ANALYTICAL EVALUATION OF INSURANCE MARKET RISK

20th Int. AFIR Coll., Madrid, June 19-22, 2011
Mean and Variance of Prospective Assets

- General Formulas
- The Black-Scholes-Merton return model
- A special case

Market Risk Target Capital

- The cost-of-capital approach
- The coherent SST approach
- Life Insurance

References
Mean and Variance of Prospective Assets (1)

Multi-period discrete time stochastic model of insurance

\begin{align*}
A_t & : \text{assets at time } t \\
L_t & : \text{actuarial liabilities at time } t \\
P_{t-1} & : \text{loaded premium paid at time } t-1 \text{ (fully invested)} \\
X_t & : \text{insurance costs paid at time } t \text{ (insurance benefits, expenses and bonus payments for period (t-1,t])} \\
R_t & : \text{accumulated rate of return on investment for period (t-1,t]} \\
\end{align*}

Equation of dynamic evolution of the random assets over the time horizon [0,T]:

\[ A_t = (A_{t-1} + P_{t-1}) \cdot R_t - X_t, \quad t \in \{1,\ldots,T\} \]

In terms of the initial capital \( A(0) \) the random prospective assets at time \( T \) equals

\[ A_T = A_0 \cdot \prod_{t=1}^{T} R_t + \sum_{t=1}^{T} \{P_{t-1} \cdot R_t - X_t\} \cdot \prod_{j=t+1}^{T} R_j \]
Mean and Variance of Prospective Assets (2)

Model Assumptions  (Multivariate normal model of logarithmic returns)

(M1) The random premiums and insurance costs are independent from the returns and their means are denoted by \( \mu_{P_i} = E[P_t] \) and \( \mu_{X_i} = E[X_t] \)

(M2) The random vector of **logarithmic returns** \( Z = (Z_1, ..., Z_T) = (\ln R_1, ..., \ln R_T) \) has a **multivariate normal** distribution with characteristic function

\[
\varphi(x) = E[\exp(ix'Z)] = \exp(ix'\mu - \frac{1}{2}x'\Sigma x),
\]

\[
x' = (x_1, ..., x_T), \quad \mu' = (\mu_1, ..., \mu_T), \quad \Sigma = (\rho_{st} \sigma_s \sigma_t; 1 \leq s \leq t \leq T),
\]

**Notation:** \( e(T - t), t \in \{1, ..., T\} \), column vector containing zeros in the first \( t \) entries followed by \( T - t \) entries with ones

**Theorem 2.1.** Under the model assumptions (M1) and (M2), the mean and variance of the random assets of an insurance portfolio are given by

\[
E[A_T] = (A_0 + P_0) \cdot \exp(e(T)'\mu + \frac{1}{2}e(T)'\Sigma e(T))
\]

\[
+ \sum_{t=1}^{T-1} (\mu_{P_i} - \mu_{X_i}) \cdot \exp(e(T - t)'\mu + \frac{1}{2}e(T - t)'\Sigma e(T - t)) - \mu_{X_T}
\]
Mean and Variance of Prospective Assets (3)

\[
\begin{align*}
Var [A_T] &= (A_0 + P_0)^2 \cdot \exp(2e(T)\mu + e(T)\Sigma e(T)) \cdot (\exp(e(T)\Sigma e(T)) - 1) \\
+ & 2(A_0 + P_0) \cdot \sum_{t=1}^{T-1} \left\{ \exp\left( e(T)\mu + e(T-t)\mu + \frac{1}{2} e(T)\Sigma e(T) + \frac{1}{2} e(T-t)\Sigma e(T-t) \right) \cdot \\
& \left( \exp(\exp(e(T-t)\Sigma e(T-t)) - 1) \right) \right\} \\
+ & \sum_{t=1}^{T-1} \exp(2e(T-t)\mu + e(T-t)\Sigma e(T-t)) \cdot \left\{ \left( \mu_{P_t} - \mu_{X_t} \right)^2 \cdot \left( \exp(e(T-t)\Sigma e(T-t)) - 1 \right) \cdot \\
& + \Var [P_t - X_t] \cdot \exp(e(T-t)\Sigma e(T-t)) \right\} \\
+ & 2 \cdot \sum_{1 \leq s < t \leq T-1} \left\{ \left( \mu_{P_s} - \mu_{X_s}, \mu_{P_t} - \mu_{X_t} \right) \cdot (\exp(\exp(e(T-t)\Sigma e(T-t)) - 1) \cdot \\
& + \Cov [P_s - X_s, P_t - X_t] \cdot \exp(e(T-t)\Sigma e(T-t)) \right\} \\
- & 2(A_0 + P_0) \cdot \sum_{t=1}^{T-1} \Cov [P_t - X_t, X_T] \cdot \exp\left( e(T-t)\mu + \frac{1}{2} e(T-t)\Sigma e(T-t) \right) + \Var [X_T]
\end{align*}
\]
The Black-Scholes-Merton return model

