When is a defined-benefit pension scheme too small for self-insurance?

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Motivation

Mortality risk components:

- Systematic risk - distribution of deaths.
- Idiosyncratic risk - random fluctuations.
Outline

1. Homogeneous pension scheme
2. Executive section
3. Risk capital allocation
Homogeneous pension scheme

- $N$ members all age 40.
- Benefit: £1 p.a. from age 65.
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$$L_N = \sum_{n=1}^{N} Y_n.$$
Homogeneous pension scheme

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- For example,

$$\mathbb{E}(L_N) = N \nu^{65-40} 25 p_{40} \bar{a}_{65}.$$
Coefficient of variation = \frac{\text{standard deviation of total liability}}{\text{expectation of total liability}}
Risk measure

Coefficient of variation $= \frac{\text{standard deviation of total liability}}{\text{expectation of total liability}}$

If $Y_1, Y_2, \ldots, Y_N$ are independent, then

$$VCo(L_N) = \frac{sd(L_N)}{E(L_N)} \rightarrow 0 \quad \text{as} \quad N \rightarrow \infty.$$
Numerical results: deterministic mortality model (PMA92C10) and $\delta = 4\%$ p.a.
Stochastic mortality model

\[
\text{age rating} = \begin{cases} 
  r & \text{with probability 0.5,} \\
  -r & \text{with probability 0.5.}
\end{cases}
\]

based on PMA92C10.
Numerical results: $\delta = 4\% \text{ p.a.}$
Here $Y_1, Y_2, \ldots, Y_N$ are not independent, and

$$VCo_r(L_N) = \frac{sd_r(L_N)}{E_r(L_N)} \rightarrow \frac{\sqrt{Cov_r(Y_1, Y_2)}}{E_r(Y_1)} \quad \text{as } N \rightarrow \infty.$$
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Interpretation:

- **Systematic risk measure:**
  $$\frac{\sqrt{\text{Cov}_r(Y_1, Y_2)}}{\mathbb{E}_r(Y_1)}.$$

- **Idiosyncratic risk measure:**
  $$VCo_r(L_N) - \frac{\sqrt{\text{Cov}_r(Y_1, Y_2)}}{\mathbb{E}_r(Y_1)}.$$
As before except

- $\alpha N$ are executives.
- Executive benefit: £$k$ p.a. from age 65.
Numerical results: deterministic mortality model (PMA92C10), $\delta = 4\%$ p.a. and $\alpha = 5\%$. 
Setting

\[ f(\alpha, k) = \frac{\alpha k^2 + 1 - \alpha}{(\alpha k + 1 - \alpha)^2}, \]

we find

\[
\text{VCor}(L_N) = \frac{1}{\mathbb{E}_r(Y_1)} \cdot \left( \frac{1}{N} f(\alpha, k) (\text{Var}_r(Y_1) - \text{Cov}_r(Y_1, Y_2)) + \text{Cov}_r(Y_1, Y_2) \right)^{1/2}.
\]
Risk capital allocation

Risk capital := \text{sd}_r(L_N)
Risk capital allocation

Risk capital := $sd_r(L_N)$

Find amounts $\pi_1, \pi_2, \ldots, \pi_N$ such that

$$\sum_{n=1}^{N} \pi_n = sd_r(L_N).$$
Euler capital allocation principle

If $X_n$ is P.V. r.v. of benefit due to member $n$ then

$$\pi_n = \frac{Cov_r(X_n, L_N)}{sd_r(L_N)}$$

is the risk capital allocated to member $n$. 
Euler capital allocation principle

If $X_n$ is P.V. r.v. of benefit due to member $n$ then

$$
\pi_n = \frac{\text{Cov}_r(X_n, L_N)}{\text{sd}_r(L_N)}
$$

is the risk capital allocated to member $n$.

Consider

$$
\sum_{\text{execs}} \frac{\pi_n}{\text{sd}_r(L_N)},
$$

the proportion of risk capital allocated to executive section.
Numerical results: \(\alpha = 5\%, N = 100\) and \(\delta = 4\%\) p.a.
Numerical results: $\alpha = 5\%, N = 500$ and $\delta = 4\%$ p.a.
Euler capital allocation principle

We find

- **systematic risk**: execs contribute $k$ times non-execs.

- **idiosyncratic risk**: execs contribute $k$ times non-execs plus

$$k(k - 1) \frac{\text{Var}_r(Y_1) - \text{Cov}_r(Y_1, Y_2)}{\text{sd}_r(L_N)}.$$
Numerical results: $k = 5$, $N = 100$ and $\delta = 4\%$ p.a.
Numerical results: $k = 5$, $N = 500$ and $\delta = 4\%$ p.a.
Future work

- More complex examples.
- Incorporate financial risks.
- Risk mitigation strategies.
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More details:

2011 C. Donnelly. *Quantifying mortality risk in a small defined-benefit pension schemes.*
Effect on VCo of change in age: $\delta = 4\%$ p.a.
Effect on VCo of executive buyout: $\delta = 4\%$ p.a.
Effect on standard deviation of executive buyout: $\delta = 4\%$ p.a.