A Comparison of Actuarial Financial Scenario Generators

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Abstract

The actuarial profession is encouraging significant work in the area of asset returns to support Risk Based Capital analysis, Dynamic Financial Analysis, pricing embedded options, solvency testing and other financial applications. One recent study is the CAS-SOA research project on Modeling of Economic Series Coordinated with Interest Rate Scenarios. The American Academy of Actuaries also recently released a set of 10,000 prepackaged scenarios for common asset classes in support of the C-3 Phase 2 RBC for Variable Annuities. Both data sets provide practitioners with a large number of iterations for key financial values, including short and long term interest rates and equity returns. This research provides a comparison of the underlying models and the output of these studies to facilitate the understanding and use of the alternative sets of output.

Keywords: Interest rates, equity returns, asset returns, economic scenarios, financial scenarios
1. Introduction

In May, 2001, the Casualty Actuarial Society (CAS) and the Society of Actuaries (SOA) jointly issued a request for proposals on the research topic “Modeling of Economic Series Coordinated with Interest Rate Scenarios.” The objectives of this request were to develop a research relationship with selected persons to investigate this topic; produce a literature review of work previously done in the area of economic scenario modeling; determine appropriate data sources and methodologies to enhance economic modeling efforts relevant to the actuarial profession; and produce a working model of economic series, coordinated with interest rates, that could be made public and used by actuaries via the CAS / SOA websites to project future economic scenarios. Categories of economic series to be modeled included interest rates, equity price levels, inflation rates, unemployment rates, and real estate price levels. In addition to providing the financial scenario generator model, this project also produced a set of output variables that could be used directly in financial analysis.

The Life Capital Adequacy Subcommittee of the American Academy of Actuaries recommended in a series of reports (December 2002, September 2003, and November 2003) that life insurers implement new tests of capital adequacy utilizing stochastic models for scenario testing of variable products with guarantees. Although the ultimate recommendation is for each insurer to develop its own models, the AAA was encouraged to provide 10,000 pre-packaged scenarios that could be used as an alternative. Practitioners are thus faced with a choice among financial scenario generators. This report will serve to explain the underlying processes used in each set of models, compare the output values for common factors, and describe the issues that should be considered when using these models.

2. Stochastic Modeling

Actuarial analysis has progressed through several stages of development regarding the use of economic variables. Initially, the standard approach was to use deterministic values for interest rates, equity returns and other key financial variables, and apply judgment to estimate the range of outcomes expected to occur and to value embedded options such as interest rate and minimum benefit guarantees. This approach led to underpricing some features of insurance policies and financial difficulties for a number of insurers.

The uncertainty surrounding economic variables began to be formally recognized by the use of predetermined scenarios that reflected specific economic conditions. For example, New York regulation 126 requires insurers to test asset adequacy under seven prescribed interest rate paths. The impact that these scenarios had on the financial condition of the insurer was then evaluated. Although an improvement over the deterministic approach, the use of a limited number of scenarios provided no indication of the relative frequency of any particular outcome, and did not reflect the full range of economic conditions that could be expected to occur. The latest stage in actuarial analysis has been the development of stochastic models to reflect the underlying uncertainty of economic and financial variables. Properly developed economic models can be used to accurately price
embedded options in insurance contracts, to set appropriate solvency margins for diverse
insurance operations and to evaluate alternative operational choices within an insurer.
Stochastic models form the basis of dynamic financial analysis (DFA), in which the
underwriting and investment components of an insurance company are evaluated, in
aggregate, under a large number of potential developments. Stochastic models are also
used in regulation to set Risk Based Capital (RBC) levels and by rating agencies to
establish company ratings. The potential value of stochastic modeling, along with recent
advances in the field of financial economics and the increased power and speed of
computers have led to an expansion of modeling tools. However, understanding the
diverse models that have been proposed is a significant challenge for actuaries.

3. Model Specifications

Two key financial variables of economic models used by insurers are the nominal interest
rate and equity returns. In the CAS-SOA model, the nominal interest rate is determined
by the combined effect of the inflation rate and the real interest rate. In the AAA
scenarios, the nominal interest rate is modeled directly.

