PENSION FUNDS:  
FUNDING INDEX, MISMATCH RISK PREMIUM AND VOLATILITY

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Abstract

The funding index is a simple indicator for broad movements in funding ratios of pension funds. This paper explains how it is calculated and illustrates how it can be used to analyze realized premiums for taking mismatch risk, the volatility of funding ratios, surplus-at-risk, and derivatives designed to protect against funding ratio risk. All numbers, tables and graphs are based on stylized information and are intended as illustrations; this is neither an empirical study nor an economic analysis.

Keywords: defined benefit; pension plan; pension fund; asset index; liability index; funding index; mismatch risk; mismatch risk premium; funding ratio volatility; surplus-at-risk

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1 — THE FUNDING INDEX

1.1 Definitions

**Pension plan and pension fund**
Consider a defined-benefit pension plan offered by an employer to its employees, funded through a pension fund. In such a plan, the benefits are determined using benefit formulae expressed in terms of salary history and years of service. They are payable upon retirement, death or disablement, typically in the form of annuities. The pension fund is assumed to be a separate legal entity.

**Assets and asset index**
We suppose that the assets of the plan are investments such as equities and bonds that have been segregated and restricted in the pension fund to secure and provide the benefits. Those assets are the property by the pension fund, and we denote their estimated market value at time $t$ by $A(t)$. The time index $t$ is expressed in years.

Given an asset allocation $AA = (\pi_1, \ldots, \pi_k)$, where the proportions $\pi_i$ satisfy the conditions $0 \leq \pi_i \leq 1$ and $\sum \pi_i = 1$, and given reinvestment indices $I_i(t)$ at time $t \geq t_0$ for the asset types $i = 1, \ldots, k$ (where $t_0$ is a date in the past, say 31 December 1955), we define the asset index $\alpha(t)$ as the combined reinvestment index reflecting the history of the total return on a hypothetical portfolio invested in those indices in accordance with asset allocation $AA$, with frequent (for example monthly) rebalancing. We ignore cash flows such as contributions deposited in the fund and benefit payments paid by the fund.

**Liabilities and liability index**
The plan’s liabilities are the accrued benefits, including pensions already in payment, deferred vested pensions, and all pensions accrued by the active participants on the basis
of their salary history and service up to the valuation date, including accrued contingent survivor pensions. We denote the value of the liabilities at time $t$ by $L(t)$. In this paper, this quantity is defined as the estimated fair market value of a theoretical replicating, or liability-matching, portfolio consisting of risk-free zero-coupon bonds — nominal bonds for nominal pensions, and bonds linked to a particular price index in the case of indexed pensions. Throughout this paper, the weighted average duration of the liability-matching portfolio is roughly 15 years. We ignore complications such as mortality risk and uncertainty about other demographic factors that may lead to actuarial gains and losses, under the assumption that such issues are best treated separately. This means for example that actual mortality is always equal to expected mortality on the basis of a given mortality table.

The liability index $\lambda(t)$ is defined as the reinvestment index based on the hypothetical liability-matching portfolio at a particular time $t_1$ (say, 30 June 2004), reflecting the history of the total return on that portfolio. This index, also used in VAN GAALLEN (2003a, b, c), is very similar to the liability index defined in BADER (1994) and the “liability benchmark portfolio” developed by SPEED ET AL. (2003).

In practice, $\lambda(t_1)$ may be calculated first on the basis of the accrued pension entitlements at time $t_1$. The history $\lambda(t)$ is then determined by going backwards from $t_1$ to the starting time $t_0$ using historical return data. Pension accruals, being the liability-side analogue of contributions on the asset side, and cash flows such as benefit payments are disregarded. In the case of indexed pensions, the matching portfolio consists of index-linked bonds, and the total return rate includes the increase of the index.

**Funding ratio and funding index**

The funding ratio at time $t$, denoted by $FR(t)$, is defined as the ratio of the assets to the liabilities at that time, that is,

$$FR(t) = A(t) / L(t).$$
We define the funding index $\phi(t)$ as the quotient of the asset index and the liability index:

$$\phi(t) = \alpha(t) / \lambda(t).$$

The funding index, although developed independently in VAN GAALLEN (2003a, b, c), is not a new concept. For one thing, the index is logically equivalent to the “funding ratio return” of LEIBOWITZ ET AL. (1996). Moreover, a “funding ratio index” appears to be used in CHUN ET AL. (1999) and a “pension asset/liability index” is discussed and illustrated in MCGALE & McINERNEY (2002).

