Selected ALM ISSUES

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Contents

Introduction

I. ALM: background and current issues
   I.1 Description
   I.2 Background
   I.3 Methods
   I.4 Current issues

II. Riding the yield curve
   II.1 Defining and analysing the strategy
   II.2 Leverage
   II.3 Future-rate expectations and persistence effect
   II.4 Empirical test
   II.5 Observations and conclusions

III. Capital allocation
   III.1 Optimising a business portfolio
   III.2 One bank, two lines of business
   III.3 Applications: Should commercial banks practise transformation?
   III.4 Implementation and criticisms
   III.5 Pertinent findings

IV. Sensitivity of a life insurer's earnings to asset allocation
   IV.1 Life insurers' portfolios: current situation in France
   IV.2 Accounting mechanisms for appropriation of earnings
   IV.3 Simulation assumptions
   IV.4 First simulation: portfolio with a 10% equity element
   IV.5 Simulation of different investment policies
   IV.6 Investing in equities

Conclusion

Bibliography
Introduction

The technique known as asset/liability management (ALM) has enjoyed remarkable popularity in recent years. From its origins as an actuarial and cashflow matching technique, ALM has grown into a conceptual framework for financial management – and a professional activity in its own right.

In a world governed by financial markets and physical commodities, it is vital to analyse objectively the economic effects of price movements on balance sheets, earnings growth and enterprise value. The constituency of ALM users goes beyond financial institutions and banks, and now includes pension funds. In the near future, companies are bound to adopt it. And individuals will surely find it attractive for investment decision-making and income planning.

ALM specialists are now recognised as fully-fledged professionals. In the banking industry, they have found a niche between capital market divisions and management control divisions (in France) or alongside strategic divisions (in other European countries). They have also carved a place for themselves in public life, as witnessed by the constant stream of seminars and conferences devoted to ALM, either directly or indirectly. Professional bodies are thriving in several countries, including AFGAP in France, NALMA in the USA and ALMA in the United Kingdom. Lastly, and significantly, ALM is now being taught not only as part of a broader curriculum but also as a discipline in its own right. It is also the subject of numerous books, including those by J. Bessis (1995) and M. Dubernet (1996), and a special review, "Balance Sheet".

Moreover, the achievements of ALM are beginning to find an audience beyond the inner circle of specialists. In France, two recent examples are significant.

When AGF, an insurance company, was privatised, the chairman stressed the importance of strategic asset allocation for boosting profitability and market value. In a totally different vein, the mainstream press covered the unfortunate consequences of Crédit Lyonnais' failure to hedge its interest-rate risk. It was reported that Consortium de Réalisation (CDR), the defeasance vehicle set up to handle the sale of the bank's assets, repaid a loan whose interest rate was pegged to short-term interest rates, which have fallen by a steep 5% in the past two years. Given the patchy information at our disposal, we cannot say with certainty whether the indexing was deliberate (unlikely, because it would imply a bet on a short-rate hike) or whether it corresponded to other areas of balance sheet exposure. Whatever the reason, the fact that the incident was attributed, rightly or wrongly, to poor ALM shows that an informed audience is also aware of this new discipline.

CCF did pioneering work in adapting ALM methods to the French environment. A feasibility study was carried out in the period 1985-86, shortly after France deregulated its money market. In 1987, a prototype unit began laying the groundwork for ALM and adapting the tools that the bank would later need. But it was not until 1989 that the asset-liability department actually implemented an original approach based on the development of proprietary information systems. CCF's Research and Innovation Department (DRI) has investigated the possibilities of either applying ALM to banking requirements or adapting the underlying concepts to other environments.
Some of the DRI's research has been outlined in various issues of Quants or at scientific financial conferences. The areas covered include securitisation and prepayment options (Quants No. 4, Andria et al., 1991), embedded options (Boulier and Schoeffler, 1992, Boulier, 1996), and the modelling of dynamic management with stochastic interest rates (Jerad and Sikorav, 1992). Quants No. 12 examined an ALM-based methodology for pension fund investment strategies (Boulier et al, 1995 and 1996) while Quants No. 23 explained how fuzzy calculus could be used to take account of unknown due-dates in a cashflow matching context.

The purpose of this issue is to examine, in the most accessible way possible, the key issues involved in ALM and its application. We provide three examples of how the technique relies on financial modelling. Through these case studies, which deal with different fields (and can be read separately), we hope to illustrate the diversity of techniques and applications of ALM. First, we discuss the advisability of exposing a company to interest rate risk – a highly topical theme in view of the current yield-curve configuration in Europe. We then give a description of a capital allocation method based on the Markowitz model. Finally, we rely on simulations to analyse the attractions and risks of increasing the equity investments of a life insurance company.

I. ALM: background and current issues

This part of our study examines, in an intentionally non-technical manner, the principal aspects of asset-liability management in its present form in Europe. Since the principal users of ALM are bankers, we intend to start with their concerns before moving on to other specialist fields such as insurance and pension financing. We have divided this part into four sections: the first attempts to define the aims of ALM; the second retraces its history and examines the role played by associations such as AFGAP; the third draws up a list of the most commonly used methods (without relying on mathematical formalisation); and the fourth examines some of the challenges currently facing practitioners of ALM, particularly in terms of methodology.

1.1 Description

Asset-liability management has three main aims, which vary in importance depending on the businesses that use the technique and on their maturity. The aims are to analyse economic risks (chiefly market risk), to choose appropriate strategies (e.g. whether to hedge exposure) and to monitor the implementation of those strategies. For companies, the main concern – in theory – is to maximise shareholder value. In practice, however, this can vary depending on how the company's shareholders, creditors and management view the process of maximisation. In other words, the company's exposure must be decided upon rationally, accepted by all those concerned, and be applied optimally. This all goes to show the importance of information processing, both internally depending on the type of products and the volumes handled, and externally depending on the market opportunities involved. Moreover, the overriding concern of ALM teams is the quality of their information, which must be exhaustive, accessible and easy to model.
a) Analysis

When analysing market risk, commercial banks traditionally focus on three types of exposure: interest rate risk, currency risk and liquidity risk. Later on, we will look at other risks (e.g. credit risk) and how they interact (counterparties, embedded options). For domestic banks, only the first and third types have any direct consequence. Interest rate risk stems from the fact that interest rates paid to depositors and yields earned on loans can change at different speeds. Take the example of a bank that grants a long-term loan at a fixed rate of interest and refinances it with deposits (which involves collection costs) and certificates of deposit (market rates). There is no certainty that the interest it pays will be lower than the interest it receives. This risk shows up in many ways: in accounting flows (net banking income), the market value of the bank's products, in futures contracts (which depend on the horizon) and, naturally, in share and bond prices. Moreover, depending on the product, interest rate risk manifests itself in very different ways. This can be seen in the simple example of fixed-rate and variable-rate loans. With fixed-rate loans, the bank can anticipate its future flows with certainty on condition that the payment dates coincide, which is not always the case. In practice, this is not always the case. As a result, the bank relies on cashflow matching methods, which we will examine in section II.3. With a variable-rate loan, the flows are not always known in advance (unless the lender uses a highly accurate matching method). However, if we look at fair market values, i.e. the value of the credit flows discounted at market rates, we observe that variable-rate loans vary less than fixed-rate ones. As regards income flows and their economic value, everything depends on the geometry of the flows and on market conditions. In Part III, we discuss and model the process of taking interest rate risk, sometimes called transformation risk.

Liquidity risk — the bane of every banker's life — is more difficult to identify and measure. In a nutshell, liquidity risk is the danger of not finding a lender in the market. In practice, the aim is to avoid paying too dearly for liquidity, with the ultimate risk being that the market will not lend money under any circumstances or at any price. What are the underlying mechanisms? The state (the benchmark issuer) borrows for a given period and at a certain rate. Private issuers such as banks borrow at a higher rate, which includes a margin. This is because they are prone to default — unlike the state, which can always raise taxes to pay off its debts. The margin, therefore, is the incremental cost to the issuing bank and reflects either investor perception of its creditworthiness or the degree to which investors are already saturated in credit risk. In an initial approximation, that margin is the annual probability of default multiplied by the collection rate (corresponding to what the issuer's default would cost the investor) plus a risk premium, which depends on the uncertainty of the information and the liquidity of the security. From a technical perspective, liquidity risk management is much less advanced and thus demands a considerable amount of common sense as well as theoretical know-how. There are three basic requirements: the issuer must make an accurate assessment of its cash requirements; it must be present in numerous markets, including foreign ones; and it must operate a financial PR policy that minimises the "risk premium".

We need to paint a broader picture of the economic risks affecting banks, even though the process of measuring and controlling such risks is not always an integral part of ALM. Clearly, the oldest
and most significant form of exposure is credit risk. Even though it differs from the risks described above, it is clearly related to interest rate risk. First, as regards variable-rate loans, a rise in interest rates can force a borrower to default. And because it eliminates expected income flows, default can create market risk and thus interfere with the management of interest rate risk. Other risks that warrant consideration include the behaviour of a bank's customers in the presence of so-called embedded options, the right to certain types of loan, the right to withdraw deposits, and the right of refund. In the case of non-bank activities, other assets such as equities or property give rise to risks. And we have not even addressed the need for insurance companies to control their liability risks, or operational risks of all sorts. Last but not least, the practice of analysing competitive pressures should become more widespread. After all, lending and borrowing margins are core considerations for any merchant, whether he is distributing or collecting.

b) Control strategies

A bank can react to (or pre-empt) a given situation in many different ways. It generally falls to the finance department of a bank or financial institution to draw up and implement suitable strategies.