Inflation Rate:

In the CAS-SOA model, inflation (denoted by q) is assumed to follow an Ornstein-
Uhlenbeck process of the form (in continuous time):

$$dq_i = \kappa(q_i - \mu_i)dt + \sigma dB_q$$

The simulation model samples the discrete form equivalent of this process as:

$$\Delta q_i = q_{t+1} - q_t = \kappa_q(q_t - \mu_t)\Delta t + \sigma_q \sqrt{\Delta t}$$

$$q_{t+1} = q_t + \kappa_q(q_t - \mu_t)\Delta t + \sigma_q \sqrt{\Delta t}$$

$$= \kappa_q \Delta t \cdot \mu_q + (1 - \kappa_q \Delta t) \cdot q_t + \sigma_q \sqrt{\Delta t}$$

From this last equation, we can see that the expected level of future inflation is a
weighted average between the most recent value of inflation ($q_t$) and a mean reversion
level of inflation, $\mu_q$. The speed of reversion is determined by the parameter $\kappa_q$. In the
continuous model, mean reversion can be seen by considering the first term on the right-
hand side of (1) (which is called the drift of the process). If the current level of inflation
($q_t$) is above the average, the first term is negative. Therefore, the first equation predicts
that the expected change in inflation will be negative; that is, inflation is expected to fall.
The second term on the right-hand side represents the uncertainty in the process.
Real Interest Rates:

In the CAS-SOA model, real interest rates are derived following the two-factor Vasicek term structure model, which is a simple case of the two-factor Hull-White model. In the two-factor Vasicek model, the short-term rate (denoted by \( r \)) reverts to a long-term rate (denoted by \( l \)) that is itself stochastic.

\[
\begin{align*}
\text{dr}_t &= \kappa_1 (l_t - r_t) \text{dt} + \sigma_1 \text{dB}_t \\
\text{dl}_t &= \kappa_2 (\mu - l_t) \text{dt} + \sigma_2 \text{dB}_2
\end{align*}
\]

In order to estimate the parameters of the model, the discrete analog of the model is used:

\[
\begin{align*}
\Delta r_t &= \kappa_1 (l_t - r_t) \Delta t + \sigma_1 \varepsilon_{l_t} \\
\Delta l_t &= \kappa_2 (\mu - l_t) \Delta t + \sigma_2 \varepsilon_{l_t}
\end{align*}
\]

\[
\begin{align*}
\Delta r_t &= \kappa_1 (l_t - r_t) \Delta t + \sigma_1 \varepsilon_{l_t} = \left(\kappa_1 l_t + \kappa_1 r_t\right) \Delta t + \sigma_1 \varepsilon_{l_t} \\
\Delta l_t &= \kappa_2 (\mu - l_t) \Delta t + \sigma_2 \varepsilon_{l_t} = \left(\kappa_2 \mu - \kappa_2 l_t\right) \Delta t + \sigma_2 \varepsilon_{l_t}
\end{align*}
\]

The stochastic factors are updated over time, analogous to the situation for inflation presented above. First, rearrange the discrete form of the two-factor Vasicek term structure model:

\[
\begin{align*}
\Delta r_t &= \kappa_1 (l_t - r_t) \Delta t + \sigma_1 \varepsilon_{l_t} \\
\Delta l_t &= \kappa_2 (\mu - l_t) \Delta t + \sigma_2 \varepsilon_{l_t}
\end{align*}
\]

\[
\begin{align*}
\text{dr}_{t+1} &= r_t + \left(\kappa_1 l_t + \kappa_1 r_t\right) \Delta t + \sigma_1 \varepsilon_{l_t} \\
\text{dl}_{t+1} &= l_t + \left(\kappa_2 \mu - \kappa_2 l_t\right) \Delta t + \sigma_2 \varepsilon_{l_t}
\end{align*}
\]

From these equations, it can be seen that the short rate is again a weighted average between the current level \( r_t \) and the mean reversion factor \( l_t \). The mean reversion factor is itself a weighted average of some long-term mean and its current value.