Movements of the funding index $\phi(t)$ account for all movements of the funding ratio $FR(t)$ except those caused by and hence attributable to

- changes in the liability structure, that is, all changes other than changes of scale;
- actuarial experience gains and losses;
- plan amendments;
- deviations of actual assets in the investment portfolio from their respective indices;
- cash outflows such as benefits paid by the fund and administrative expenses;
- cash inflows such as contributions deposited in the fund; and
- accrual of additional benefits.

**A neutral funding policy**

In a complete analysis, the above influences should not be ignored, but examined separately. From this purely analytical perspective, a neutral funding policy is such that the funding ratio $FR(t)$ is not affected by the combination of all cash flows and the accrual of new benefits. The policy implications of a surplus or deficit in comparison with any given funding target are a separate issue beyond the scope of this paper.
1.2 Stylized data

**FIGURE 1A**

Funding index for an American pension fund with nominal pensions  
1955 – 2004  
Pensions without indexation in US$  
Bonds: US government; 5 year duration  
Equities: MSCI gross world index in US$ (estimated index before 1970)  
Initial funding index value: 1.000 at 31 December 1955

**FIGURE 1B**

Funding index for a European pension fund with nominal pensions  
1955 – 2004  
Pensions without indexation in EUR (NLG before 1999)  
Bonds: European government; Dutch government before 1999; 5 year duration  
Equities: MSCI gross world index in EUR (NLG before 1999; estimated index before 1970)  
Initial funding index value: 1.000 at 31 December 1955
Funding index for an American pension fund with indexed pensions
1998 – 2004
Pensions with indexation in US$
Bonds: US government; 5 year duration
Equities: MSCI gross world index in US$
Initial funding index value: 1.000 at 31 December 1998

Funding index for a European pension fund with indexed pensions
1998 – 2004
Pensions with indexation in EUR
Bonds: European government; 5 year duration
Equities: MSCI gross world index in EUR
Initial funding index value: 1.000 at 31 December 1998
1.3 Comment

At first glance, figures 1A and B might seem to imply that the American pension fund has somehow achieved a much better investment performance on its equity portfolio than the European fund. But this is nonsense, since they invested in exactly the same worldwide equity index.

Consider the case of the 100% worldwide equity portfolio.

<table>
<thead>
<tr>
<th>TABLE 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Funding index for nominal pensions</td>
</tr>
<tr>
<td>Assets: 100% equities (MSCI gross world index)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\phi_{US}(t)$</th>
<th>$\phi_{EUR}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 December 1955</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>30 June 2004</td>
<td>5.524</td>
<td>2.314</td>
</tr>
</tbody>
</table>

How is it possible that in 2004 the American funding index was 2.4 times as large as the European funding index?

For one thing, the dollar (US$) lost approximately 50% of its value in terms of the Dutch guilder (NLG) and its successor currency, the euro (EUR) between 1955 and 2004 — the relevant currency equivalents were as follows: US$ 1.00 = NLG 3.81 at the end of 1955; EUR 1.00 = NLG 2.20 when the Dutch guilder was converted into the euro at the end of 1998; and US$ 1.00 = EUR 0.82 on 30 June 2004.

Furthermore, note that the liability index is nothing but a reinvestment index with respect to a notional liability-matching portfolio consisting of long-term government bonds. The average total annual return was approximately 6.4% on the American version and 6.7% on the European one, both in nominal terms in their respective currencies. The corresponding liability indices at $t = 30$ June 2004 were $\lambda_{US}(t) = 20.2$ and $\lambda_{EUR}(t) = 23.0$. 
Given the depreciation of the dollar by approximately 50%, this means that the European liability-matching portfolio outperformed the American one by a factor of more than two, when both are calculated on the basis of the same currency.

