A Note on Life-Cycle Funds
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Agenda

Motivation

Modeling approach

Results

Conclusion
Motivation

Life-cycle funds’ assets under management have tremendously increased in recent years and are especially applied within old age provision products.

Life-cycle funds are (planned to be) set as default investment option in many defined contribution plans.

→ Life-cycle funds’ performance will have a huge impact on retirement wealth and living standard after the active working phase.

→ The risk-return profile of life-cycle funds has to be assessed appropriately for sustainable financial planning.
Motivation

Contribution

- Compare the risk-return profile of life-cycle funds to the risk-return profile of simple balanced funds

- Derive balanced funds exactly matching the risk-return profile of life-cycle funds assuming a Black-Scholes model and challenge these approximations using more sophisticated asset models
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Modeling approach

Asset models considered

- Black-Scholes (BS)
  - closed form solutions
- hybrid Cox-Ingersoll-Ross – Heston (CIR-SV)
  - Monte-Carlo approach
- hybrid Cox-Ingersoll-Ross – Heston + jumps (CIR-SVJD)
  - Monte-Carlo approach

Analyze single and regular contributions to the considered funds
Modeling approach

Life-cycle and balanced funds

- Both funds invest in equity (risky asset) and zero-bonds (riskless asset).
- Management fees reduce the funds’ performance.

Life-cycle funds

- apply a time-dependant (not path-dependant) switch from risky to riskless assets following the so-called glide path.

Balanced funds

- apply a constant mix of risky and riskless assets.
Modeling approach

Life-cycle funds’ glide paths under consideration (for numerical analyses)

– classical (A)
  • Starting with 100% investment in equity, the equity exposure is linearly decreased up to 0% equity portion in the last year.

– contrarian (B)
  • Starting with 0% investment in equity, the equity exposure is linearly increased up to 100% equity portion in the last year.

– “alternating” (C)
  • Starting with 100% investment in equity, the equity exposure is alternated on a yearly basis between 0% and 100%.
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Results – single contribution

Black-Scholes model

→ For any given life-cycle fund, there exists a balanced fund allowing
for exactly the same risk-return profile as the life-cycle fund.

→ A balanced fund stochastically dominating the life-cycle fund is
constructed using the balanced fund above but setting its
management fee equal to the life-cycle fund’s management fee.
## Results – single contribution

### CIR-SVJD model

<table>
<thead>
<tr>
<th>Strategy</th>
<th>5%</th>
<th>25%</th>
<th>Median</th>
<th>75%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balanced A</td>
<td>-3.03%</td>
<td>1.14%</td>
<td>3.89%</td>
<td>6.71%</td>
<td>10.63%</td>
</tr>
<tr>
<td>Life-cycle A</td>
<td>-3.17%</td>
<td>1.09%</td>
<td>3.96%</td>
<td>6.75%</td>
<td>10.67%</td>
</tr>
<tr>
<td>Balanced B</td>
<td>-3.03%</td>
<td>1.14%</td>
<td>3.89%</td>
<td>6.71%</td>
<td>10.63%</td>
</tr>
<tr>
<td>Life-cycle B</td>
<td>-3.15%</td>
<td>1.17%</td>
<td>3.94%</td>
<td>6.63%</td>
<td>10.48%</td>
</tr>
<tr>
<td>Balanced C</td>
<td>-4.74%</td>
<td>0.22%</td>
<td>3.52%</td>
<td>6.88%</td>
<td>11.62%</td>
</tr>
<tr>
<td>Life-cycle C</td>
<td>-4.88%</td>
<td>0.14%</td>
<td>3.54%</td>
<td>6.84%</td>
<td>11.57%</td>
</tr>
</tbody>
</table>

Statistical tests (Kolmogorov-Smirnov, Anderson-Darling) do not neglect the null hypothesis of above samples being drawn from the same original probability distribution.

*12 year single premium investment
Results – regular contribution

Black-Scholes model

→ For any given life-cycle fund, there exists a balanced fund matching the first two moments of the life-cycle fund investment.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>5%</th>
<th>25%</th>
<th>Median</th>
<th>75%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balanced A</td>
<td>1.00%</td>
<td>3.34%</td>
<td>4.97%</td>
<td>6.60%</td>
<td>9.08%</td>
</tr>
<tr>
<td>Life-cycle A</td>
<td>1.27%</td>
<td>3.34%</td>
<td>4.92%</td>
<td>6.57%</td>
<td>9.11%</td>
</tr>
<tr>
<td>Balanced B</td>
<td>-3.27%</td>
<td>1.53%</td>
<td>4.96%</td>
<td>8.45%</td>
<td>13.83%</td>
</tr>
<tr>
<td>Life-cycle B</td>
<td>-3.63%</td>
<td>1.45%</td>
<td>4.96%</td>
<td>8.52%</td>
<td>13.81%</td>
</tr>
<tr>
<td>Balanced C</td>
<td>-2.62%</td>
<td>1.64%</td>
<td>4.67%</td>
<td>7.74%</td>
<td>12.45%</td>
</tr>
<tr>
<td>Life-cycle C</td>
<td>-2.53%</td>
<td>1.65%</td>
<td>4.64%</td>
<td>7.79%</td>
<td>12.46%</td>
</tr>
</tbody>
</table>

Differences in the lower tail yield to a rejection of the null-hypothesis by the statistical tests considered.

*12 year regular (i.e. annual) premium investment
Results – regular contribution

CIR-SVJD model

→ statistical tests reject the null-hypothesis, but

*12 year regular (i.e. annual) premium investment
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Conclusion and further research

Conclusion

– Life-cycle funds’ risk-return profile can to a major part be appropriately assessed by a corresponding balanced fund
  • Supports financial planning and clients’ awareness of the risk they are taking
  • Challenges the very existence of life-cycle funds

Further research

– Extend the analysis using historical data
– Clarify the reason for the very existence of life-cycle funds
– Analyze more sophisticated (e.g. path-dependant) life-cycle strategies
Thanks for your attention

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