Some of those strategies are purely financial, such as the decision to rely on maturity transformation. Similarly, selecting the percentage of equities in an insurance company's portfolio involves the same type of decision. Other strategies give greater emphasis to the distribution side. For example, in the case of leasing transactions, if the "tax advantage market" is prepared to offer higher margins on a durable basis, having taken into account the cost of funds, then it may be worth the bank's while to develop this activity. We note that the decision involves operational factors that do not come within the bailiwick of the finance department, even though that department will assess the broad economic interest. A more proactive strategy may consist in designing new products. For example, when the yield curve is steeply sloped, it offers the opportunity of selling capped variable-rate loans to a certain segment of the customer base.

Responsibility for ALM is generally devolved to two entities: on the one hand, a department tasked with analysing, preparing and implementing strategy; on the other hand, a committee comprising representatives of general management and the various specialist arms, which is in charge of financial decision-making and, in certain cases, product development. Dealing rooms are entrusted with the task of putting these decisions into practice; depending on the institution, they either deal only for their own account or have a discrete investment banking operation. The entity in charge of operations can, in certain cases, tolerate a degree of latitude, in the same way that investors will accept a tracking error from a fund manager who manages their money by means of a benchmark. Taken to its extreme, ALM can be a profit centre with a delegation of market risk.

All this comes at a cost in terms of expertise and information technology, not to mention a slew of procedures that limit the flexibility of an institution's departments. But is obviously vital in a competitive world where, for private companies, unearned income is a thing of the past. ALM teams, like the practitioners of any other applied discipline, must strike the right note of compromise in their analyses (given the cost of obtaining exhaustive information). Likewise, their
recommendations must be easily understood throughout the company and have a reasonable chance of success.

1.2 Background

So recent is the technique of ALM that the word "history" seems inappropriate. However, its roots can be found in the practice of planning and matching cashflows, commonly found in project financing and long employed by specialised financial institutions.

The approach was formalised by the British and, more particularly the American financial world. In terms of risk modelling, it dates back to the early 1980s and the introduction of measures such as duration and cashflow volatility. In terms of corporate management, too, it can be found in the concept of decentralised banking, which is diametrically opposed to one-stop, universal banking. In France, ALM first appeared in the late 1980s and, as we recalled in the introduction, was originally developed by medium-sized private banks before spreading to the rest of the economy. Interestingly, the advance of ALM was not prompted by regulatory constraints. Life insurance companies tried to adapt the concept, but ran into several difficulties. The most significant hurdle is the accounting "maze" specific to the insurance industry. The second is the fact that insurance-company assets and liabilities are diversified and non-fungible, meaning that they cannot naturally be represented as flows, as they can in the banking world. The third problem – without wishing to be overly polemical – is the relevance of ALM in state-run companies. Recently, retirement funds, private pension funds and employee benefit organisations – often acting at the behest of consultants – have looked into ALM with a view to using it for strategic asset allocation (see Quants No. 12). The actuaries conferences organised by the AFIR show the extent to which the pension industry is becoming increasingly aware of the importance of ALM.

Much of the credit for spreading the word about ALM lies with the Association Française des Gestionnaires Actif Passif (AFGAP), founded in 1989 by CCF, Compagnie Bancaire and a handful of other pioneers. Today, AFGAP has one 120 members, two-thirds of whom are drawn from the banking industry, one-fifth from insurance companies and the remainder from businesses and consultancies. AFGAP's contacts with its counterparts abroad show that France and the UK are probably among the most advanced nations in terms of ALM for banking applications. However, such is the banking culture in countries like the Netherlands and Switzerland that institutions in those countries have already adopted approaches based on asset/liability management, albeit under a different name. Several of the topics covered in this issue of Quants have already been discussed at AFGAP meetings.

1.3 Methods

Strictly speaking, there is no economic or financial theory of asset/liability management. Having subsumed a number of complementary methods, ALM can be likened to a box of tools used by practitioners of the art. For a more detailed description of these methods, see J. Bessis (1995) and M. Dubernet (1996). We intend to examine the three main approaches: gap analysis, duration analysis and simulations.

The first method, gap analysis, is widely used. It consists in taking the present balance sheet situation and projecting two factors into the future: one the one hand, the capital flows received or paid out; on
the other hand, the interest received or paid. The diagram of the differences for the different maturities is called the treasury gap. Products are usually pooled into categories that are as homogenous as possible, with ranges of dates that are as detailed as necessary. The analysis then seeks to update future financing or investment requirements. The gap can be narrowed by means of transactions in the "physical" market (issuance, investment). And although interest rate risk can be eliminated by the use of derivatives, liquidity risk may persist if gap analysis reveals a borrowing requirement.

Second, duration analysis aims to identify the impact of a change in market conditions—chiefly the term structure of interest rates—on several items on the balance sheet or the P&L account. Consequently, it forms an excellent fit with the first method. However, it is often considered to be "theoretical", either because it has not yet been properly adapted to actual requirements, or because it constitutes an "economic" viewpoint that has no direct consequences in accounting terms. At least, not in the immediate future...

The third method, simulation, is rapidly becoming the only tool that allows the future to be introduced into factors that are either internal (e.g. production) or market-related (interest rate movements). Initially deterministic and scenario-based, simulations rely increasingly on stochastic modelling of environment variables. This involves generating a large number of random samplings in order to quantify the probability distribution of a result or an accounting (or financial) quantity at a certain horizon. This approach is widely used by consultants as a basis for the asset allocation recommendations they make to pension funds or insurance companies. It allows the user to take into objective consideration the uncertainty related to equities, which, while offering higher yields than bonds, are nevertheless more volatile.

I.4 Current issues

Aside from the operational management aspects, which are not addressed in this study, there are several questions of principle which, in our view, tend to exercise users of ALM techniques.

First and foremost is the question of capital allocation, which will be discussed in Part III. Another key issue, covered in Part IV, is the selection of the proportion of equities that insurance companies should hold in their investment portfolios. But there are many other questions: the correlation (or lack thereof) between accounting data and economic information, transfer pricing (i.e. how to represent the cost of banks' raw material), the use of derivatives by insurance companies and pension funds, and the links with credit risk and with the regulatory approach to market risk. Then there are time-honoured issues such as the modelling of demand deposits, which will become increasingly important—at least for French banks—in the context of the euro.

Before tackling the two main issues, and at the risk of over-simplification, we thought it would be useful to examine the subject most traditionally associated with ALM, namely interest rate risk. We have approached it from a practical angle that makes it possible to identify the attractions and risks of maturity transformation. Moreover, the current slope of the yield curve (with a gap of more than 3% in early October between money market rates and 10-yr zero coupon yields) makes this topic all the more relevant.
II. Riding the yield curve

Riding the yield curve is a strategy that enables an investor to earn a higher rate of return by buying longer-dated bills and selling them before they mature. The attraction of maturity transformation is that yield curves are generally upward-sloping: there is a yield "premium" for longer-dated investments. If interest rate conditions remain unchanged, or if they are sufficiently stable, an investor may find it more profitable to choose a maturity that exceeds his holding period. By riding the yield curve, investors get a higher starting rate and a capital gain when they sell their investment...on condition that interest rates have not risen in the meantime.

The aim of this part of our study is analyse the returns and risks associated with maturity transformation from the viewpoint of a corporate treasurer assessing his results on the basis of market value. We first compute the rate of return, the risks and the Sharpe ratio of this well-known strategy using the term structure of French interest rates at the beginning of October. We then consider the impact of leverage and examine how the "persistence" of the yield curve affects results. We conclude with a short, empirical test to examine the efficacy of the strategy in the past.

II.1 Defining and analysing the strategy

When an investor's maturity differs from his horizon, he is exposed to two types of risk: reinvestment and/or capital gain and loss.

Consider the example of treasurer who has a certain sum at his disposal for 6 months. If he buys a 3-month CD, he runs a reinvestment risk because he does not know the rate at which he will invest his funds three months hence. Conversely, if he buys a 1-yr CD, he does not know the price at which he can sell it when the six months are up and he is therefore exposed to the risk of capital gain/loss.

Riding the yield curve is a strategy that consists in buying paper with a maturity longer than the investment horizon. The aim to realise an "extra" return vis-à-vis an investment with a maturity coinciding with his holding period. The investor must take into consideration not only the upward slope of the current yield curve for the maturities in question, but also the expected level of interest rates when the position is closed out.

a) Example

The data used in this example are those for 1 October 1996. The treasurer buys a 4-yr bond with a par value of FF1,000. Interest is paid annually at a rate of 5%. Given the slope of the yield curve, the price paid for the bond is:

\[
\frac{50}{1.0382} + \frac{50}{(1.0415)^2} + \frac{50}{(1.0457)^3} + \frac{1050}{(1.05)^4} = 1001.82
\]

Consider the situation one year later, when he receives his first FF50 coupon. We assume that there has been no change in the yield curve. Our treasurer then decides to liquidate his position, which he does at the following price:

\[
\frac{50}{1.0382} + \frac{50}{(1.0415)^2} + \frac{1050}{(1.0457)^3} = 1012.52
\]

He has therefore received: 1012.52 + 50 = FF1062.52. Reasoning in terms of the return over one year, it is as though he invested FF1001.82 for a year at a rate equivalent to (1062.52-1001.82)/1001.82 =
6.06%. Over the same period, a one-year investment would have returned just 3.82%: our treasurer has made a profit out of his ride.