Nominal Interest Rates:

Fisher (1930) provides a thorough presentation of the interaction of real interest rates and inflation and their effects on nominal interest rates. He argues that nominal interest rates compensate investors not only for the time value of money, but also for the erosion of purchasing power that results from inflation. In the model presented here, the underlying movements in inflation and real interest rates generate the process for nominal interest rates. If bonds are priced using expectations of inflation and real interest rates until the bond’s maturity, then nominal interest rates are implied by combining the term structure of inflation and the term structure of real interest rates. Therefore:

\[
P^i(t, T) = P^r(t, T) \times P^g(t, T)
\]
where \( i \) refers to nominal interest rates and the superscripts on the bond prices correspond to the underlying stochastic variables.

Unfortunately, the parameters for the real interest rate process shown above generate a distribution that severely restricts the range of potential future nominal interest rates. For example, the 1st percentile of the distribution for the 20-year nominal rate is 5.9% and the 99th percentile is 8.2%. There are several candidates for problems with real interest rates that may lead to this seemingly unrealistic distribution of future nominal rates: (1) the use of ex post real interest rate measures is unsuitable, (2) monthly measurement of real interest rates contains mean reverting errors which exaggerate mean reversion speed, or (3) the time period used to measure real interest rates is too short.

As a result, the parameters for real interest rates in the CAS-SOA model were altered to allow nominal interest rates to more accurately reflect historical volatility. Specifically, mean reversion speed was dramatically reduced. Given that mean reversion speed and volatility work together to affect the range of interest rate projections, volatility was also reduced. The following parameters are used as the “base case” in the model.

<table>
<thead>
<tr>
<th>( \kappa_r )</th>
<th>( \mu_r )</th>
<th>( \sigma_r )</th>
<th>( \kappa_l )</th>
<th>( \sigma_l )</th>
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</thead>
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<tr>
<td>1.0</td>
<td>2.8%</td>
<td>1.00%</td>
<td>0.1</td>
<td>1.65%</td>
</tr>
</tbody>
</table>

The AAA RBC C-3 scenarios model nominal interest rates directly by the following processes:

1) Long term, 10-Year Treasury Yield
\[
i_t^* = [(1 - \alpha) i_{t-1}^* + \alpha \tau^*] e^{Z_1}
\]

2) Short term, 3-Month Treasury Yield
\[
i_t = [(1 - \phi) i_{t-1} + \phi (i_{t-1}^* + \gamma)] e^{Z_2}
\]

\( Z_1, Z_2 \) are correlated normal variants with standard deviations \( \sigma_1 \) and \( \sigma_2 \) respectively.

\[
\mu_1 = -\frac{1}{2} \sigma_1^2
\]

\[
\mu_2 = -\frac{1}{2} \sigma_2^2
\]

The monthly parameters for the two processes are:

Long term mean-reversion strengths: \( \alpha = 0.0034, \quad \tau = 6.764\% \),

Short term mean-reversion strengths: \( \phi = 0.0447, \quad \gamma = -1.299\% \),

\( \sigma_1 = 0.03542, \quad \sigma_2 = 0.07843. \)

The correlation co-efficient between the processes is \( \rho = 0.54271. \).
The medium-term rate, represented by the yield on 7 year Treasury notes, is determined:

\[ i_t^m = \frac{1}{2} (i_t^* + \lambda i_t + \xi_t) + Z3 \]

Z3 is a normal variant with mean 0 and standard deviation \( \sigma \).

The monthly parameters are:

\[ \lambda = 1.046282 \]
\[ \sigma = 0.004069 \]

\( \xi_t \) depends on the relationship between \( i_t^* \) and \( i_t \) whereby \( \xi_t = 0.011628 \) when \( i_t^* \geq i_t \) and -0.008914 otherwise.

The starting values for the interest rate model are \( i_0^* = 4.03\% \) (long rate), \( i_0 = 1.21\% \) (short rate) and \( i_0^m = 3.63\% \) (medium-term rate). These values roughly correspond to the yield curve at 31 December 2002.

Equity returns:

Both the CAS-SOA model and the AAA scenarios use a regime switching equity return model, following the approach of Hardy (2001). In this model, Hardy assumes that stock prices are lognormally distributed under each of two regimes, one with low volatility and one with high volatility. The CAS-SOA model uses the excess equity returns \( x \) over the nominal interest rate. The modeled excess returns are then added to the modeled nominal interest rate for each period. The AAA prepackaged scenarios are based on the total equity returns, rather than excess returns.