The American asset and liability index series are normally developed in American currency, but can of course be converted to European currency. Similarly, the European series are normally developed in European currency and can be converted to American currency. Since in this case the two funds invest in exactly the same assets,

\[ \alpha_{\text{US}}(t) \text{ on US$ basis} = \alpha_{\text{EUR}}(t) \text{ on US$ basis}, \]

\[ \alpha_{\text{US}}(t) \text{ on EUR basis} = \alpha_{\text{EUR}}(t) \text{ on EUR basis}. \]

Moreover,

\[ \phi_{\text{US}}(t) = \{ \alpha_{\text{US}}(t) \text{ on US$ basis} \} / \{ \lambda_{\text{US}}(t) \text{ on US$ basis} \} \]

and

\[ \phi_{\text{EUR}}(t) = \{ \alpha_{\text{EUR}}(t) \text{ on EUR basis} \} / \{ \lambda_{\text{EUR}}(t) \text{ on EUR basis} \} \]

\[ = \{ \alpha_{\text{EUR}}(t) \text{ on US$ basis} \} / \{ \lambda_{\text{EUR}}(t) \text{ on US$ basis} \}. \]

Hence

\[ \phi_{\text{US}}(t) / \phi_{\text{EUR}}(t) = \{ \lambda_{\text{EUR}}(t) \text{ on US$ basis} \} / \{ \lambda_{\text{US}}(t) \text{ on US$ basis} \} \]

\[ = \{ \lambda_{\text{EUR}}(t) \text{ on EUR basis} \} / \{ \lambda_{\text{US}}(t) \text{ on US$ basis} \} \]

\[ \times \text{cumulative exchange rate effect EUR/US$} \]

\[ = (23.0 / 20.2) \times 2.1 = 2.4 \]

\[ = 2.4 \]

at time \( t = 30 \text{ June 2004} \), as was to be demonstrated. Of course, this arithmetic is not an economic explanation, but only a technical reconciliation of our index numbers.
Even though all this is based on stylized data and intended only to illustrate the usefulness of the funding index, the question arises in what sense and to what extent the American nominal pensions may have lost value in comparison with European nominal pensions. With respect to a plan participant who works and retires in one and the same currency area, it is far from obvious whether an unambiguous answer can be given in a world without purchasing power parity between currencies. Moreover, in this subsection we are comparing funding indices of identical funded pension plans, both of which are purely hypothetical, completely static and without any kind of discretionary cost-of-living adjustments. In our simple model we follow indices based on one unit of pension which is being rolled forward indefinitely, as it were; we are not comparing actual pension plans, actual pension policies, actual asset allocations, and actual funding levels. Obviously, there is no justification for jumping to the conclusion that an average American plan participant has in some meaningful sense become better or worse off in comparison with an average European plan participant.
2 — THE REALIZED MISMATCH RISK PREMIUM

2.1 Definitions

Mismatch risk
If all outgoing future cash flows constituting the plan’s liabilities are exactly and with complete certainty matched by future incoming cash flows generated by its assets, then there is no mismatch risk. But if such a match cannot be realized, then a shortfall may occur in the future. Apart from the consequences of any initial surplus or deficit, this is called mismatch risk. This comprehensive concept of risk plays a central role in the methodology for assessing the financial adequacy of pension funds and insurance companies that is described in PVK (2001) and has been developed further in related PVK and Dutch government publications.

Realized mismatch risk premium
LEIBOWITZ ET AL. (1996) contains extensive analysis focusing on the funding ratio $FR(t)$ and the rate of return with respect to the funding ratio (“funding ratio return”). We define the realized mismatch risk premium between times $s$ and $t$ somewhat analogously as the observed geometric average rate of return on the funding index $\phi(t)$:

$$\rho(s, t) = \left[ \frac{\phi(t)}{\phi(s)} \right]^{1/(t-s)} - 1.$$
### 2.2 Stylized data

**TABLE 2A**

Realized mismatch risk premium for an American pension fund  
*Geometric average, per annum*

<table>
<thead>
<tr>
<th>Asset allocation</th>
<th>100%</th>
<th>50%</th>
<th>0%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bonds</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US government</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 year duration</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Equities</strong></td>
<td>0%</td>
<td>50%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>MSCI gross world index, in US$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) **Realized mismatch risk premium**  
for nominal pensions in US$  

<table>
<thead>
<tr>
<th>Date</th>
<th>0.2%</th>
<th>2.1%</th>
<th>3.6%</th>
<th>0.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 December 1995 – 30 June 2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31 December 1995 – 31 December 1969</td>
<td>2.4%</td>
<td>6.0%</td>
<td>9.7%</td>
<td>0.0%</td>
</tr>
<tr>
<td>31 December 1969 – 30 December 1989</td>
<td>0.3%</td>
<td>2.5%</td>
<td>4.2%</td>
<td>0.0%</td>
</tr>
<tr>
<td>31 December 1989 – 31 December 1998</td>
<td>-3.3%</td>
<td>-2.3%</td>
<td>-1.7%</td>
<td>0.0%</td>
</tr>
<tr>
<td>31 December 1998 – 30 June 2004</td>
<td>0.2%</td>
<td>-1.6%</td>
<td>-4.3%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