The calculation is based on the assumption that future interest rates are unchanged. If rates had risen, then the investment would have returned less than 6.06% and could even have fallen below the one-year rate. In this case, a riding strategy would have resulted in a loss.

Reciprocally, the steeper the curve's upward slope at the outset, the lower the interest rates when the position is liquidated (the horizon), and the higher the return to the strategy.

b) Computing the return to the strategy

Generally speaking, we consider an investor who has a certain sum for a period $H$. He could invest it immediately at the spot rate $R_H$ over this period. However, he makes forecasts concerning $R(H,M)$, the rate of maturity $M-H$ that will prevail at date $H$. He thus anticipates that the rate will be $R$, which is low\(^1\).

So he decides to invest in an instrument with maturity $M$, greater than $H$. At $H$, his expectation proves to be correct so he closes out his position, selling his security for the residual period at the price prevailing in the market at $H$. Assuming actuarial rates (see also Figure II.1), the return on this transaction over the period $H$, noted \(^2\) $R_g$ verifies:

$$(1 + R_g)H \times (1 + R_o)^{M-H} = (1 + R_M)^M. \quad (I.1)$$

\[\begin{align*}
R_g &= (1 + R_o) \times \left[ \frac{1 + R_M}{1 + R_o} \right]^{M/H} - 1. \quad (I.2)
\end{align*}\]

Observing that the returns are small relative to 1, we can write a linear formula that approximates $R_g$:

$$R_g = R_M + (R_M - R_o) \frac{M-H}{H}, \quad (I.3)$$

This is valid for money market rates and actuarial rates.

c) The future yield curve and the ride strategy

To make formula (I.3) more meaningful, we can draw the yield curve that would result in a preset return curve at the end of the year. Using Equation (I.3), we express the future rate as a function of the return:

$$R_o = R_M + (R_M - R_g) \frac{H}{M-H}. \quad (I.4)$$

---

\(^1\) $o$ as in anticipate
\(^2\) $g$ as in gain
Equation (11.4) allows us to establish a link between the future curve and the return to the strategy. It is illustrated in the following three examples, which illustrate how the choice of strategy is implicitly linked to a precise expectation of future rates.

- **Reproducing the yield at maturity (Figure II.2)**

We will now try to determine what the shape of the curve must be in one year's time in order for the return (for different maturities \( M \)) to coincide with the initial yield on an instrument with maturity \( M \). As Equation (11.4) shows, the configuration in question is the initial curve, with a one-year shift to the left along the x-axis. We want \( R_s = R_M \), which gives us \( R_s = R_M \). This is equivalent to a shift of a period \( H \).

- **Identical return (Figure II.3)**

In order for the return to the riding strategy to be the same for all maturities, what shape must the yield curve be one year later? This corresponds to the situation where an investor is indifferent between the "horizon = maturity" strategy and riding the curve. Thus the future yield curve is the forward curve defined by the Equation (11.4), with \( R_s = R_M \).

- **Naive forecast of yield curves (Figure II.4)**

We will now examine what the return curve looks like if the one-year curve remains unchanged. This scenario results in a return curve that sits above the initial yield curve. We obtain it by shifting the initial curve to the left and adding a term that is proportional to the difference \( (R_M - R_H) \).

**d) Risk measurement**

The risk of riding the curve is that the investor will get a lower return than he
would from a buy-and-hold strategy. Our risk quantification method is based on the value $R_i^*$ of the future rate at date $H$ that ensures a zero payoff from the riding technique. With $R_y = R_m$ in (II.3), we have:

$$R_i^* = R_m + (R_M - R_H) \frac{H}{M - H}.$$  (II.5)

The threshold future rate $R_i^*$ (for example) corresponds to the initial value of the forward rate between $H$ and $M$. Ex post, we find that the more $R_i^*$ exceeds $R_y$, the higher the real rate of return on the transaction, and vice versa.

We have two more strategy evaluation tools at our disposal. The first is a trading performance index known as the Sharpe ratio, which takes the form of a risk-return ratio. The return to the strategy is the average excess return over the riskless rate of interest. Risk is measured by the variability or standard deviation $\sigma_y$ of the strategy's return. The higher the ratio, the better the performance. The Sharpe ratio is defined as follows:

$$SR = \frac{E(R_y) - R_H}{\sigma_y}.$$  (II.6)

The second indicator is called the margin of safety (see Dyl and Joehnk, 1981), which is a relative deviation:

$$MS = \frac{R_i^* - R_y}{R_y}.$$  (II.7)

The margin of safety measures the fluctuation of the future rate that would eliminate the profit derived from the strategy.

e) Risk associated with a simple curve

To introduce our first interest rate risk model, we assume that the initial yield curve is an ascending straight line with slope $\rho$ per year. With $R_y$ as the overnight rate, the yield on maturity $m$ is given by:

$$R_m = R_y + \rho \times m.$$  (II.8)

On the future curve, we make the following assumption regarding the yields on the maturity $M - H$:

$$R_y = R_{M-H}^* + \epsilon,$$  (II.9)

where $\epsilon$ is a random variable with zero expectation and standard deviation $\sigma$. In other words, we suppose that the current curve is the best possible forecast of the future curve, and that the future curve is subject to translation risk. Thus, the average return is written:

$$\bar{R} = R_y + \rho \times (2M - H).$$  (II.10)

The future rate that ensures a zero payoff is:

$$R_i^* = R_y + \rho \times (M + H).$$  (II.11)

The Sharpe ratio is:

$$RS = \frac{2\rho H}{\sigma}.$$  (II.12)

The margin of safety is:

$$MS = \frac{2\rho H}{R_y + \rho \times (M - H)}.$$  (II.13)

As the above formulae show, the steeper the curve (while remaining positive), the more compelling the strategy. This is because the return, the margin of safety and Sharpe ratio all increase as the curve steepens.

\[3\] Meaning an incremental return relative to a buy-and-hold investment.
Also, when the yield curve is upward-sloping, the average return can be increased by extending the maturity $M$ of the investment, while the Sharpe ratio remains unchanged. However, we must not jump to hasty conclusions as a result of this observation, which depends heavily on our modelling of the curve and its attendant risks. If the riding strategy is to be successfully fine-tuned, then those risks must be carefully analysed.

f) Risk associated with a real curve

The yield curve is now described by an Equation such as: $R_w = f(m)$. Also, we will assume a naive expectation, as defined by Equation (II.9).

Noting $p_{a,b}$ as the slope of the chord joining the points of the curve for maturities $a$ and $b$, the mean return is:

$$\bar{R}_r = f(M) + p_{M-H,M} \times (M - H).$$

(II.14)

In this case, the Sharpe ratio is written:

$$RS = \frac{H}{\sigma} \times (P_{M-H,M} + P_{H,M}).$$

(II.15)

We applied the above calculations to the curve at 1 October 1996, once again under the naive expectation assumption. Table II.1 shows the ex ante results of the strategy. The horizon $H$ is one year and the maturity $M$ of the investment varies.

<table>
<thead>
<tr>
<th>Maturity $M$ (yrs)</th>
<th>Yield at $M$ $R_w$</th>
<th>Return on ride $\bar{R}_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4.2%</td>
<td>4.5%</td>
</tr>
<tr>
<td>3</td>
<td>4.6%</td>
<td>5.4%</td>
</tr>
<tr>
<td>4</td>
<td>5.0%</td>
<td>6.2%</td>
</tr>
<tr>
<td>5</td>
<td>5.3%</td>
<td>6.7%</td>
</tr>
</tbody>
</table>

II.2 Leverage

Thus far, we have focused solely on the example of an investor with no liability constraints. His choices are based on the maturity of the securities that form his asset base and on the fact that he must liquidate the position at a given date. We can apply the same approach to a more realistic situation: an investor with something more than shareholders' equity on the liabilities side of his balance sheet. We will model those liabilities (excluding equity) by means of a loan with a maturity $H$, which is our investor's horizon. The initial amount $A$ of the assets can be decomposed thus:

$$A = \alpha A + (1 - \alpha)A_{\text{equity}}.$$ 

(II.16)

Once again $R_g$ is the return on the riding strategy associated with a horizon $H$ and an investment with an initial maturity $M$. At $H$, the date on which the positions are closed out, we have:

$$\text{Assets} = A \times (1 + R_g)^H$$

(II.17)

$$\text{Liabilities} = (1 - \alpha) \times A \times (1 + R_H)^H$$

(II.18)
If $R_g'$ is the return to the lending and borrowing transaction (I.16), we can write:

$$
(1+R_g')^n A - (1 - \alpha (1 + R_h')^n A
= \alpha (1 + R_h')^n A.
$$

(I.19)

Which, after linearisation, gives us:

$$
R'_g = R_h + \frac{1 - \alpha}{\alpha} (R_g - R_h).
$$

(I.20)

Equation (I.20) shows that, if the rate of return $R_g'$ is greater than $R_h$ (the liability rate), then the return is greater than that earned by investing the equity alone. The supplemental terms come from the loan. This produces leverage.