4. Comparison of Results

Output from the CAS-SOA economic model and the AAA RBC C-3 pre-packaged scenarios are compared in several ways. Tables list the basic statistics for 10,000 iterations of the CAS-SOA model and the full sample of the 10,000 prepackaged scenarios provided by the AAA RBC C-3 project, along with historical data for the relevant variable. Two types of graphs illustrate the relationship between the two sets of output. One set of displays Redington’s funnel of doubt graphs over time, showing the 1st, 25th, 50th, 75th and 99th percentile values for the output distributions from their initial values out through 20 years. This format allows easy comparison of the dispersion reflected in each model. The other type of graph displays a histogram of the model values one year after the start of the simulation, along with historical values of the appropriate variable. These graphs provide a more detailed analysis of the distribution of each variable at a particular point on the funnel of doubt graphs, in this case after one year.
This comparison indicates significant differences between the two models, especially for interest rates. Based on Table 1, the mean value of the 3 month nominal interest rate for the CAS-SOA model after one year, at 3.28%, demonstrates much greater reversion to the long run mean than is reflected in the AAA RBC C-3 scenarios, at 1.88%. The standard deviation of the CAS-SOA model is 2.7%, which is also much higher than the AAA RBC C-3 scenarios, at 0.4%. Although the skewness of both models is very similar (.620 to .654), the kurtosis of the AAA scenarios at .538 is much closer to the historical value of .886 than the CAS-SOA at -.230.

Figure 1 illustrates the differences in levels and dispersion for the 3 month nominal interest rate of the two models over a 20 year horizon. Both models start at approximately the same level (CAS-SOA at 1.1% and the AAA at 1.21%), but the CAS-SOA model increases faster and to a higher level with greater dispersion. Even after 20 years, the AAA scenarios indicate a less than 1% chance of 3 month interest rates exceeding 13%, even though this value has been as high as 16% within the last 30 years. The histogram displayed on Figure 2 shows the distribution of 3 month nominal interest rates after one year. The values for the AAA scenarios are almost entirely within the range of 1-3%. The CAS-SOA model has a much wider distribution, although not as wide as the historical values.

Comparing the results for the 7 year nominal interest rates produces a similar pattern. From Table 2, the CAS-SOA values have a higher mean, 4.9% to 3.6%, and standard deviation, 1.6% to 0.6%, than the AAA scenarios. Actual values are only available for this data series from 1969-2004, which primarily reflects a period of relatively high interest rates, with a mean of 7.7%. Neither model generates values for kurtosis or skewness that are close to the limited period of historical values that are available, but the signs of the AAA scenarios are both positive, in line with actual values. The funnel of doubt graphs on Figure 3 are both higher and wider for the CAS-SOA values than the AAA scenarios. After 20 years, the AAA scenarios do indicate 99th percentile values almost as high as the CAS-SOA model produces, but the 50th and 75th percentile values are still much lower. Figure 4 again shows the distribution of the CAS-SOA model is wider than the AAA scenarios. Both are much lower than the historical values shown on the figure. However, this discrepancy might not be a problem. Current interest rates, which are low by historical standard, are used as the starting point for both interest rate models. Despite mean reversion, high interest rates are less likely to occur within a year than they would be if interest rates were starting at a higher level.

The same patterns and relationships are indicated on Table 3 and Figures 5 and 6 for 10 year nominal interest rates. The results from the CAS-SOA model are higher and more widely dispersed than the AAA scenarios. Both models generate values lower than historical 10 year interest rates. The limited range of interest rate dispersion for both models is evident from the maximum values on Table 3. Of the 10,000 iterations, the largest value for the CAS-SOA model was 9.98%, and the largest value for the AAA scenarios was 6.56%. The starting value for ten year interest rates in the CAS-SOA model was 5.0, and for the AAA scenarios was 4.03%. Current (July 2004) yields on 10 year Treasury notes are 4.49%. In essence, the CAS-SOA model indicated that there was
essentially chance of an increase in interest rates of more than 498 basis points over the
next year; the AAA scenarios indicated there was essentially no chance the interest rates
could increase by more than 253 basis points. Given the 403 basis point rise from July,
1980 to July, 1981, these assumptions might be considered too limited.