(b) **Realized mismatch risk premium**  
for indexed pensions in US$  

<table>
<thead>
<tr>
<th>Date</th>
<th>-5.0%</th>
<th>-6.8%</th>
<th>-9.3%</th>
<th>0.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 December 1998 – 30 June 2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 2B**

Realized mismatch risk premium for a European pension fund  
*Geometric average, per annum*

<table>
<thead>
<tr>
<th>Asset allocation</th>
<th>100%</th>
<th>50%</th>
<th>0%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bonds</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>European government</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dutch government before 1999</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 year duration</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Equities</strong></td>
<td>0%</td>
<td>50%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>MSCI gross world index, in EUR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In NLG before 1999</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimated index before 1970</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) **Realized mismatch risk premium**  
for nominal pensions in NLG/EUR  

<table>
<thead>
<tr>
<th>Date</th>
<th>-0.1%</th>
<th>1.1%</th>
<th>1.7%</th>
<th>0.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 December 1955 – 30 June 2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31 December 1955 – 31 December 1969</td>
<td>2.7%</td>
<td>5.5%</td>
<td>8.3%</td>
<td>0.0%</td>
</tr>
<tr>
<td>31 December 1969 – 30 December 1989</td>
<td>-0.7%</td>
<td>0.2%</td>
<td>0.5%</td>
<td>0.0%</td>
</tr>
<tr>
<td>31 December 1989 – 31 December 1998</td>
<td>-2.5%</td>
<td>-1.5%</td>
<td>-1.2%</td>
<td>0.0%</td>
</tr>
<tr>
<td>31 December 1998 – 30 June 2004</td>
<td>-0.6%</td>
<td>-2.1%</td>
<td>-4.6%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

(b) **Realized mismatch risk premium**  
for indexed pensions in NLG/EUR  

<table>
<thead>
<tr>
<th>Date</th>
<th>-1.7%</th>
<th>-3.2%</th>
<th>-5.7%</th>
<th>0.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 December 1998 – 30 June 2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.3 Comments

Nominal long-term government bonds are arguably not a good benchmark for a truly risk-free position. After all,

- actual inflation, expectations about inflation and inflation risk
- actual currency movements, expectations about currency movements and currency risk

are likely to affect government bond returns in real terms. Moreover, the influence of these factors may vary considerably among currency areas. Long-term index-linked government bonds should be a better benchmark, at least within their own currency areas. However, their actual performance, too, will be affected by the inflation and currency factors.

Thus, whether in comparison with a nominal or a real benchmark, realized mismatch premiums are not natural and universal constants. Although realized risk premiums (ex post) and theoretical anticipated risk premiums (ex ante) are different concepts, this simple observation appears to undermine the notion of an anticipated risk premium as a single, uniform, worldwide parameter.

Note that all numbers, tables and graphs presented in this paper are based on stylized information and are intended only to illustrate the usefulness of the funding index as an analytical tool in a simple model. This is not an empirical study. Moreover, we have kept our analysis deliberately at a superficial level, and consequently it should not be considered an investigation into “the” equity risk premium.
3 — FUNDING RATIO VOLATILITY

3.1 Definition

Let

$$r_i = \log \left[ \frac{\phi(t_{i+1})}{\phi(t_i)} \right]$$

for subsequent times $t_0, t_1, \ldots, t_n$. Suppose that the time intervals $t_{i+1} - t_i$ are all equal to one $N$th of a year, so that for example $n = N = 12$ in the case of monthly data over the single year ending at time $t_n = t$.

We define the volatility $\sigma(t)$ of the funding index $\phi(t)$ as the observed annualized standard deviation of $r_1, \ldots, r_n$:

$$\sigma(t) = \left\{ \sum \left[ r_i - \left( \sum r_i / n \right) \right]^2 \cdot N / (n - 1) \right\}^{\frac{1}{2}}.$$

Since movements of the funding index $\phi(t)$ account for all movements of the funding ratio $FR(t)$ apart from the disturbance factors mentioned in section 1.1, $\sigma(t)$ may in that sense be viewed as an estimate of the volatility of the funding ratio $FR(t)$.