Naturally, a higher rate of return goes hand in hand with greater risk. Not only that, but the Sharpe ratio of this type of transaction is identical to a non-leveraged transaction.

Returning to example a) in chapter I.1, we will take a proportion of shareholders' equity such that $\alpha = 10\%$. The excess return obtained through leverage is:

$$
(1 - 0.1) \times (6.06\% - 3.82\%) / 0.1 = 2016\%,
$$

in other words, three times greater than the non-leveraged return (6.06%). Remember, however, that we made this calculation within the framework of a naive forecast. Further, the volatility of this result is ten times greater than it would be without leverage.

II.3 Future-rate expectations and persistence effect

As we have seen, choice of strategy is directly related to expectations of future rates. The most common expectations call for no change in the yield curve or, alternatively, a future curve identical to the current forward curve. In the latter case, the transformation is based on Equation (II.4) with $K_g = K_h$. When the initial curve is upward-sloping, the forward yield curve is above the initial curve. Conversely, when the curve is inverted, forward rates are lower than initial rates.

The persistence factor quantifies the curve's tendency to keep its initial shape. A persistence factor of 1 indicates an expectation of an unchanged future curve whereas a factor of 0 indicates that we are anticipating the forward curve.

Generally speaking, we assume that the future yield curve can be characterised by a persistence factor $k$ as per the equation:

$$
\text{future rate} = k \times \text{initial rate}
= (1-k) \times \text{forward rate}
$$

When the persistence factor varies from 0 to 1, it generates a continuous spectrum of curves within an envelope bounded by the initial curve and the forward curve (the persistence factor can also be negative or greater than one).

![Figure II.6: Returns to the riding strategy as a function of maturity, with different persistence factors](image)

We will now return to the material covered in earlier chapters. When an investor's expectations can be summarised by a
persistence factor \( k \), the expected return \( R_e(k) \) is easy to compute: it is equal to the average of the returns on the two preceding curves, weighted by the coefficients \( k \) and \( k - 1 \). Noting that \( R_e(0) = R_H \), we have:

\[
R_e(k) = k \times R_e(1) + (1 - k) \times R_H, \tag{II.22}
\]

where \( R_e(t) \) corresponds to the return earned under a naïve-expectation scenario. Figure II.5 shows the returns on riding strategies for three persistence factors: 0, 0.75 and 1.

II.4 Empirical test

To test the efficacy of the riding strategy, we carried out an empirical study over two discrete periods: 1988-1992 and 1992-1996. We smoothed the yield curves with the Vasicek-Fong method and used weekly data. We then computed the return to the strategy using actual rather than expected rates. Thus, the study is based on ex post returns. An inequality condition on the margin of safety acts as a sort of filter through which we examined the success rate of the riding strategy (i.e. the number of times that the excess return is positive). Table II.2 shows the statistics for each period. The first column gives the frequency, or number of times that the margin of safety was positive. Under this condition, we then counted the success rate, or the number of times the strategy really paid off. The table also shows the average returns and the Sharpe ratios, once again under the condition \( MS > 0\% \). Table II.3 shows how the choice of filter influences the strategy’s success rate (in terms of both frequency and average return).

The period 1988-1992 was not ideal for riding: it is characterised by high but mainly flat yield curves. Better results were obtained in the second period, 1992-1996, chiefly towards the end of 1995 and during 1996. Here, the curves are generally steeply sloped, a factor reflected in the frequency of the event \( MS > 0\% \).

Table II.2: Actual results of the strategy with the filter \( MS > 0\% \)

<table>
<thead>
<tr>
<th>Maturities (yrs)</th>
<th>Frequency</th>
<th>Success rate</th>
<th>Excess return</th>
<th>Sharpe ratio</th>
<th>Frequency</th>
<th>Success rate</th>
<th>Excess return</th>
<th>Sharpe ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M=1 )</td>
<td>8.2%</td>
<td>44.5%</td>
<td>-0.7%</td>
<td>-0.39</td>
<td>77.5%</td>
<td>53.3%</td>
<td>0.5%</td>
<td>+0.25</td>
</tr>
<tr>
<td>( M=2 )</td>
<td>9.4%</td>
<td>53.3%</td>
<td>-0.4%</td>
<td>-0.25</td>
<td>81.4%</td>
<td>56.8%</td>
<td>1.3%</td>
<td>+0.28</td>
</tr>
<tr>
<td>( M=3 )</td>
<td>10.8%</td>
<td>68.6%</td>
<td>0.6%</td>
<td>+0.33</td>
<td>78.8%</td>
<td>58.8%</td>
<td>2.0%</td>
<td>+0.27</td>
</tr>
<tr>
<td>( M=5 )</td>
<td>12.8%</td>
<td>65.7%</td>
<td>2.8%</td>
<td>+0.93</td>
<td>82.3%</td>
<td>64.4%</td>
<td>3.4%</td>
<td>+0.26</td>
</tr>
<tr>
<td>( M=10 )</td>
<td>17.7%</td>
<td>51.8%</td>
<td>4.7%</td>
<td>+1.23</td>
<td>85.3%</td>
<td>67.5%</td>
<td>7.0%</td>
<td>+0.44</td>
</tr>
</tbody>
</table>

Table II.3: Impact of the filter on the success rate. Horizon \( H \) is 1 month, maturity \( M \) is 1 year

<table>
<thead>
<tr>
<th>Margin of safety</th>
<th>Frequency of observation</th>
<th>Success rate</th>
<th>Average return over the period</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( MS=0% )</td>
<td>77.3%</td>
<td>71.5%</td>
<td>3.2%</td>
<td>5.6%</td>
</tr>
<tr>
<td>( MS=5% )</td>
<td>32.4%</td>
<td>72.6%</td>
<td>3.3%</td>
<td>3.1%</td>
</tr>
<tr>
<td>( MS&gt;10% )</td>
<td>3.1%</td>
<td>66.7%</td>
<td>0.7%</td>
<td>6.4%</td>
</tr>
</tbody>
</table>
II.5 Observations and conclusions

The strategy we have just described is both well-known and time-served – in the early 1990s, for example, it enabled US banks to return to profitability. However, although efficient, it nevertheless involves an element of risk. Adopting an accounting method based on rediscount rates rather than market values naturally modifies the perception of risk and return. This study goes beyond the framework that we initially adopted. However, we are still dealing with the same phenomenon, construed in two different universal sentences. Naturally, the smoothing that results from the accounting method helps to attenuate the short-term variability of the results. For periods that are much longer than the smoothed period, we can expect comparable volatilities.

In short, the return obtained through maturity transformation depends on the slope and persistence of the yield curve. Whereas slope can be observed, persistence is more random. At current levels, the Sharpe ratios raise the question of the degree of maturity transformation that a bank can introduce into its management policies. This depends on other opportunities that may arise and on the bank's capital allocation strategy, which we discuss in the next section.

III. Capital allocation

Allocating factors of production is a key concern for any company. Moreover, in order to protect the currency and the financial market, which are public goods, the regulatory authorities in charge of banks and insurance companies have imposed capital adequacy requirements. For many years, the French financial industry (banks and insurance companies alike) was under the wing of the state and was therefore less concerned with return on equity than other sectors of the economy, more exposed to foreign competition. But with privatisation, gradual deregulation and the prospect of a unified market underpinned by a single currency, France's bankers and insurers have become acutely aware of the need for capital allocation strategies and measures of ROE.

We intend to look at the principle of capital allocation, by which we mean the process whereby a bank apportions its equity among different business units. Such an approach also makes it possible to gauge the level of profitability that each unit must achieve in order to ensure the optimum business structure. We will discuss the underlying principles and compute, on the basis of simple cases, the resulting allocations. This will be done for didactic purposes, using a loan portfolio, and also to assess the advisability for a commercial bank of engaging in maturity transformation. We will conclude by discussing the difficulties and the limits of this method.

III.1 Optimising a business portfolio

The basic idea is simple: commercial banks, universal banks and financial holding companies have to manage portfolios of business activities. Since managers seek to satisfy the demands of private shareholders, it is only natural that they should use classic portfolio optimisation methods. For example, the aim of a particular allocation structure will be to maximise profit for a given level of risk. Our first task is to define the requisite level of profitability for each line of business.

The profitability of an activity is the net income from activity $I$ divided by the
capital $C$ allocated to it over a given period of time (one quarter, one year or more).

$$r = \frac{I}{C} \quad \text{(III.1)}$$

Net income is equal to total income minus capital losses and expenditure. Expenditures include: interest expense, overheads and outlays on plant and equipment (property, IT systems, etc.). In the case of a loan, the rate of return is equivalent to the net interest rate minus current expenditure due to the loan, minus capital expenditure, minus the default rate for the life of the loan.

Now, assuming the presence of competition and random economic factors, we will assume that the rates of return of the different lines of business are random variables. We will assume, for example, that they are Gaussian multivariate. Consequently, they are wholly determined by their mean and their variance-covariance matrices.

