension system, financial risk, compound interest, retirement planning, Colombia, México.
1. Mathematical Model
The model discussed in this section follows the accrual process of periodical contributions designed to finance retirement. In a social security system, retirement can also occur after a disability or death, but these contingencies will not be considered. The assumptions for the proposed model are the following:

- The affiliate saves every month, in other words, once his/her career begins, he/she continues being engaged with the system until he/she retires. This also means that the affiliate has a stable job, in such a way that savings for retirement are possible.

- Contributions are usually shared between employer and employee (e.g. matching plans); the model is focused on money, which enters to the system and therefore, no considerations about the money’s source will be made. We also exclude system costs, such as management commissions, taxes and any other kind of contributions with a purpose different than financing retirement.

- The interest rate is fixed for each year, but it changes from year to year. It is also assumed that the rate for one year is independent from the rate of any other year.

- Salary, and therefore contributions may increase more than inflation. This happens usually for professional careers.

- There are not any other savings than the periodical contributions from the salary. In other words, no contributions different than the periodical ones are made.

- Projections are carried out using constant currency (i.e. currency after inflation). By following this approach, no future inflation is required and results obtained for different periods of time are comparable.

Because of the last assumption, real rates must be used. For every year, a monthly contribution and a fixed interest rate are used. Every month, a contribution is made;
the balance account statement of the prior year accrues according to interest and contributions. Therefore, if the affiliate is \( n \) years old, the balance is the following:

\[
Bal_n = Cont_n \ Fac(i_n) + Bal_{n-1}(1 + i_n)
\]

Where:

- \( Bal_n \): Balance of the affiliate when he/she is \( n \) years old (end of year figure, EOY). When the process starts, the balance is null.
- \( Cont_n \): Monthly contribution when the affiliate is \( n \) years old.
- \( Fac(i_n) \): Interest factor that projects the monthly contribution along the year in the equivalent EOY value, under an interest rate \( i_n \).
- \( i_n \): Interest yield rate, when the affiliate is \( n \) years old.

This model describes the capital accrual of an affiliate to the system, given the stated assumptions.

**Plain Model**

For this first model, the monthly contribution is even along the career and a real rate of 5\% is proposed; it is assumed that it might randomly fluctuate between 4\% and 6\% every year.

The affiliate may begin to contribute at any time, thus, several time periods of continuous savings are considered. For this process, simulated 1,000,000 times for different years of savings, the results are shown in graph 1, where every dot corresponds to a realization of the process; all saving processes for a given period are shown in the same color (e.g. 45 years process is displayed in light purple):
The horizontal axis shows the average real rate. In the vertical axis appears the ratio between the final balance and savings. If the ratio between the final balance and savings is 3.0, it means that for every saved dollar, two additional dollars are obtained as a result of the saving process, i.e. as financial gain. For the saving process of 40 years the average ratio is 3.10, which means that the interest is responsible for 2.10 and the saved capital is 1.

From figure 1, the following conclusions can be obtained:

- With the same average interest, different final balances may be obtained. In other words, it is not correct to evaluate an accrual process by using the average rate as the sole criteria. The variability is visualized as the range of results that may be obtained when a vertical line is drawn in figure 1. For the process of 40 years, if the rate were 4% during the first 20 years and 6% during the following 20 years, the final result would be 3.39, but if we
switched the rate for these periods, the result would be 2.84. The final balance would be higher if during the last years of savings, the rates were higher.

- The same final balance might be obtained as a result of accrual processes with different average rates. In other words, different accrual processes may reach the same final balance.
- Short term accrual processes (5 years in this case) results do not change substantially as a result of changes in the interest rate.
- The longer the accumulation period, the more centered on the average interest rate is the accrual process. In statistical terms, we would say that the process reverts to the mean.
- The variability in the final balance will be greater, if the future time horizon is greater.

The time in this model may also be interpreted as the future time horizon, therefore, the longer the remaining saving horizon is, the greater the dispersion of the final result. To emphasize this point, more future of investing, means greater uncertainty in the final result.

The variability in the final balance is the result of interest rate risk exposure; therefore, the shape of “Bananas” shown in the graph is a depiction of the uncertainty in a DC system. The interest rate risk is represented in the horizontal axis; the vertical axis shows the consequence for an affiliate of being exposed to this risk, when an accrual process takes place.

In other words, the system uncertainty is depicted in the horizontal axis, and the uncertainty faced by the affiliate, in the vertical axis.