Table 4 provides the basic statistics for large stock returns for both models and for
historical values, based on the S&P 500 and its predecessor, the Cowles Index. The mean
of the CAS-SOA model is 8.7%, close to the historical values. The mean for the AAA
scenarios is 12.5%. The standard deviation of the CAS-SOA model, at 22.1%, is higher
than the AAA scenarios, at 17.5%, and the historical values, at 17.8%. The effect of the
larger standard deviation is evident in the 99th and 1st percentile values (largest 100 and
smallest 100). These values are 62.7% and -52.6% for the CAS-SOA model, but only
53.5% and -31.5% for the AAA scenarios. The AAA scenarios are in line with the
largest and smallest returns of the S&P 500 over the 132 year period, 51.4% and -36.5%.
The different shape of the funnel of doubt graphs of Figure 7 reflects the different method
this information is generated in the two models. In the CAS-SOA model, the values
represent the cumulative annual returns from investing in large stocks over the indicated
time period. This produces a portfolio effect over time, as large gains, or losses, in one
year are likely to be moderated by the returns of the remaining years in the investment
horizon. Thus, the funnel of doubt is inverted, with returns much more predictable for a
20 year investment horizon than for a single year. The AAA scenarios generate the
results of each year separately. Thus, there is no time portfolio effect, and each year
indicates a similar dispersion. The histogram on Figure 8 for large stock returns after one
year (which are measured similarly for both models) illustrate the greater dispersion of
the CAS-SOA model and the higher values of the AAA scenarios.

The results for small stock returns are indicated on Table 5. Again the mean values for
the CAS-SOA model are lower than the AAA scenarios, 13.6% to 15.4%, but the AAA
value is closer to the historical mean of 17.5%. The standard deviation of the CAS-SOA
model, at 35.7% is closer than the AAA scenarios, at 24.5%, to the historical value of
33.3%. The 99th and 1st percentile values are 129.7% and -61.8% for the CAS-SOA and
74.6% and -45.3% for the AAA. In this case the CAS-SOA results are closer to the
historical range of 142.9% and -58.0% over a 78 year period. The greater initial
dispersion of the CAS-SOA model is evident on Figure 9, despite the different displays
of the additional years. The histogram Figure 10 provides another illustration of the
greater dispersion of the CAS-SOA model.

5. Conclusions

Both the CAS-SOA models and the AAA prepackaged scenarios provide values for
interest rates and equity returns that can be used in actuarial modeling. The different
approaches used in each procedure lead to a differences in the resulting output. The
CAS-SOA models lead to a wider set of distributions, especially for interest rates, than
the AAA scenarios. Before adopting either approach, the user should understand the
factors considered by each model and the use of the output.
References


### Table 1
Descriptive Statistics - 3 Month Nominal Interest Rates

<table>
<thead>
<tr>
<th></th>
<th>CAS-SOA Economic Model</th>
<th>AAA RBC C-3 Scenarios</th>
<th>Actual (01/34-06/04)</th>
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<tr>
<td>Mean</td>
<td>0.0328</td>
<td>0.0188</td>
<td>0.0394</td>
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<tr>
<td>Standard Error</td>
<td>0.0003</td>
<td>0.0000</td>
<td>0.0011</td>
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<td>Median</td>
<td>0.0298</td>
<td>0.0183</td>
<td>0.0354</td>
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<td>Mode</td>
<td>0.0000</td>
<td>0.0138</td>
<td>0.0038</td>
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<td>Standard Deviation</td>
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<td>0.0042</td>
<td>0.0320</td>
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<tr>
<td>Sample Variance</td>
<td>0.0007</td>
<td>0.0000</td>
<td>0.0010</td>
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<td>Kurtosis</td>
<td>-0.2302</td>
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<td>0.8861</td>
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<td>Skewness</td>
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<td>0.6539</td>
<td>0.9207</td>
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<tr>
<td>Range</td>
<td>0.1528</td>
<td>0.0362</td>
<td>0.1629</td>
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<td>Minimum</td>
<td>0.0000</td>
<td>0.0092</td>
<td>0.0001</td>
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<td>Maximum</td>
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### Table 2
Descriptive Statistics - 7 Year Nominal Interest Rates