Under these circumstances, the average return on a loan will be:

\[
\text{(average interest rate - current and capital expenditure - average default rate) / capital requirement}
\]

What are the risks? Under reasonable assumptions, randomness can stem only from changes in interest rates and the default rate. The covariance between two lines of business is simply a measure of the random factors common to both.

For example, in the case of a loan to two different borrowers, we will consider the covariances of the interest rates and of the default rate. If there is no interest rate risk and if the margin is stable (i.e. no variation due to competition on the margin), the only relevant factor will be the covariance between the default rates of the two borrowers. Here, that covariance is equal to the standard deviations of the borrowers' default rates multiplied by their default correlation coefficients.

Consider a bank with $N$ lines of business. We denote:

\[
\bar{r}_i \quad \text{average profitability of business } i \\
( i=1, \ldots, N ) \\
\sigma_{ij} \quad \text{covariance of businesses } i \text{ and } j \\
( \sigma_{ii} = \sigma_{jj} \text{ is the variance of business } i ) \\
\alpha_i \quad \text{proportion of total capital allocated to business } i.
\]

Under these circumstances, the optimal allocation in the case of risk-averse management is obtained by solving the maximisation of the utility:

\[
\text{Max} \sum \alpha_i \bar{r}_i - \frac{a}{2} \sum \alpha_i \alpha_j \sigma_{ij} \quad \text{(III.2)}
\]

\[
\sum \alpha_i = 1 \text{ et } \alpha_i \geq 0
\]

where $a$ is the coefficient of the manager's aversion to risk. If we do not know this risk aversion, but we do know the maximum level of risk, then $a$ represents the coefficient of the constraint (maximal risk) in the Lagrangian.

This optimal allocation principle corresponds to Markowitz's theory, discussed in *Quants* No. 2. In practice, the indicator we are seeking to optimise is the expected return on a portfolio of businesses minus a cost corresponding to the uncertainty of the return. That cost stems from two factors, one objective (the variability of the results), the other subjective and specific to the shareholder, measuring his or her fear of uncertainty.
III.2 One bank, two lines of business

We will illustrate this approach using a simplified example of a bank with two lending activities. We will assume that the two business units in question lend to large companies, considered to be reliable, and to smaller firms, which are more risk-prone. The loans made to large companies carry a margin of 80 basis points (net of default), reflecting a higher level of safety than small-company loans, which have a margin of 120 bp. Conversely, the uncertainty on the net margin is higher for smaller companies— we have assumed a normal distribution (implying a well diversified portfolio) with a standard deviation of 1.6%— whereas the standard deviation for the large companies is assumed to be two times smaller (0.8%). We have also assumed that the two uncertainties are decorrelated. From the standpoint of the present Cooke ratio, there is no difference between the two types of loan, which both require a capital adequacy ratio of 8%.

When establishing the ratio of margins to equity, we deduce from these assumptions that the returns to the two lines of business are characterised by:

\[ r_1 = 10\% \quad \sigma_1 = 10\% \]
\[ r_2 = 15\% \quad \sigma_2 = 20\% \]
\[ \rho = 0 \]

The solution to the problem of maximisation, presented in III.1, is expressed by a proportion \( \alpha \) invested in large-company loans such that:

\[ \alpha = \frac{(r_1 - r_2 + \alpha(\sigma_1^2 - \rho \sigma_1 \sigma_2))}{a(\sigma_1^2 + \sigma_2^2 - 2 \rho \sigma_1 \sigma_2)} \]  

\[ (III.3) \]

\[ a = 0.8 - 1/a \]  

(III.4)

Thus, for a low level of risk aversion, i.e. 2, 30% of the capital is invested in loans to large companies and 70% in loans to small ones. If risk aversion is high (we will assume it to be infinite) then 80% is invested in the first line of business. For the two businesses respectively, the return on capital invested is 13.5% and 11% while the volatility of the results is 14.3% and 8.9%. In the second case, diversification (the portfolio effect) made it possible to achieve an activity with an overall level of profit higher than that earned from the first business alone, with a lower level of risk. To that end, 20% of the portfolio consisted of loans to small companies.

III.3 Applications: Should commercial banks practise transformation?

The economic principle of capital allocation will now be applied to an issue of concern to retail and investment banks. We will look at the attraction of maturity transformation for a bank that has earned a certain margin on its loan. To do this, we will analyse the relative appeal of interest rate risk and credit risk.

Interest rate risk is often considered to be superfluous in a commercial bank's conventional lending business. Using the results obtained in Part II, we will try to optimise a commercial bank's exposure to interest rate risk. We have assumed that the capital of our bank exceeds the mandatory level imposed by the Cooke ratio in view of the loans it has extended. The margins the bank earns on its credit business are exposed to default risk, modelled in the same way as in example III.2. We are interested in the refinancing aspect: should the bank borrow in the market, matching the maturities of its loans with those of its
borrowings, or should it transform the maturities? And if so, in what proportions?

This yield curve model has been simplified deliberately. We assume that the curve remains positive with a slope $dS_m$ (see Figure III.1) and that the short-term rate is random, normally distributed with mean $i$ (its initial value) and a standard deviation $\sigma_i / S_m$. $S_m$ is merely a point of reference on the curve. A difference (gap) in the duration of the assets and liabilities produces a gain related to the slope of the curve. However, it also carries an interest rate risk that we can identify by means of the current values of the asset and liability streams. We are therefore dealing with an economic approach to maturity transformation, not an accounting approach.

Denoting the nominal amount of the assets as $A$, the nominal amount of the liabilities as $B$, shareholders’ equity as $C=A-B$ and their actuarial sensitivities as $S_A$, $S_B$ and $S_C$, we have:

$$S_c = (S_A - S_B) / (A - B) \quad (III.5)$$

The return on capital $C$ is the difference between the income stream from the assets and the costs, that is to say, in terms of income:

$$A(i + dS_A / S_m + m) - AS_A \Delta i - A \Delta m \quad (III.6)$$

where $m$ is the margin, $\Delta i$ the (random) variation of short rates and $\Delta m$ the (random) variation of the margin, which is assumed to have zero mean and standard deviation of $\sigma_m$.

And in terms of cost:

$$B(i + dS_B / S_m) - BS_B \Delta i + E \quad (III.7)$$

where $E$ is the overhead, which for expositional ease we will not take into account. Hence the return is computed as:

$$r = i + dS_C / S_m + (mA / C - S_c \Delta i - (A / C) \Delta m) \quad (III.8)$$

The terms can be easily interpreted. The margin on the loan is enhanced by the leveraged impact $A/C$ (in terms of return and risk) and the payoff from the transformation is proportional to $S_C / S_m$ with an interest rate risk proportional to $S_C$. Consequently, the sensitivities to interest rate risk and credit risk are $A/C$ and $S_C$. We assume them to be decorrelated.

On average:

$$\bar{r} = i + dS_C / S_m + mA / C \quad (III.9)$$

and the volatility $\sigma$ of the results is such that:

$$\sigma^2 = (A / C)^2 \sigma_m^2 + (S / S_m)^2 \sigma_c^2 \quad (III.10)$$

Lastly, we assume that there are regulatory constraints affecting credit risk and "structural" interest rate risk (not currently the case). Hence:
The problem remains the maximisation of $\bar{r} - 0.5a \sigma^2$. The solution can be found in Boulier (1994). When $t$ is zero (no capital for the structural risk), the optimal sensitivity of the equity is:

$$S_C^* = S_m d / a \sigma^2.$$

That sensitivity corresponds to the situation where the capital is invested entirely in a portfolio with sensitivity $S_c$. Only when the capital is "rationed" will a comparison between risks and returns come into play.

Let us look at some numerical values:

- $d = 1\%$, $\sigma_d = 7\%$, $S_m = 7$ (corresponding to an annualised standard deviation of 1% for $d$),
- $m = 1\%$, $\sigma_m = 0.5\%$.

$b = 8\%$ and $t = 8\%$ (the first corresponding to Cooke risks with a 100% weighting, the second corresponding broadly to two times the equity requirement calculated as a market risk according to BIS standards).

Table III.1 supplies optimal values for the sensitivity of equity $S_C$ as a function of the level of equity $C$ expressed as a percentage of assets (lines) and aversion to risk $a$ (columns).

As we can deduce, it is mainly $a$ that influences our exposure to interest rate risk. If $a$ is low, then sensitivity to interest rates is strong. Conversely, if $a$ is high, then interest rate risk is negligible. Note, however, that $S_c = 6.3$ corresponds to a very small difference $S_B - S_d = 0.13$ for $S_d = 5$ yrs (i.e. an asset with a maturity of around 12 years for loans with constant monthly repayments).

<table>
<thead>
<tr>
<th>$a$</th>
<th>0.5</th>
<th>1</th>
<th>2.5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_C$</td>
<td>6.3</td>
<td>6.4</td>
<td>6.5</td>
<td>6.6</td>
</tr>
</tbody>
</table>

The substitution effect between interest rate risk and credit risk is clearly in evidence in Table II.2, where we have varied the parameter $\sigma_m$, the uncertainty of the credit margin. For a strong aversion ($a = 5$), the exposure $S_C$ increases significantly with $\sigma_m$. We would have obtained the same result (from a qualitative perspective) if $m$ had decreased or if $d$ had increased. In periods when the yield curve is steeply sloped and visibility on interest rates is good, the bank is encouraged to increase its reliance on maturity transformation.