(M1) The random premiums and insurance costs are independent from the returns

(M2) The random accumulation factors are independent and identically log-normally distributed such that

\[ Z_t = \ln\{R_t\}, t \in \{1, \ldots, T\} \]

is normally distributed with mean \( \mu \) and standard deviation \( \sigma \). The quantity

\[ r = \exp\left(\mu + \frac{1}{2} \sigma^2\right) \]

is the one-period expected accumulation factor over the time horizon \( (t-1, t) \), \( t \in \{1, \ldots, T\} \).

Corollary 2.1. Under the model assumptions (M1) and (BSM), the mean and variance of the random assets of an insurance portfolio are given by

\[
E[A_t] = r^T \cdot \left\{ A_0 + P_0 + \sum_{i=1}^{T-1} r^{-i} \cdot (\mu_{P_i} - \mu_{X_i}) \right\} - \mu_X,
\]

\[
Var[A_t] = r^{2T} \cdot \left\{ \left( A_0 + P_0 \right)^2 \cdot \left( e^{T\sigma^2} - 1 \right) + 2 \left( A_0 + P_0 \right) \cdot \sum_{i=1}^{T-1} r^{-i} \cdot (\mu_{P_i} - \mu_{X_i}) \cdot \left( e^{(T-i)\sigma^2} - 1 \right) + \sum_{i=1}^{T-1} r^{-2i} \cdot \left\{ (\mu_{P_i} - \mu_{X_i})^2 \left( e^{(T-i)\sigma^2} - 1 \right) + \text{Var}[P_i - X_i] \cdot e^{(T-i)\sigma^2} \right\} \ight. \\
\left. + 2 \cdot \sum_{1 \leq s < t \leq T-1} r^{-(s+t)} \cdot \left\{ (\mu_{P_s} - \mu_{X_s}) (\mu_{P_t} - \mu_{X_t}) \left( e^{(T-s)\sigma^2} - 1 \right) + \text{Cov}[P_s - X_s, P_t - X_t] \cdot e^{(T-s)\sigma^2} \right\} \right\}
\]

\[-2 \left( A_0 + P_0 \right) \cdot r^T \cdot \sum_{i=1}^{T-1} r^{-i} \cdot \text{Cov}[P_i - X_i, X_T] + \text{Var}[X_T] \]
Black-Scholes-Merton return model: a special case

Model assumptions:

(A1) Insurance cash-flows are valued with constant discount rate \( v_0 = r_0^{-1} \)

The quantities \( CF_{t-1} = v_0 \cdot X_t - P_{t-1} \), \( t = 1, \ldots, T \), represent the future (net) insurance cash-flows at time \( t - 1 \) over the period \( (t-1,t] \).

(A2) Insurance cash-flows and premiums are deterministic quantities (i.e. focus on market risk)

(A2) \( \Rightarrow \) insurance costs are deterministic, i.e. \( X_t = E[X_t] \)

Negative of insurance cash-flows = premium loadings:

Vector notations:

\[ \Theta_{t-1} = -CF_{t-1} = P_{t-1} - v_0 \cdot E[X_t] \], \( t = 1, \ldots, T \),

\[ P = (P_0, \ldots, P_{T-1}) \), \( \Theta = (\Theta_0, \ldots, \Theta_{T-1}) \), \( \mu_P = (0, P_1, \ldots, P_{T-1}) \), \( \mu_X = r_0 \cdot (0, P_0 - \Theta_0, \ldots, P_{T-2} - \Theta_{T-2}) \)

Present value functions:

\[ PV_T(a, r) = \sum_{t=0}^{T-1} r^{-t} \cdot a_t \]

\[ PV_T(a, r_a; b, r_b) = \sum_{0 \leq s < r \leq T-1} r_a^{-s} r_b^{-t} a_s b_t \]

\[ a = (a_0, \ldots, a_{T-1}) \]

\[ b = (b_0, \ldots, b_{T-1}) \]
Mean and Variance of Prospective Assets (6)

Mean of discounted assets:

\[ E[r^{-T} A_T] = A_0 + PV_T \left( P - \frac{r_0}{r} (P - \Theta), r \right), \]

