Policyholder Exercise Behavior for Variable Annuities including Guaranteed Minimum Withdrawal Benefits\(^1\)

Thorsten Moenig\(^2\)

Department of Risk Management and Insurance, Georgia State University
35 Broad Street, 11th Floor; Atlanta, GA 30303; USA

Email: thorsten@gsu.edu

June 2011

\(^1\)We gratefully acknowledge sponsorship by the Society of Actuaries.

\(^2\)Joint work with Dr. Daniel Bauer, Georgia State University
Overview

1. Introduction

2. A Lifetime Utility Model for Variable Annuities

3. Results

4. Conclusions and Future Research
Introduction

Motivation
Risk-Neutral Valuation Approach

A Lifetime Utility Model for Variable Annuities

Results

Conclusions and Future Research
Introduction

Motivation

- Variable Annuities:
  - Popular long-term investment vehicles
  - Tax-deferred growth
  - Investment evolves according to underlying (risky) portfolio
  - Uncertain payout

- Guaranteed Minimum (Death / Income / Accumulation / Withdrawal) Benefits
  - Insurers offer guaranteed payments
  - Policyholders can purchase security
  - Similar to (combination of) financial options
Motivation

- Withdrawal uncertainty
  - Could mitigate or intensify insurer’s exposure to investment and/or mortality risks
  - Interactions non-trivial
  - Affects pricing and risk management

- Insurers in trouble
  - Disintermediation in 1970s
  - Equitable Life closed to new business in 2000
  - The Hartford accepted $3.4B in TARP money in June 2009 after losing $2.75B in 2008, hurt by investment losses and the cost of VA guarantees
Risk-Neutral Valuation Approach

- Used in actuarial literature to price variety of options:
  - Ulm (2006): “Real option to transfer”
  - Zaglauer and Bauer (2008): Participating life insurance contracts

- To analyze withdrawal behavior for GMWBs:
  - Milevsky and Salisbury (2006)
  - Bauer, Kling and Russ (2008)
  - Optimal stopping problem, akin to pricing American put option
  - Exercise / Withdraw if exercise value exceeds continuation value
  - Worst-case scenario, calculate correct upper bound

- VA market incomplete: cannot sell – or repurchase – policy at its risk-neutral value
  - Withdrawing means giving up possible guarantees and tax benefits
1 Introduction

2 A Lifetime Utility Model for Variable Annuities
   The Model
   Bellman Equation
   Implementation in a Black-Scholes Framework
   Parameter Assumptions

3 Results

4 Conclusions and Future Research
The Model

- Consider withdrawal decisions in life-cycle model with outside investment.
- PH maximizes expected lifetime utility:
  - Consumption and bequests
  - Initial wealth $W_0$
  - Annual (deterministic) income $I_t$
- Invests $P_0$ in VA with finite maturity $T$, remainder in outside account:
  - Includes GMWB, possibly other guarantees
  - Return-of-investment guarantees
  - Other types possible, at cost of larger state space
  - All guarantee accounts identical to benefits base, $G_t$
  - Annual guarantee fee $\phi$ as percentage of concurrent account value
The Model

- VAs grow tax-deferred
  - Withdrawals taxed on last-in first-out basis
  - Early withdrawal tax (10%) if PH withdraws prior to age 59.5

- Restrict all actions to policy anniversary dates
  - Four state variables
    - VA account $X_t$
    - Outside account $A_t$
    - Benefits base $G_t$
    - Tax base $H_t$
  - Three choice variables
    - Withdrawal amount $w_t$
    - Consumption $C_t$
    - Risk allocation in outside account $\nu_t$
A Lifetime Utility Model for Variable Annuities

Bellman Equation

\[ V_t(y_t) = \max_{C_t, w_t, \nu_t} u_C(C_t) + e^{-\beta} \cdot E_t [q_{x+t} \cdot u_B(b_{t+1} | S_{t+1}) + p_{x+t} \cdot V_{t+1}(y_{t+1} | S_{t+1})] , \quad (1) \]