<table>
<thead>
<tr>
<th>$\sigma_m$ (%)</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>6.2</td>
<td>6.2</td>
<td>6.5</td>
<td>6.5</td>
<td>7.1</td>
</tr>
</tbody>
</table>

### III.4 Implementation and criticisms

The capital allocation process described above involves several types of difficulty. We will not go into detail regarding implementation *stricto sensu*; instead, we will try to identify the most important points and then to take a more critical look at the process.
a) Difficulties of implementation

The economic approach we adopted was, from the beginning, outside the field of accountancy. The task of reconciling the two fields can prove difficult, for several reasons. First, the perception of interest rate varies according to whether assets are marked to market. Second, it seems necessary to allocate default provisions for each entity. In France, this is generally done ex post, which reduces the visibility of economic results. Lastly (although this is not particular to the problem we are describing), we must consider the accounting mechanisms used to amortise goodwill on new acquisitions as well as any exceptional profit (or loss) generated when a business is discontinued. This is important inasmuch as all these items are relevant to an analysis of return on assets.

The second type of difficulty stems from information and communication. Allocation is based on the outlook for profitability and risk. Practitioners of the Markowitz optimisation technique know that results are sensitive to inputs. This constitutes an important issue, to which operational management must make as objective a contribution as possible. This is almost certainly easier in a mature line of business, where margins are stable. However, in today’s highly competitive environment, such situations are infrequent. Much skill is required to get across the underlying logic and to acquire and process relevant information (fuzzy calculus, discussed in Quants No. 23, can be useful for this). However, the figures are unlikely to be extremely precise. True, frequent monitoring and analysis of the (economic) results of the businesses can help. But, like all information relating to the past, it is not enough because the allocation is naturally be made ex ante and not ex post.

The third type of difficulty stems from employee incentive and compensation schemes. This is best exemplified by the bonuses paid to trading room staff. These are clearly useful, but they can lead to a markedly different application of the bank’s choice. The same problems doubtless apply to compensation based on credits "sold". First, the cost of such compensation is not always taken into account: if the managers of a subsidiary receive stock options, for example, how does the parent company calculate this cost? Such sums can in some cases be identified – witness the case of Microsoft, which has declared an amount corresponding to nearly 10% of its market capitalisation!

b) Criticisms

A number of criticisms can be levelled at this approach. From an economic viewpoint, at least three objections are germane.

First, is there any point in considering that the bank is risk-averse and must operate a risk management policy risk that is consistent with its objectives? As theory shows, maximising shareholder value involves maximising the risks taken by the company. In contrast, the regulatory or market authorities restrict or penalise risk-taking to some extent. Answering that criticism does not come within the ambit of this study. It is nevertheless a fact that a growing number of bank chairmen are trying to limit earnings volatility.

Another critical question involves the investment horizon. Should a bank reject a new line of business on the grounds that it will take three years to reach maturity? On what joint horizon can shareholders’ equity be compared? The question also arises when determining the risk-aversion
coefficient, the importance of which was seen in III.2 and III.3.

Lastly, there are bound to be quasi-monopolistic situations in which the approach would no longer be appropriate because profitability can increase until such time as a competitor arrives.

Under these circumstances, the nature of the risk differs enormously. The same remark applies in the case of cartelisation. When determining strategy in such situations, it is necessary to consider the way in which the other party reacts to the bank's decision. This, too, lies outside the scope of our study.

III.5 Pertinent findings

With competitive pressures encouraging practitioners to take risks, which are the sole source of performance, it is necessary to find a discipline to govern risk-taking. We have just presented a rational method for achieving that objective, a method well-known to investors because it forms the underpinnings of business portfolio optimisation.

However, adapting it to the context of capital allocation raises a number of difficulties - accounting practices, acquisition of information on choices, criteria, etc. - which must be dealt with by management.

The most pertinent findings of our simplified examples are three-fold: that diversification (in a known business area) is beneficial, that the activity mix changes depending on the market (e.g. margin, credit risk, yield curve) and that the skill of the workforce (ability to profit from market anomalies, to screen borrowers, etc.) must be taken into account. While this may seem obvious, an approach such as ours makes it possible to pin numbers on what would otherwise be an intuitive understanding.

IV. Sensitivity of a life insurer's earnings to asset allocation

While the above considerations may help to answer questions of strategy, it must not be forgotten that, on a day-to-day basis, ALM depends heavily on the regulatory framework and accounting mechanisms. To assess the impact of these two factors, we will analyse the balance sheet and P&L account of a life insurance company in order to determine how the percentage of equities in the asset portfolio can affect profitability in the short and long term. To do so, we must take into account the regulatory and accounting constraints that affect the choice of asset allocation. However, our aim is not to model French insurance accounting mechanisms as a whole, but only those that impact on the asset allocation strategy. In this part, we present the initial results obtained from the simulation tool used by the Financial Engineering Department of CCF Gestion.

For savers, the attraction of life insurance products - tax-advantaged, flexible and profitable - remained undimmed for many years. In 1994, they generated total premium income of FF331bn. Today, with the decline in bond yields, returns on life-insurance media are less attractive and insurance companies are now looking for ways of making up for loss of yield. They are motivated by the fact that a decline in returns on life insurance products will, in the medium to long term, result in an outflow of funds. A number of industry professionals are now taking a cautious interest in the equity market. In France, equities have traditionally earned much higher returns than fixed-income
instruments (Figure IV.1). Moreover, current economic conditions suggest that the yield gap is likely to widen still further in the coming years.

![Yields on equities and long bonds in France, 1980 to 1995](image)

Figure IV.1: Yields on equities and long bonds in France, 1980 to 1995

After a brief look at insurance companies' current portfolios, we will examine the accounting constraints that govern investment choices. We will also consider the security mechanisms required by the regulatory authorities in order to smooth the profits distributed both to policyholders and insurers. In the last section, we present the results of our simulations.

IV.1 Life insurers' portfolios: current situation in France

a) Structure of insurance company investments

Looking at the insurance industry (life and non-life) in France, we observe that bond holdings have risen sharply since 1980 and now account for between 50% and 60% of total investments. Conversely, property investments have been on a relative downtrend, dropping from 20% to 10% between 1980 and 1993. During the same period, the proportion of equities has hovered at around 20%.

Regarding the asset allocation of life insurers alone, bond holdings are larger (64% in 1993). Non-life insurance, in contrast, has an equivalent proportion of equities (37%) and bonds (38%) whereas property accounts for just 14% of the portfolio.

This segmentation can be fine-tuned still further according to the age of the insurance company. Older companies have substantial reserves (a profit participation reserve and a special depreciation reserve, the objectives of which are discussed later on), which give them greater latitude to invest in high-yielding media such as equities without compromising returns to policyholders. Conversely, younger companies will have a larger proportion of fixed-income instruments because they have to ensure sustainable returns throughout the life of their policies.

A technical and legal conception of the insurance business still prevails, even today, and this tends to relegate investment activities to second place. Moreover, the state intervenes in different ways, influencing insurers' investment policies with a view to ensuring guaranteed financing. Recently, however, the context has changed. And, with the development of cross-border service provision, insurers now have to add a financial dimension to their competitiveness strategies.

Within the European Union, regulations have changed radically in the past few years. Our aim is not to examine the content of the new framework; merely to stress that, owing to regulatory constraints, insurers now have greater freedom.
b) Causes of overweighting of fixed-income investments

There are several reasons for the overweighting of fixed-income products. The main ones are outlined below:

- **Treasury management:** assets and liabilities are easier to match when the portfolio is invested in bonds. Also, there has been strong demand in recent years for guaranteed-return contracts. These are designed around fixed-rate bonds with the same maturity as the contracts themselves because the future cashflows correspond to defined amounts.

- **Flexibility:** policyholders can terminate their contracts and pocket the proceeds at any time. This situation is easier to manage through fixed-income investments. For this reason, life insurers limit their investments in volatile instruments.

- **Accounting mechanisms:** the absence of provisions for unrealised capital losses on bonds, negotiable debt instruments, loans and time deposits (an accounting mechanism that we will describe later on) encourages insurers to opt for this type of investment. This is particularly true for younger companies, which have virtually no unrealised capital gains.

c) Consequences of overweighting

The long-term yields of insurers investing heavily in the bond markets could decline because equities outperform fixed-income investments in the long run. Moreover, with the recent downtrend in interest rates, bond yields will become less and less attractive.

For these reasons, insurance companies must identify precisely the proportion of equities that they can hold (while respecting the interests of their shareholders and insureds) in view of the prevailing regulatory and accounting regulations.

IV.2 Accounting mechanisms for appropriation of earnings

We will attempt to describe simply the regulatory and accounting framework within which French life insurers conduct their financial management. The accounting and underwriting rules are restrictive to the point that they constitute the main factor influencing asset allocation. For example, current accounting practices require that insurance companies set aside a provision for unrealised losses on equities, but there is no equivalent requirement for bonds. This rule clearly favours fixed-income investments. Consequently, it is vital for fund managers working on behalf of insurance companies to be aware of the relevant accounting mechanisms in order to make the appropriate asset allocation.

a) Insurers' regulatory commitments to policyholders

Life insurance contracts fall into two broad categories: those denominated in francs, which offer guaranteed returns, and those that are unit-linked; which in general do not contain a guaranteed revaluation clause. In this study, we will concentrate solely on the first type of contract, which have attracted higher levels of outstandings.