Fair value assumption: \( r_0 = r \)

\[ E[r^{-T} A_T] = A_0 + PV_T \left( \Theta, r \right) \]

(initial capital + PV of all premium loadings)

Variance of discounted assets:

\[ Var[r^{-T} A_T] = (A_0 + P_0)^2 \cdot \left( e^{T \sigma^2} - 1 \right) + S_1 + S_2 + S_3, \]

\[ S_1 = 2(A_0 + P_0) \cdot \left[ e^{(T-1)\sigma^2} \cdot PV_T \left( e^{\sigma^2} P - \frac{r_0}{r} (P - \Theta), r e^{\sigma^2} \right) \right. \]
\[ \left. - PV_T \left( P - \frac{r_0}{r} (P - \Theta), r \right) - P_0 \cdot \left( e^{T \sigma^2} - 1 \right) \right] \]

\[ S_2 = e^{T \sigma^2} \cdot PV_T \left( (\mu - \mu_X)^2, r^2 e^{\sigma^2} \right) - PV_T \left( (\mu - \mu_X)^2, r^2 \right) \]

\[ S_3 = 2 \cdot \left( e^{T \sigma^2} \cdot PV_T \left( \mu - \mu_X, r; \mu - \mu_X, r e^{\sigma^2} \right) - PV_T \left( \mu - \mu_X, r; \mu - \mu_X, r \right) \right) \]
The Cost-of-Capital approach

\[ C_t = A_t - L_t \] : risk-bearing capital (RBC) at time \( t \)
\[ SC_t = -C_t \] : shortfall risk-bearing capital at time \( t \) (negative of RBC)

- Discounted shortfall RBC and its time period change
  \[ SC_t^d = v^t \cdot SC_t = v^t \cdot (L_t - A_t), \quad t = 1, 2, \ldots, T \]
  \[ \Delta SC_t^d = SC_t^d - SC_{t-1}^d = v^t \cdot (L_t - rL_{t-1}) - v^t \cdot (A_t - rA_{t-1}), \quad t = 1, 2, \ldots, T \]

- Economic Capital (EC) to risk measure \( R(\cdot) \), e.g. VaR / CVaR
  \[ EC := R[\Delta SC_1^d] \]

- Risk Margin (RM) / Market value Margin (MvM)
  \[ RM := i_{CoC} \cdot \sum_{t=2}^{T} R[\Delta SC_t^d] \]

- Target Capital (TC) : \( TC = EC + RM \)
Market Risk Target Capital (2)

\[ TC^{CoC/VaR} = C_0 + VaR_\alpha \left[ v \cdot (L_1 - A_1) \right] + i_{CoC} \cdot \sum_{t=2}^{T} VaR_\alpha \left[ v^t \cdot (L_t - rL_{t-1}) - v^t \cdot (A_t - rA_{t-1}) \right] \]

CVaR Target Capital = SST Target Capital \hspace{1em} (FOPI(2004/2006))

\[ TC^{CoC/CVaR} = C_0 + CVaR_\alpha \left[ v \cdot (L_1 - A_1) \right] + i_{CoC} \cdot \sum_{t=2}^{T} CVaR_\alpha \left[ v^t \cdot (L_t - rL_{t-1}) - v^t \cdot (A_t - rA_{t-1}) \right] \]

Goal: Analytical evaluation for deterministic liabilities

Justification for simplifying assumption: best- and worst-case insurance scenarios s.th.

\[ L_t^b \leq L_t \leq L_t^w, \hspace{1em} t = 1,2,...,T \]

\[ \Rightarrow L_t^b - rL_{t-1}^w \leq L_t - rL_{t-1} \leq L_t^w - rL_{t-1}^b, \hspace{1em} t = 2,3,...,T \]

\[ \Rightarrow TC^b \leq TC \leq TC^w, \hspace{1em} t = 1,2,...,T. \]
Market Risk Target Capital (3)

**Economic Capital** (= Solvency II 1\textsuperscript{st} year SCR for the market risk)

\[ EC = C_0 - E[vC_1] + R[E[vA_1] - vA_1] \]

with \[ L_1^A = E[vA_1] - vA_1 \] (1\textsuperscript{st} year asset loss)