<table>
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<th></th>
<th>CAS-SOA Economic Model</th>
<th>AAA RBC C-3 Scenarios</th>
<th>Actual (07/69-05/04)</th>
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<tbody>
<tr>
<td>Mean</td>
<td>0.04862073</td>
<td>0.036275915</td>
<td>0.077150476</td>
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<td>Standard Error</td>
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<td>Median</td>
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<td>0.03618095</td>
<td>0.07255</td>
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<td>Mode</td>
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<td>0.036589322</td>
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<td>Standard Deviation</td>
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<tr>
<td>Sample Variance</td>
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<td>Kurtosis</td>
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<td>Skewness</td>
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<td>Range</td>
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<td>Minimum</td>
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<td>Maximum</td>
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<td>Largest(2500)</td>
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<tr>
<td>Smallest(2500)</td>
<td>0.037906304</td>
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### Table 3
Descriptive Statistics - 10 Year Nominal Interest Rates

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<tr>
<th>Scenario</th>
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<th>AAA RBC C-3 Scenarios</th>
<th>Actual (04/53-06/04)</th>
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<td>Mean</td>
<td>0.051310433</td>
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<tr>
<td>Standard Error</td>
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<td>Median</td>
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<td>0.0393</td>
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<tr>
<td>Standard Deviation</td>
<td>0.013066543</td>
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<td>Sample Variance</td>
<td>0.000170735</td>
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<td>Smallest(2500)</td>
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### Table 4
Descriptive Statistics - Large Stock Returns

<table>
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<tr>
<th>Scenario</th>
<th>CAS-SOA Economic Model</th>
<th>AAA RBC C-3 Scenarios</th>
<th>Actual (1871-2002)</th>
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<tbody>
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<td>Mean</td>
<td>0.087207955</td>
<td>0.125021464</td>
<td>0.081652505</td>
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<td>Median</td>
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### Table 5

**Descriptive Statistics - Small Stock Returns**

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<tr>
<th></th>
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<th>Actual (1926-2003)</th>
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<td><strong>Mean</strong></td>
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### Table 6

**Comparison of Model Parameters - Equity Markets**

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<td>CAS-SOA Large Stock</td>
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<td>3.9%</td>
<td>1.1%</td>
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<td>5.9%</td>
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</tr>
<tr>
<td>AAA Diversified</td>
<td>1.3%</td>
<td>3.5%</td>
<td>3.4%</td>
<td>-1.0%</td>
<td>6.4%</td>
<td>15.4%</td>
<td>82.0%</td>
<td>18.0%</td>
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<tr>
<td>CAS-SOA Small Stock</td>
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<td>15.4%</td>
<td>82.0%</td>
<td>18.0%</td>
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Figure 1
Funnel of Doubt Graphs
3 Month Nominal Interest Rates (U. S. Treasury Bills)

CAS-SOA Economic Model

AAA RBC C-3 Scenarios
Figure 2
Histogram of 3 Month Nominal Interest Rates
Model Values and Actual Data (01/34-05-04)
Figure 3
Funnel of Doubt Graphs
7 Year Nominal Interest Rates (U. S. Treasury Bonds)
Figure 4
Histogram of 7 Year Nominal Interest Rates
Model Values and Actual Data (07/69-05/04)
Figure 5
Funnel of Doubt Graphs
10 Year Nominal Interest Rates (U. S. Treasury Bonds)

CAS-SOA Economic Model

AAA RBC C-3 Scenarios
Figure 6
Histogram of 10 Year Nominal Interest Rates
Model Values and Actual Data (04/53-05/04)
Figure 7
Funnel of Doubt Graphs
Large Stock Return (US Equity)

CAS-SOA Economic Model

AAA RBC C-3 Scenarios
Figure 8
Histogram of Large Stock Return Model Values and Actual Data (1872-2004)
Figure 9
Funnel of Doubt Graphs
Small Stock Return (Intermediate Risk Equity)

CAS-SOA Economic Model

AAA RBC C-3 Scenarios
Figure 10
Histogram of Small Stock Return
Model Values and Actual Data (1926-2003)