Life insurance contracts in French francs provide a benefit (capital or annuity) with a fixed term in return for a premium. This benefit is computed by means of a technical interest rate. Owing to the dual impact of regulatory pressures and intense competition, companies are seeking to offer policyholders more than just the guaranteed
minimum yield (which, moreover, is capped by the regulations currently in force). This extra return is not simply a plus for policyholders; it is also a key factor in growing the business: policyholders are the first to benefit from higher yields, which will be used to revalue their investment.

An insurer's liabilities are primarily composed of mathematical reserves (MR), which represent the contracts that bind the company to its policyholders. Every year, these contracts are revalued on the basis of the minimum guaranteed rate, at least.

In terms of commitments, the insurer offers its policyholders a minimum return composed of two elements: a minimum guaranteed rate and a share of the company's profits. In France, the minimum rate is capped by law in order to protect both sides from unrealistic promises. For contacts in excess of eight years, the rate cannot exceed a predetermined level (in 1995, this was 3.5% or 60% of the yield on government paper, or TME). Profit sharing is mandatory: an insurance company is required to pay out to policyholders at least 85% of the financial income generated during the year (coupons, dividend and capital gains). This Profit Share (PS) is immediately credited to policyholders and is then transferred to Mathematical Reserves. It can also be added to reserves by adding to the Profit Participation Reserve (PPR).

The regulations also impose solvency requirements to ensure that the insurer adopts a responsible approach to management. The aim of the solvency margin is to require insurance companies to hold a minimum level of wealth, proportional to their business volume, in order to cope with contingencies or with an unexpected shortfall in underwriting reserves. The margin is computed on the basis of shareholders' equity, the Réserve de Capitalisation (i.e. the provision for unrealised capital loss exposures) and potential capital gains. It must be equivalent to 4% of MR for franc-denominated contracts and 1% for unit-linked contracts.

b) Provisions used to smooth earnings

To help insurers to manage their appropriation of earnings, the regulatory framework depends primarily on two notions: the Réserve de Capitalisation and the Profit Participation Reserve.

- Réserve de Capitalisation

The purpose of this reserve is to provide for the depreciation of an insurance company's assets and a decline in its income. It covers depreciable transferable securities that come within the scope of Article R332-19 of the French Insurance Code, i.e. securities that will be redeemed at a certain value and that generate a fixed level of income. They are valued at their acquisition price. When such securities are sold, the reserve is adjusted so that the rate of return is equivalent to that expected when they were acquired.

Pro memoria, the Réserve de Capitalisation at end 1995 represented between 0.5% and 1% of provisions, and may account for as much as 3% of MR.

- Profit Participation Reserve

The other balance sheet item that can be used to smooth earnings is the PPR, (which in volume terms is larger than the Réserve de Capitalisation). It is used by most insurers that do not distribute the bulk of their financial income each year.
Accordingly, they allocate the portion of undistributed Profit Share to the PPR. Broadly, the PPR is topped up either by capital gains on equities or whenever the returns on the overall portfolio are sharply higher than the company's or the market's expectations. The reserve forms a sort of safety cushion, which is used when adverse market conditions prevent the insurer from revaluing policyholders' contracts at the agreed rate. Notwithstanding this arrangement, each insurance company has considerable latitude when setting up its PPR. The only legal constraint is that the contents of the reserve must be redistributed to policyholders within eight years.

*Pro memoria*, the PPR at end 1995 represented between 1% and 3% of MR. It can be used to offset the effects of a decline in value of riskier assets such as equities.

c) Mechanism of apportioning earnings between insurers and policyholders

An insurance company can fulfil its role only to the extent that policyholders entrust it with their funds. For this reason, both its underwriting results (i.e. results related to output, including commissions and overheads) and its financial results should logically be apportioned between the company and its policyholders on an equitable basis. Further, life insurers are required by law to allocate to policyholders a minimum portion of the financial income from investments.

If companies have considerable latitude in allocating the amount remaining once the minimum guaranteed rate has been paid to policyholders, then they can use the PPR to amortise capital losses on equities. Conversely, it can be topped up solely with part of the potential gains realised on the equity market.

IV.3 Simulation assumptions

After describing briefly the reasons that prompted us to develop a simulation tool, we will present the principal assumptions and operating rules of our model. We then go on to describe the accounting behaviour of an insurance company over a 15-year period with two scenarios for the equity and bonds markets (portfolio ageing analysis). We conclude by commenting on how the level of equity investment impacts the policy revaluation rate (the policyholder's gains) and the solvency margin (the insurer's perspective) in view of the specific features of life insurance accounting practices.

a) The need to develop a specific model

A number of asset allocation techniques exist, but none of them takes into consideration the specific accounting mechanisms of the life insurance industry. Two examples that spring to mind are the Markowitz portfolio optimisation method and the use of shortfall constraints associated with a minimal rate of return or a given level of surplus risk. Useful as they may be, these techniques cannot encompass the full range of accounting constraints. Further, they do not allow us to simulate the way in which portfolios change over time. For these reasons, it is necessary to develop a model that takes all these aspects into account.
b) Financial and accounting mechanisms

When introducing equities into a portfolio, two factors must be borne in mind: first, equities are much more volatile than bonds; second, insurance companies' accounting principles favour bonds over stocks insofar as provisions must be set up for capital losses on equity investments. Figure IV.2 shows the principal flows involved in the apportionment of financial income. The underlying mechanisms will be described in the paragraph below.

![Figure IV.2: Distribution of financial income](image)


c) Commercial assumptions

- Commercial features of contracts

The contracts in question are single-premium policies with a maturity of ten years. We have assumed that investors do not redeem their policies before they mature and that they liquidate them after ten years. This may seem somewhat simplistic, for two reasons. First, we are dealing with one specific type of contract only; however, it would be possible to apply the model to all the contracts offered by the company and then to reconstitute a global view of the balance sheet by a linear combination of its constituent parts. Second, our simulation does not so far take into account the risk of early redemption. However, we do have the means to simulate commercial scenarios involving subscriptions and redemptions influenced by bond market trends.

- Loading policy

Policyholder inflows and outflows are effected at year's end, and a constant loading commission (e.g. 5%) is added to inflows. We will ignore commissions on outstandings because they have no fundamental influence on the results generated by the model.

d) Financial assumptions

- Type of investment

We consider that the portfolio is composed solely of shares and bonds. For the bond portion, the investment universe comprises fixed-rate instruments with a 10-yr maturity and annual coupons. At any given moment, the portfolio will contain 10 bond issues purchased at par, each with a maturity corresponding to one of the ten subsequent years. The equity portion of the investment will track an index such as the SBF 120. It will be simulated by a normally distributed expected return of 9% and volatility of 15%. The 10-yr bond yields are simulated by means of a random variable resulting from a return-to-the-mean model (Orstein-Uhlenbeck) based on the French market.

It is also possible to correlate the behaviour of equities and bonds with a correlation coefficient. Lastly, we have assumed that investments and disinvestments are made at year's end.
e) Accounting assumptions

- Global structure of balance sheet and P&L account

![Figure IV.3: Simplified balance sheet of an insurance company](image1)

![Figure IV.4: Simplified P&L a/c of an insurance company](image2)

Shares and bonds are carried on the balance sheet at cost. It is therefore important to model the accounting portfolio and the financial portfolio simultaneously.

The balance sheet and P&L shown in Figures IV.3 and IV.4 have been simplified, but they nevertheless reflect the basic accounting mechanisms.

Our assumption for return on equity is as follows: the assets representing shareholders' equity will be invested on the basis of the same policy as those representing MR.

- Distribution policy

For portfolios invested solely in bonds, we will adopt a policy of immediate and maximum payout of financial income to policyholders. In contrast, whenever a portfolio contains stocks, we will use the PPR as a cushion to smooth out any fluctuations in the equity market. To some extent, we will use the PPR in the same way as the Réserve de Capitalisation is used to smooth income streams from bond investments.

- Minimum guaranteed rate

Part of the financial income will be used to revalue in-force contracts (allocation to MR). Thus the revaluation rate used for the MR can be defined as the ratio of financial income to MR. In addition to being revalued on the basis of this rate, the MR will be increased to reflect the value of new contracts signed during the year and decreased by the value of contract outflows. Note also that these outflows correspond to the inflows recorded ten years earlier, capitalised each year at the revaluation rate over the past ten years.

In the event that the company's financial income is not sufficient to pay the guaranteed minimum rate (set at 3.5% in the model), the amount needed to make up the difference will be drawn on the PPR. When the amount available in the PPR is zero or insufficient to allow the desired writeback, then the equity reserves will be used.
• **P&L income streams**
  
  - *financial income from bonds and equities*: coupons and dividend
  
  - *capital gains and losses realised on equities in order to allow for portfolio readjustment*: recall that our investment strategy is to hold a constant percentage of shares in the financial portfolio or the accounting portfolio.
  
  - *capital gains on equities generated on potential gains or losses that we want to realise when the equity markets perform in line with our expectations* (i.e. when the annual returns to the equity market are higher than the expected return parameterised in the model). We will designate the rate of realisation by $K$ ($0 < K < 1$).
  