**1\textsuperscript{st} year Solvency II SCR** (VaR measure, lognormal approx.)

\[ \text{VaR}_\alpha[L_1^A] = \rho_{\alpha}^{\text{VaR}}(\sigma A_1) \cdot (A_0 + \Theta_0), \quad \rho_{\alpha}^{\text{VaR}}(x) = 1 - \frac{\exp\{\Phi^{-1}(1-\alpha) \cdot \sqrt{\ln(1+x^2)}\}}{\sqrt{1+x^2}} \]

\[ \sigma A_1 = \left( \frac{A_0 + P_0}{A_0 + \Theta_0} \right) \cdot \sqrt{e^{\sigma^2} - 1} \]

(up to sign change & exchange of confidence level/loss probability = SCR Non-Life)

**SST market risk EC** (CVaR measure, lognormal approx.)

\[ \text{CVaR}_\alpha[L_1^A] = \rho_{\alpha}^{\text{CVaR}}(\sigma A_1) \cdot (A_0 + \Theta_0), \quad \rho_{\alpha}^{\text{CVaR}}(x) = 1 - \frac{\Phi\left(\Phi^{-1}(1-\alpha) - \sqrt{\ln(1+x^2)}\right)}{1-\alpha} \]

(up to sign change & exchange of confidence level/loss probability = SST Non-Life)
Comparison of solvency capital ratios \( \rho^*_\alpha(\sigma_A)/\sigma_A \)

<table>
<thead>
<tr>
<th>Confidence level percentile</th>
<th>VaR Method</th>
<th>CVaR Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>0.995</td>
<td>0.99612</td>
</tr>
<tr>
<td>-2.326</td>
<td>-2.576</td>
<td>-2.662</td>
</tr>
<tr>
<td>0.995</td>
<td>0.98720</td>
<td>0.99</td>
</tr>
<tr>
<td>-2.232</td>
<td>-2.326</td>
<td>-2.576</td>
</tr>
<tr>
<td>Return volatility ( \sigma )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.0%</td>
<td>2.216</td>
<td>2.438</td>
</tr>
<tr>
<td>2.194</td>
<td>2.409</td>
<td>2.482</td>
</tr>
<tr>
<td>2.183</td>
<td>2.395</td>
<td>2.468</td>
</tr>
<tr>
<td>2.172</td>
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<td>7.5%</td>
<td>2.162</td>
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<td>2.151</td>
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<tr>
<td>2.140</td>
<td>2.341</td>
<td>2.410</td>
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<tr>
<td>2.129</td>
<td>2.328</td>
<td>2.395</td>
</tr>
<tr>
<td>2.118</td>
<td>2.314</td>
<td>2.381</td>
</tr>
<tr>
<td>2.108</td>
<td>2.301</td>
<td>2.367</td>
</tr>
</tbody>
</table>

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Market Risk Target Capital (5)

**Risk Margin**

\[
R[\Delta SC^d_t] = E[v^t (rC_{t-1} - C_t)] + R[L^A_t], \quad t = 2,\ldots,T
\]

\[
L^A_t = E[v^t (A_t - rA_{t-1})] - v^t (A_t - rA_{t-1}), \quad t = 2,\ldots,T
\]

(increase of the \(t\)-th year discounted change in assets with respect to the mean)

**Solvency II Risk Margin** (VaR measure, lognormal approx.)

\[
VaR_\alpha[L^A_t] = \rho^{VaR}_\alpha (\sigma^A_t) \cdot E[v^t (A_t - rA_{t-1})], \quad \sigma^A_t = \sqrt{\frac{Var[A_t] - r^2 Var[A_{t-1}]}{E[A_t - rA_{t-1}]}} , \quad t = 2,\ldots,T
\]

**SST Risk Margin** (CVaR measure, lognormal approx.)

\[
CVaR_\alpha[L^A_t] = \rho^{CVaR}_\alpha (\sigma^A_t) \cdot E[v^t (A_t - rA_{t-1})],
\]

**Remark.** \(A_t, A_t - rA_{t-1}\) only approximately lognormal

Can consider improved comonotone approx. and log-elliptical extensions
Market Risk Target Capital (6)

SST target capital via SST risk measure

\[ TC^{SST} = C_0 + R^{SST}_\alpha [SC^d] \]

with the **SST risk measure**

\[ R^{SST}_\alpha [SC^d] := CVaR_\alpha [SC^d_1] + i_{CoC} \cdot \sum_{t=2}^{T} CVaR_\alpha [SC^d_t - SC^d_{t-1}] \]

_not a coherent_ multi-period risk measure (Filipovic and Vogelpoth(2008)):

\[ X \geq Y \quad \text{with probability one does not imply} \quad R^{SST}_\alpha [X] \geq R^{SST}_\alpha [Y] \]