We present the above volatility formula pro forma, without assuming a stochastic model such as geometric Brownian motion, and also without claiming any kind of optimality. Moreover, we are not advocating $\sigma(t)$ as a sufficient or even appropriate risk measure for policy evaluation, whether from a short-term or from a long-term perspective.

It is worth noting that it is not necessary to first determine a matrix with variances and covariances for equities, bonds and pension liabilities to calculate this volatility.
3.2 Stylized data

FIGURES 3A AND B

One-year volatility for a pension fund with nominal pensions 1999 – 2004

A Nominal pensions in US$
Bonds: US government; 5 year duration
Equities: MSCI gross world index in US$

B Nominal pensions in EUR
Bonds: European government; 5 year duration
Equities: MSCI gross world index in EUR

FIGURES 4A AND B

One-year volatility for a pension fund with indexed pensions 1999 – 2004

A Indexed pensions indexation in US$
Bonds: US government; 5 year duration
Equities: MSCI gross world index in US$

B Indexed pensions in EUR
Bonds: European government; 5 year duration
Equities: MSCI gross world index in EUR
4 — **Surplus-at-Risk**

### 4.1 Definition

A surplus-at-risk methodology for solvency testing with a one-year horizon is proposed in PVK (2003), albeit without using the funding index and its volatility. In this connection, a maximum one-year shortfall probability of \( p = 2.5\% \) will be written in the proposed new Pensions Act, which is expected to become law of the land in the Netherlands by 2005. This requirement, in combination with a maximum allowed recovery period of fifteen years, will apply to the funding ratio with respect to the unconditional pension entitlements, including all guaranteed elements, but excluding conditional indexation to be granted in the future.

Against this background, we define surplus-at-risk \( SaR_p(t) \) as

\[
SaR_p(t) = \left\{ \exp [ k_p \cdot \sigma(t) ] - 1 \right\} \times 100\%,
\]

where \( k_p \) is the lower \( p \)-quantile of Student’s \( t \) distribution with \( n - 1 \) degrees of freedom, where \( n \) is as in section 3.1. If \( p = 2.5\% \) and \( n = 12 \), then \( k_p = 2.2 \); if \( n = 120 \) then \( k_p = 2.0 \). All this is inspired by elementary statistical theory, of course, which we are applying mechanically, without pretending that we necessarily have a realistic model.

Let \( MFR(t) \) denote a minimum funding requirement, defined by

\[
MFR(t) = 100\% + SaR_p(t) = \exp [ k_p \cdot \sigma(t) ] \times 100\%.
\]

It is possible to formulate a stochastic model in which \( FR(t) \) and \( \phi(t) \) follow essentially the same process of geometric Brownian motion. In such a model, it is easy to show that,
apart from the effect of using an approximation based on non-experimental observed data,

\[
\text{Probability}\ \{ FR(t+1) < 100\%, \text{ given } FR(t) > MFR(t) \} < p,
\]

if the expected mismatch premium for one year is negligible. Note that the quality of this approximation is not the point of this illustration and beyond the scope of this paper.

Using the so-called reflection principle from the elementary theory of stochastic processes, one can also prove that

\[
\text{Probability}\ \{ \min_{0 \leq u \leq 1} [ FR(t+u) ] < 100\% , \text{ given } FR(t) > MFR(t) \} < 2p.
\]
4.2 Stylized data

**FIGURES 5A AND B**

Surplus-at-risk for a pension fund with nominal pensions using one-year volatility \((n = 12, \ p = 2.5\%)\)

\[ MFR(t) = 100\% + SaR_p(t) \]

1998 – 2004

**A**

**Nominal pensions in US$**

- Bonds: US government; 5 year duration
- Equities: MSCI gross world index in US$

**B**

**Nominal pensions in EUR**

- Bonds: European government; 5 year duration
- Equities: MSCI gross world index in EUR

**FIGURES 6A AND B**

Surplus-at-risk for a pension fund with nominal pensions using ten-year volatility \((n = 120, \ p = 2.5\%)\)

\[ MFR(t) = 100\% + SaR_p(t) \]

1998 – 2004

**A**

**Nominal pensions in US$**

- Bonds: US government; 5 year duration
- Equities: MSCI gross world index in US$