  - *new inflows during the course of the year.*

• **P&L expenses**

  - *outflows corresponding to inflows of contracts ten years earlier.* This corresponds to the value of the contracts revalued each year according to actual profit.

  - *profit participation reserve*: This allocation will be incremented by a portion of the capital gains realised during the period. We can define a coefficient ($K'$) representing those realised gains that are capitalised immediately and those that are capitalised at a later date. The amount [$K' \times$ realised capital gains] will be factored into the PPR the following year and will thus form a safety cushion. If necessary, a capital loss on equities can be made up by reversing a provision from the PPR and allocating it to the MR as per the minimum guaranteed rate.

  - *mathematical reserves*: an allocation is made to the MR in order to increase them by the revaluation rate. That allocation represents the difference between the MR for the present year and the MR of the previous year.

  - *... and a number of simplifications*: We will ignore the *Réserve de Capitalisation* as well as underwriting income and charges.

**IV.4 First simulation: portfolio with a 10% equity element**

When an equity element is introduced into a portfolio, it tends to decrease the level of regular income while at the same time increasing the income related to capital gains in the equity market.

We will analyse the ageing of our accounting portfolio under two scenarios in order to illustrate the importance of ensuring that a company's allocation of equities is proportional to the level of its reserves (including the PPR). In our example, we have invested 10% in equities. Taking a scenario based on equity-market performance over the next fifteen years, we will examine the evolution of the reserves at our disposal.

a) **Bullish scenario**

Consider a pattern over the next fifteen years where the equity markets are bullish at first, then become more volatile before moving onto a steep downtrend at the end of the period. We will look at the case of a company with no reserves or PPR in year 0.
Owing to the equity market run-up, the gains realised on equities and not distributed to policyholders can be capitalised in the PPR, making it possible to build substantial reserves. During the last two years of the downtrend, provisions must be set aside for capital losses on equities. In this case, it will be necessary to draw on the PPR — and then, if necessary, on the reserves — an amount that will allow the company to pay the minimum guaranteed rate to its policyholders (see Figure IV.6). That same amount was used to offset the provision made for capital losses on equities in years 14 and 15!

The insurer increases the rate paid on savings because the equity element helps to increase reserves (cumulative earnings not paid out to policyholders). Further, the PPR has acted as a shock absorber in the event of capital losses on equities. Looking at the P&L, we see how the company built up a safety cushion during the bull run and then used it during the lean years.

The profit smoothing method thus allows the company to cope with capital losses on equities. This is reflected in changes in the reserves on the balance sheet and financial income on the P&L.

**b) Bearish scenario**

Consider a bleak scenario in which the equity markets perform poorly for 15 years.

To meet its contractual commitments, an insurer will have to draw regularly on its PPR and then on its reserves before resorting to a capital injection from shareholders. Since the PPR and the equity reserve are both nil at the beginning of the period, recapitalisation is inevitable.

Consequently, the company needs an equity investment policy proportionate to its existing reserves. By analogy, the portfolio
manager will think of the constant proportion portfolio insurance method (or CPPI, a portfolio insurance technique). Or, by transposition, the PPR can be considered as the cushion.

We will now consider two investment policies.

- **Scenario with bond portfolio only**

The bond-only scenario is the least risky of the two, and its returns stem solely from regular income streams (cf our model).

- **Scenario with a constant equity element in the financial portfolio**

A riskier scenario, from a financial and accounting perspective, involves holding a constant proportion of equities during the period under review. This strategy should generate higher returns than the bond-only scenario: even though the regular income stream declines in relative terms when bonds are replaced by equities, it is nevertheless possible to realise substantial capital gains on share disposals, which in turn will easily offset the drop in regular income.

a) **The policyholder's perspective**

Figure IV.10 plots the changes in the average revaluation rate computed over the next 15 years. For each level of equity allocation, we analyse the results corresponding to 2,000 randomly generated scenarios for the equity and bond markets (Monte Carlo method). In our model, we chose to apply the market scenarios in exactly the same way to both investment strategies (viz. the all-bond and the equity-element portfolios). This choice was necessary in order to allow us to compare the accounting result. The horizontal axis of Figure IV.10 represents the percentage of equities in the portfolio according to the chosen scenario. The vertical axis represents the average revaluation rate used.
for the MR over 15 years. These quantities have been computed with an expected return to equities of 9%, i.e. a relatively small risk premium in view of the average level of long rates (6.7%). The results are consistent with theory: they simply illustrate the fact that, in the long run, equities offer higher returns than bonds.

Figure IV.10: Average revaluation of MR according to percentage of equities in the portfolio (over 15 yrs)

b) The insurer's perspective

The insurer has to weigh up the impact of asset allocation decisions on its solvency margin (i.e. shareholders' equity plus unrealised capital gains as a percentage of MR). Figure IV.11 shows how the solvency margin changes according to the percentage of equities in the portfolio.

Interestingly, once a certain percentage of equities has been reached (around 10%), unrealised capital gains no longer help to increase the solvency margin. This seems logical: on one hand, the MR revaluation rate increases in line with the percentage of equities; and on the other hand, the total of reserves and unrealised gains increases in nonlinear fashion. This can be explained by the fact that some of the financial income goes to policyholders (MR revaluation), some to the Internal Revenue (tax) and some to the insurer (dividend). As a result, reserves and unrealised gains increase rapidly at first and then less quickly thereafter, which explains this local optimum.

The size of the equity investment corresponding to this optimum value is closely related to the company's payout policy. In our case, the positioning of that optimum is determined by the coefficients $K$ and $K'$ (respectively, the realisation of potential gains and the apportionment of gains between MR and PPR). The model used by CCF Gestion allows $K$ and $K'$ to be dynamically managed (depending on the percentage of equities) in order to optimise the distribution of equity gains. It is therefore possible for portfolios with an equity element of more than 10% to choose an earnings appropriation method that will be satisfactory to insurers and policyholders alike.

Figure IV.11: Solvency margin as a function of the equity element (over 15 yrs)

IV.6 Investing in equities

Should the percentage of equities be increased for contracts denominated in French francs? To answer that question, we need precise rules for appropriating income and managing reserves in a manner consistent with the chosen investment strategy. Based on the rules and assumptions we have adopted, our study
shows that, whereas an equity allocation allows a substantial increase in earnings for both the insurer and the policyholder, the returns to both parties can be optimised by dynamically managing the appropriation of unrealised capital gains. Consequently, an ALM simulation tool can be used to integrate both commercial and financial scenarios with a view to defining tactical allocation choices based on strategic allocation. One complementary aspect that bears examination is the transposition of the portfolio insurance method (the constant proportion method) for use by insurance companies in the field of asset allocation. The results look promising.

Conclusion

ALM is fashionable, and its popularity is a good sign. Given the pressing need for many financial institutions to evolve, and the importance of the ALM function for the corporate sector, it was clearly necessary to establish teams of trained professionals. Nevertheless, ALM is no panacea.

ALM practitioners generally agree on analyses and diagnostics. They also share the same arsenal of models and financial methods, which they wield with varying degrees of sophistication. In contrast, there is no such consensus on management policies. There are two basic reasons for this divergence. First, the status of the various institutions: banks, mutual insurance companies and pension funds do not share the same goals or priorities. Second, within the banking community, policies on maturity transformation, credit and capital market activities will depend on strategies, investment horizons and the appetite for risk of the management and shareholders.

In this issue of Quants, we have sought to explain the attractions and risks of two basic strategies, maturity transformation and equity investment. We approached our subject primarily from a financial angle, while touching on the question of the accounting impact. The main lessons we learned are: (1) that the efficacy of transformation strategies is directly related to the slope of the yield curve, and that successful implementation depends on the curve's stability; (2) that the methods used by insurance companies to account for equity holdings puts a strong damper on investment; this exerts downward pressure on policy returns, which are particularly sensitive to a low interest-rate environment.

We also showed how, using a capital allocation method based on portfolio theory, it is possible to obtain a target return while limiting the volatility of a company's results.

A host of other questions warrant closer examination. Banks themselves have raised several issues – how to evaluate and manage liquidity; how to evaluate and manage sight deposits (interest-bearing and non-interest bearing); how credit risk interacts with ALM, etc. Further, ALM practitioners are now faced with new problems stemming from banking innovations, such as capped adjustable-rate loans, time deposits with incremental interest pegged to maturity, new insurance products, and new open-ended, guaranteed funds generating heterogeneous liabilities. Financial solutions can be found for many of these problems. But it requires the skill and the adaptability of ALM teams for those solutions to be translated into accounting practices or to be processed through IT systems.
Interesting questions also arise from investment banking techniques, including the management of market risk and compensation schemes for traders.

Corporate financial management, which is highly developed in modern economies, also shares certain areas of concern with ALM specialists. But only the future can tell whether this commonality will lead to new perspectives and management approaches.

With the hindsight afforded by several years' practice, it seems that ALM specialists are asking an increasing number of basic questions. What management guidelines should be adopted? Should income be smoothed? What is the optimum investment horizon? (One tongue-in-cheek suggestion is: the same as the chairman's term of office). To answer these questions, more macroeconomic and financial research would be useful. For our part, we have shown how research-driven methods can be used to solve the investment management problems of financial institutions. Much has still to be done. But in the meantime, assets and liabilities have still to be managed.
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