Coherent SST target capital via coherent SST risk measure

\[ TC^{SST,c} = C_0 + R^{SST,c}_\alpha [SC^d] \quad \text{with the **coherent SST risk measure**} \]

\[ R^{SST,c}_\alpha [SC^d] := (1 - i_{CoC} \cdot CVaR_\alpha [SC^d_1]) + i_{CoC} \cdot CVaR_\alpha [SC^d_T] \]

\[ TC^{SST,c} = C_0 - E[vC_1] + i_{CoC} \cdot E[vC_1 - v^T C_T] \]

\[ + (1 - i_{CoC}) \cdot \rho^{CVaR}_\alpha (\sigma A_1) \cdot (A_0 + \Theta_0) + i_{CoC} \cdot \rho^{CVaR}_\alpha (\sigma A_T) \cdot (A_0 + PV_T(\Theta, r)) \]

\[ \sigma A_1 = \left( \frac{A_0 + P_0}{A_0 + \Theta_0} \right) \cdot \sqrt{e^{\sigma^2} - 1}, \quad \sigma A_T = \frac{\sqrt{(A_0 + P_0)^2 \cdot (e^{T\sigma^2} - 1) + S_1 + S_2 + S_3}}{A_0 + PV_T(\Theta, r)} \]

Coeff.'s of variation
**Market Risk Target Capital (7)**

**Example: portfolio of identical life insurance policies**

\[ \pi_0 = \pi \quad : \text{pure level premium} \]
\[ \theta_t = \theta, \; t = 0, \ldots, T - 1 \quad : \text{constant premium loading factor} \]
\[ P_{t-1} = t_{t-1} \; p_x \cdot (1 + \theta) \pi, \; t = 1, \ldots, T \quad : \text{loaded premiums} \]
\[ t \; p_x \quad : \text{survival probabilities} \]

One has \( \Theta_t = t \; p_x \cdot \theta \pi \), \( \mu_p - \mu_{x_t} = ((1 + \theta)_t \; p_x - r \cdot t_{t-1} \; p_x) \cdot \pi, \; t = 1, \ldots, T - 1 \)

\[ TC_{SST,c} = C_0 - E[vC_1] + i_{CoC} \cdot E[vC_1 - v^T C_t] + R^SST,c_\alpha (\sigma, T, A_0, \pi, \theta, t \; p_x, r, i_{CoC}), \]
\[ R^SST,c_\alpha (\sigma, T, A_0, \pi, \theta, t \; p_x, r, i_{CoC}) \]
\[ = (1 - i_{CoC}) \cdot \rho^{CVaR}_\alpha (\sigma A_t) \cdot (A_0 + \Theta_0) + i_{CoC} \cdot \rho^{CVaR}_\alpha (\sigma A_t) \cdot (A_0 + PV_T (\Theta, r)) \]

Comparison of coherent SST risk measure ratio \( R^SST,c_\alpha / A_0 \) with approximate ratio obtained from approximation \( \sigma A^*_T = \left( \frac{A_0 + P_0}{A_0 + PV_T (\Theta, r)} \right) \cdot \sqrt{e^{T \sigma^2}} - 1 \)
## Market Risk Target Capital (8)

**Example 1:** \( \alpha = 0.99, \sigma = 7.5\%, \pi = 100, \theta = 10\%, A_0 = 1000, r = 1.025, i_{CoC} = 6\% \)

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>0.99</th>
<th>( a = \mu_P - \mu_X )</th>
<th>( PV(\Theta, v) )</th>
<th>( PV(a, v) )</th>
<th>( PV(a, v_0) )</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>( \sigma_A )</th>
<th>( \sigma_{\alpha T} )</th>
<th>( \rho(\sigma_A) )</th>
<th>( \rho(\sigma_{\alpha T}) )</th>
<th>( R_{c/A_0} )</th>
<th>( R_{c^*/A_0} )</th>
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<td>( a = \mu_P - \mu_X )</td>
<td>( PV(\Theta, v) )</td>
<td>( PV(a, v) )</td>
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<td>( S_1 )</td>
<td>( S_2 )</td>
<td>( \sigma_A )</td>
<td>( \sigma_{\alpha T} )</td>
<td>( \rho(\sigma_A) )</td>
<td>( \rho(\sigma_{\alpha T}) )</td>
<td>( R_{c/A_0} )</td>
<td>( R_{c^*/A_0} )</td>
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<td>8.3%</td>
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Example 2: $\alpha = 0.99$, $\sigma = 5\%$, $\pi = 100$, $\theta = 10\%$, $A_0 = 1000$, $r = 1.025$, $i_{CoC} = 6\%$

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