**B**

**Nominal pensions in EUR**

- Bonds: European government; 5 year duration
- Equities: MSCI gross world index in EUR
5 — Final Comments

5.1 Alternative to surplus-at-risk: a flexible mismatch cushion

Instead of using the surplus-at-risk methodology, one could define a minimum funding requirement by a formula such as

\[
MFR(t) = \max \{ 100\%, \min [ a \times FR_{mod}(t), b ] \}.
\]

where \(0 < a < 1\), \(b > 100\%\), and \(FR_{mod}(t)\) is a modified version of \(FR(t)\), developed on the basis of the funding index and modified by the release of assets whenever \(FR(t)\) is increasing and remains above a sufficiently high upper boundary \(b\). This flexible mismatch cushion is advocated in VAN GAALEN 2003 (a, b, c), inspired by a proposal by EUVERMAN (1997) for a solvency cushion on an equity portfolio on the basis of the history of an equity index. Essentially, this is tantamount to being required to save during years of plenty and being allowed to consume one’s savings during years of famine.

5.2 An armchair actuary’s collar

Example

A stand-alone pension fund is at time \(t_1\) concerned about the possibility of its funding ratio \(FR(t)\) falling below a given minimum level \(mfr\) at some given future time \(t_2\). It may wish to sacrifice some upward potential to obtain downside protection by entering into a derivatives contract with a third party. Such a contract is called a collar. The question whether this is a good idea or not is debatable and far beyond the scope of this paper.

Unfortunately, \(FR(t)\) with \(t > t_1\) itself is not a useful parameter for a derivatives contract signed at time \(t_1\), since it will be influenced by the pension fund’s actions and policies,
such as its actual investment policy, its administrative expenses and any discretionary benefit payments, not to mention plan amendments. On the other hand, the asset index $\alpha(t)$, the liability index $\lambda(t)$ and funding index $\phi(t)$ depend only on their initial definitions and the subsequent market movements in equity prices, interest rates, and possibly the applicable price index if index-linked bonds and indexed pensions are involved.

Therefore, unlike $FR(t)$, the indices $\alpha(t)$, $\lambda(t)$ and $\phi(t)$ can be used to construct derivatives. Suppose without loss of generality that $\alpha(t_1) = \lambda(t_1) = 1$, and hence that $\phi(t_1) = 1$. Consider, for example, a derivative consisting of the single positive or negative payment of $D(t_2)$ at time $t_2$, defined by

$$D(t_2) = \lambda(t_2) \cdot L(t_1) \cdot \{ \max [ mfr - \phi(t_2) \cdot FR(t_1), 0 ] - \max [ \phi(t_2) \cdot FR(t_1) - ub, 0 ] \}.$$  

Here $ub$ denotes an upper boundary such that the initial value of this contract is zero. Such a boundary is likely to exist for sufficiently low values of $mfr$ in comparison with $FR(t_1)$. Note that $FR(t_1)$ is of course known at time $t_1$, so its presence in the formula does not cause any problems.

Let $FR_u(t)$ denote the unprotected funding ratio without the collar. Then

$$FR(t_2) = A(t_2) / L(t_2)$$

$$\approx \frac{[\alpha(t_2) A(t_1) + D(t_2)]}{[\lambda(t_2) L(t_1)]}$$

$$= \phi(t_2) FR(t_1) + \max [ mfr - \phi(t_2) \cdot FR(t_1), 0 ] - \max [ \phi(t_2) \cdot FR(t_1) - ub, 0 ]$$

$$\approx \begin{cases} 
mfr & \text{if } FR_u(t_2) < mfr \\
ub & \text{if } FR_u(t_2) > ub \\
FR_u(t_2) & \text{otherwise.} 
\end{cases}$$

These approximations can be refined. This is of course only a simple illustration, not a policy recommendation. Actual mileage may vary.
REFERENCES


APPENDIX: DATA

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<tr>
<th>Asset allocation = ( AA )</th>
<th>Proportion ( \pi_1 ) in equities and proportion ( \pi_2 ) in bonds, as indicated; rebalanced at the beginning of each month</th>
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<td>Bond index = ( I_2(t) )</td>
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<tr>
<td>Asset index = ( \alpha(t) )</td>
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<td>American nominal yield rates</td>
<td>Until 1999: 10-year constant maturity rates used for bond index; monthly data obtained from the US Federal Reserve System</td>
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<td>American real interest rates</td>
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<td>European price index</td>
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