\[ y_t \equiv (A^-_t, X^-_t, G_t, H_t), \]
\[ X^+_t = \left( X^-_t - w_t \right)^+, \]
\[ A^+_t = A^-_t + I_t + w_t - C_t - \text{fee}_I - \text{fee}_G - \text{taxes}, \]
\[ \text{fee}_I = s \cdot \max \left\{ w_t - \min(g^W_t, G^W_t) \right\}, \]
\[ \text{fee}_G = s^g \cdot (w_t - \text{fee}_I) \cdot \mathbb{I}_{\{x+t<59.5\}}, \]
\[ \text{taxes} = \tau \cdot \min\{w_t - \text{fee}_I - \text{fee}_G, (X^-_t - H_t)^+\}, \]
\[ G^-_{t+1} = \begin{cases} (G^i_t - w)^+ & : w \leq g^W_t \\ \left( \min \left\{ G^i_t - w, G^i_t \cdot \frac{X^+_t}{X^-_t} \right\} \right)^+ & : w > g^W_t \end{cases} \]
\[ H_{t+1} = H_t - \left( w_t - (X^-_t - H_t)^+ \right)^+, \]
\[ A^-_{t+1} = A^+_t \cdot \left[ \nu_t \cdot \frac{S^-_{t+1}}{S^+_t} - \kappa \cdot \left( \nu_t \cdot \frac{S^-_{t+1}}{S^+_t} - 1 \right)^+ \right], \]
\[ X^-_{t+1} = X^+_t \cdot e^{-\phi} \cdot \left[ \nu^X \cdot \frac{S^-_{t+1}}{S^+_t} \right], \]
\[ b_{t+1} = A^-_{t+1} + \max\{X^-_{t+1}, G^D_{t+1}\}, \]
\[ \nu_t \geq 0, \quad \sum_i \nu_t(i) = 1, \]
Solve by recursive dynamic programming:

(I) Create appropriate state space grid

(II) For \( t = T \): for all grid points \((A, X, G, H)\), compute \( V_T^- (A, X, G, H) \).

(III) For \( t = T - 1, T - 2, \ldots, 1 \): Given \( V_{t+1}^- \), calculate \( V_t^- (A, X, G, H) \) recursively for each \((A, X, G, H)\) on the grid using an approximation of the integral in (1)

- Discretize return space and evaluate via Green’s function
- Gauss-Hermite quadrature

(IV) For \( t = 0 \): For the given starting values \( A_0 = W_0 - P_0, X_0 = P_0, G_0 = G_1 = P_0 \) and \( H_0 = H_1 = P_0 \), compute \( V_0^- (W_0 - P_0, P_0, P_0, P_0) \) recursively from Equation (1)
A Lifetime Utility Model for Variable Annuities

Parameter Assumptions

- Policyholder is 55 years old, \( T = 15 \) years to maturity
- \( P_0 = 100K; \ W_0 = 2 \cdot P_0 = 200K; \ l_t = 40K \)
- CRRA(\( \gamma = 3 \)) utilities; \( B = 1; \ \beta = r \)
- \( \tau = 30\%, \ \kappa = 15\% \)
- Guarantee fee \( \phi = 50 \) bps
- Surrender fee \( s = 5\% \),
  \[ g_t^W = \begin{cases} 
  0 & : t \leq 5 \\
  20,000 & : t > 5 
\end{cases} \]
- \( r = 4\%, \ \mu = 8\%, \ \sigma = 15\% \)
  - Merton ratio: \( \frac{\mu - r}{\sigma^2 \cdot \gamma} = \frac{0.08 - 0.04}{0.15^2 \cdot 3} \approx 0.5926 \)
- \( \nu^X = 100\% \) equity exposure in VA
1 Introduction

2 A Lifetime Utility Model for Variable Annuities

3 Results
   Withdrawal Behavior
   Pricing and Sensitivities

4 Conclusions and Future Research
Results

Withdrawal Behavior

- Little withdrawal activity (approx. 12K per PH on average)
  - No withdrawals during accumulation period
  - No premature withdrawals in 67% of cases
  - PH empties guarantee account in 6% of cases
  - < 1% chance of excessive withdrawal during contract phase

- Two main reasons to withdraw prematurely:
  - VA account below tax base (approx. 7K on average)
    - Nuanced patterns
    - Interaction of in-the-moneyness of guarantee, tax considerations and excess withdrawal charge
  - VA account much greater than outside account (approx. 5K on average)
    - To reduce overall risk exposure (Merton ratio)
Fig. 1:  $t=14, A_t = 180K, G_t = 100K, H_t = 100K$. 

$w_t \max(Guarantee, X_t^-)$
Withdrawal Behavior

**Fig. 3:** $t=10$, $A_t = 180K$, $G_t = 100K$, $H_t = 100K$.

$X_t^-$

$w_t$

$max(Guarantee, X_t^-)$

$w_t$
Results

Withdrawal Behavior

Fig. 7: $t=10$, $A_t = 20K$, $G_t = 100K$, $H_t = 100K$.