**Statistical Results**
Given the shape of the “Bananas”, it is almost intuitive to claim normality. For the processes of 45 years (we maintain the color convention for each process), the histograms are the following (similar results are obtained for the other processes):
We can now state the results in statistical terms. We may say that 90% of the results lie in the following ranges:

The final balance is quite centered on the mean, but this variable relates to a multiple of the final balance, therefore, this “centering effect” could be apparent.
A similar result may be obtained for the average real rate:

![Graph showing average yield rate with 90% confidence interval.](image)

**Figure 4: 90% Confidence Interval for the Average Yield Rate (Plain Model)**

It is also tempting to fit a straight line for each “Banana”; the statistical tests are significant. For each process, the slope and the difference between successive slopes, is the following:

<table>
<thead>
<tr>
<th>Process</th>
<th>Slope</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>15</td>
<td>0.12</td>
<td>0.05</td>
</tr>
<tr>
<td>20</td>
<td>0.19</td>
<td>0.07</td>
</tr>
<tr>
<td>25</td>
<td>0.28</td>
<td>0.09</td>
</tr>
<tr>
<td>30</td>
<td>0.40</td>
<td>0.12</td>
</tr>
<tr>
<td>35</td>
<td>0.56</td>
<td>0.16</td>
</tr>
<tr>
<td>40</td>
<td>0.77</td>
<td>0.21</td>
</tr>
<tr>
<td>45</td>
<td>1.05</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Therefore, for a longer investment period, the odds magnify to obtain a greater final balance.
2. Model for Colombia

Law 1328/2009 enacted the multifund scheme, but it was implemented since 2011. This law states that the yield of the funds shall be related to the age of the affiliate. Given the fact that the implementation is quite recent, in this article a single fund will be considered. The model of the previous section will be used, with the following adjustments:

- The nominal rate is obtained using the historical rate for one of the AFPs (Administradoras de Fondos de Pensiones). When this rate is contrasted with the inflation for every year, the real rate is obtained.

- The salary grows according to the “National Average Salary”\(^1\), therefore, the salary grows as the affiliate becomes older.

- The retirement age is 62|57 for men|women\(^2\), and the savings period will be the same of the previous section; a savings period of 40 years means that men|women are engaged in the system since they are 22|17 years old.

A real rate of 6.22% is obtained, it may fluctuate 1.72% around this mean.

With all these considerations, the results are the following:

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\(^1\) Decree 2779/1994.

\(^2\) The usual age in this regime, according to Law 797/2003.
These results have the same structure than the plain model; thus this DC system behaves statistically talking, in the same way than the plain model.

The corresponding confidence intervals for the ratio and the average yield are the following:
Figure 6: 90% Confidence Interval for Final Balance/Saving (Colombia)
The successive slopes of the “Bananas”, are the following:

<table>
<thead>
<tr>
<th>Process</th>
<th>Slope</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>15</td>
<td>0.14</td>
<td>0.06</td>
</tr>
<tr>
<td>20</td>
<td>0.23</td>
<td>0.09</td>
</tr>
<tr>
<td>25</td>
<td>0.35</td>
<td>0.12</td>
</tr>
<tr>
<td>30</td>
<td>0.51</td>
<td>0.16</td>
</tr>
<tr>
<td>35</td>
<td>0.72</td>
<td>0.20</td>
</tr>
<tr>
<td>40</td>
<td>0.97</td>
<td>0.25</td>
</tr>
<tr>
<td>45</td>
<td>1.26</td>
<td>0.29</td>
</tr>
</tbody>
</table>

The first slopes do not differ from the ones obtained from the plain model, but after 15 years of savings, the slopes are greater and the “magnification effect” of time (i.e. the difference between successive slopes is greater for longer investment periods), increases the odds that better results can be obtained for long term saving processes.
3. Model for México
The DC multifund scheme was implemented in 1997. The system is publicly mandated but funds are privately managed; the private companies operating the pension funds are the “AFORES” (Administradoras de Fondos para el Retiro), which run investment funds called “SIEFOREs” (Sociedad de Inversión en Fondos de Retiro). Each “Basic Siefore” or BS is associated with the affiliate’s age:

<table>
<thead>
<tr>
<th>Basic Siefore</th>
<th>Minimum Age</th>
<th>Maximum Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS5</td>
<td>...</td>
<td>26</td>
</tr>
<tr>
<td>BS4</td>
<td>27</td>
<td>36</td>
</tr>
<tr>
<td>BS3</td>
<td>37</td>
<td>45</td>
</tr>
<tr>
<td>BS2</td>
<td>46</td>
<td>55</td>
</tr>
<tr>
<td>BS1</td>
<td>56</td>
<td>...</td>
</tr>
</tbody>
</table>