Thorsten Moenig
Policyholder Exercise Behavior for Variable Annuities including GMWBs
Results

Pricing and Sensitivities

- Guarantee fee of $\phi = 50$ bps sufficient to cover expected costs
- In-the-moneyness appears to be OK proxy for pricing
  - Different source to withdrawals
- Eliminating excess withdrawal fee increases net profits (win-win)

<table>
<thead>
<tr>
<th></th>
<th>Base Case</th>
<th>w/d if $X_t^- \leq G_t$</th>
<th>$s = 0%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^Q[\text{Fees}]$</td>
<td>6,252</td>
<td>5,925</td>
<td>5,907</td>
</tr>
<tr>
<td>$E^Q[\text{Excess-Fee}]$</td>
<td>19</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$E^Q[\text{Guarantee}]$</td>
<td>4,558</td>
<td>4,761</td>
<td>2,136</td>
</tr>
<tr>
<td>$% (\text{Guarantee} &gt; 0)$</td>
<td>24.34%</td>
<td>33.97%</td>
<td>31.94%</td>
</tr>
<tr>
<td>$E[\text{agg. w/d}]$</td>
<td>12,084</td>
<td>14,180</td>
<td>16,374</td>
</tr>
<tr>
<td>$E[\text{agg. w/d} &amp; t \leq 6]$</td>
<td>0</td>
<td>0</td>
<td>4,374</td>
</tr>
<tr>
<td>$E[\text{agg. w/d} &amp; X_t^- \leq H_t]$</td>
<td>6,953</td>
<td>14,119</td>
<td>6,047</td>
</tr>
<tr>
<td>$E[\text{agg. w/d} &amp; X_t^- &gt; H_t]$</td>
<td>5,030</td>
<td>0</td>
<td>5,816</td>
</tr>
</tbody>
</table>
## Results

### Pricing and Sensitivities

- Withdrawal patterns highly sensitive to volatility
- Considering taxes important

<table>
<thead>
<tr>
<th></th>
<th>BC: $\sigma = 15%$</th>
<th>$\sigma = 20%$</th>
<th>$\sigma = 25%$</th>
<th>No Taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^Q[\text{Fees}]$</td>
<td>6,252</td>
<td>6,047</td>
<td>5,152</td>
<td>4,734</td>
</tr>
<tr>
<td>$E^Q[\text{Excess-Fee}]$</td>
<td>19</td>
<td>62</td>
<td>1,006</td>
<td>64</td>
</tr>
<tr>
<td>$E^Q[\text{Guarantee}]$</td>
<td>4,558</td>
<td>8,384</td>
<td>11,533</td>
<td>4,746</td>
</tr>
<tr>
<td>$%(\text{Guarantee} &gt; 0)$</td>
<td>24.34%</td>
<td>36.48%</td>
<td>45.57%</td>
<td>28.16%</td>
</tr>
<tr>
<td>$E[\text{agg. w/d}]$</td>
<td>12,084</td>
<td>29,914</td>
<td>85,879</td>
<td>88,791</td>
</tr>
<tr>
<td>$E[\text{agg. w/d } &amp; t \leq 6]$</td>
<td>0</td>
<td>54</td>
<td>13,356</td>
<td>82</td>
</tr>
<tr>
<td>$E[\text{agg. w/d } &amp; X_t^- \leq H_t]$</td>
<td>6,953</td>
<td>17,615</td>
<td>27,877</td>
<td>19,500</td>
</tr>
<tr>
<td>$E[\text{agg. w/d } &amp; X_t^- &gt; H_t]$</td>
<td>5,030</td>
<td>10,674</td>
<td>31,221</td>
<td>69,033</td>
</tr>
</tbody>
</table>
Conclusions and Future Research

1. Introduction

2. A Lifetime Utility Model for Variable Annuities

3. Results

4. Conclusions and Future Research
Conclusions and Future Research

- Develop lifetime-utility model to analyze withdrawal behavior for VA with guarantees
- Numerically solve policyholder’s decision making problem in Black-Scholes framework
  - Return-of-investment GMWB
- Infrequent withdrawals
- PH withdraws when VA account is below tax base
  - Interaction of in-the-moneyness of guarantee, tax considerations and excess w/d fee
- PH withdraws when VA account is large
  - To lower overall risk exposure
Conclusions and Future Research

- Extend policyholder environment
  - Unemployment Risk
  - Subjective mortalities

- Withdrawal patterns highly sensitive w.r.t. volatility $\sigma$
  - Stochastic volatility framework

- Alternatives to EUT
  - Epstein-Zin preferences
  - Correlation Aversion
THANK YOU!