We use the weighted net yield for each BS, according to the Consar’s calculations\(^3\). This yield is weighted by the managed assets, and is net of costs. In order to obtain a real rate, the nominal rate must be deflected by inflation. The inflation for the previous years\(^4\), was as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>3.57</td>
</tr>
<tr>
<td>2010</td>
<td>4.40</td>
</tr>
<tr>
<td>2011</td>
<td>3.82</td>
</tr>
</tbody>
</table>

The average of these three years is 3.93%\(^5\). With all of the previous information, we obtain an average real rate for each BS. The historical yield\(^6\), gives information about the deviation for each BS, the results are as follows:

<table>
<thead>
<tr>
<th>BS</th>
<th>Yield</th>
<th>Real Rate</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS5</td>
<td>13.17</td>
<td>8.89</td>
<td>8.90</td>
</tr>
<tr>
<td>BS4</td>
<td>12.28</td>
<td>8.03</td>
<td>8.23</td>
</tr>
<tr>
<td>BS3</td>
<td>11.07</td>
<td>6.87</td>
<td>6.82</td>
</tr>
<tr>
<td>BS2</td>
<td>10.00</td>
<td>5.84</td>
<td>6.16</td>
</tr>
</tbody>
</table>

\(^3\) Information gathered from the Consar’s website (www.consar.gob.mx) accessed last July 13\(^{th}\). The available information is the average net yield of the previous 39 months, up to June 2012.

\(^4\) Information taken from the central bank website (www.banxico.org.mx), accessed last July 13\(^{th}\). The inflation is called “Indice Nacional de Precios al Consumidor”, INPC.

\(^5\) The yield information is up to June 2012 and uses information of the previous 3.25 years, we use inflation of the last 3 years (2009 up to 2011); there is a mismatch in these periods that we consider negligible.

\(^6\) From 2009 there is information for all the BSs, for the previous years there is no information for BS3, BS4 and BS5. We use information from 2009 up to 2011.
The real rate and the deviation make sense for a multifund scheme, i.e. greater yields are associated with greater deviations around the mean, and the funds for the younger affiliates are more aggressive than for older affiliates.

We consider that the salary increases according to a factor (FASA) and with this, we have all the elements required by the “Banana” model: A stochastic interest rate for each age, and a salary increase assumption. With all of these elements, for 1,000,000 simulations, the following results are obtained:

![Final Balance/Savings Vs. Average Yield Rate](image)

In this case, the results are not normally distributed, but this is good news, because the upside part of the “Banana” is much better than the downside portion. In other words, the additional risk that exists in a multifund scheme gives a generous reward; the “magnification effect” favors the affiliated in a greater order of magnitude.
The first processes seems to fit to the normal, but the “Bananas” for the longer accrual periods are much more interesting than for shorter ones and indeed, much better than the results from the previous models. As time increases, the dispersion for the ratio has a heavy tail.

If we work with the ratio's logarithm, the results conform to a normal distribution, therefore it can be said that the ratio is lognormal distributed. The results that we obtained are the following:

![Figure 9: Log Model for México](image)

The statistical test on these results, fit to the normal distribution, the 90% confidence intervals are the following:
Figure 10: 90% Confidence Interval for the Ln(Final Balance/Savings), (Log México)
We omit the increasing effect of successive slopes in the linear regression for this logarithmic model, because the results are not so intuitive.

**Conclusions**

- The “Banana” model depicts the interest risk inherent in a DC system, and allows the comparison between different DC systems.
- The plain model may describe a DC system with a single fund.
- Short term accrual processes (5 years) results do not change substantially as a result of changes in the interest rate.
- The longer the accumulation period, the more centered on the average interest rate is the accrual process. In other words, the process reverts to the mean.
• For a longer investment period, the odds magnify to obtain a greater final balance. For the normal models, the difference between successive “Banana” slopes is greater for longer investment periods; this can be coined as a “magnification effect”, and is the consequence of the compound interest and the passage of time.

• The multifund scheme results in a “Banana” that fits to a log normal, the upside part of the “Banana” is much better than the downside portion. The additional risk that exists in a multifund scheme gives a generous reward; the “magnification effect” favors the affiliated in an exponential order of magnitude.

• The dispersion for the ratio of Final Balance/Saving do not seems so great (at least for the saving processes of few years), but it relates to a ratio, therefore, the dispersion in the final result that the affiliate would perceive is greater, and it is proportional to the sum of the savings done through his career. The dispersion for the log normal model is still greater than the one for the normal models.

Appendix
TBD

Bibliography
TBD