To hedge or not to hedge
that is the problem

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The crisis of confidence

In 2008 the crisis of confidence begins when the financial institutions, like banks, pensions funds and insurance companies can’t comply with the depositors demand or catastrophes liquidations

The lack of solvency appeared due to the lack of balance between assets and liabilities
Asset Liability Management

The ALCO committee of each company not only must established conditions on the balance between Assets and Liabilities, also must determine a good polite on reserves hedging portfolio risks

The actuaries must estimate the risks that directly affect the investment portfolio as a result of:

Deposits

Mathematical and technical reserves
# Investment Portfolio and Risks

<table>
<thead>
<tr>
<th>Investment Portfolio</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed income investment</td>
<td>Variable income investment</td>
</tr>
<tr>
<td>• Bonds</td>
<td>• Equities</td>
</tr>
<tr>
<td>• Notes</td>
<td>• Foreign exchange rate</td>
</tr>
<tr>
<td>• Mortgages</td>
<td>• Indices</td>
</tr>
<tr>
<td>• Loans with guarantee</td>
<td>• Commodities</td>
</tr>
<tr>
<td>• Long term debt</td>
<td></td>
</tr>
<tr>
<td>• Other loans</td>
<td></td>
</tr>
</tbody>
</table>

## Risk involved

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>• Market Risk</td>
<td>• Market Risk</td>
</tr>
<tr>
<td>• Credit Risk</td>
<td></td>
</tr>
</tbody>
</table>
To estimate the Risks we use VaR models

There are two problems with the use of VaR

1) The model may pass the back testing and reflects the reality market movements

2) Once we estimate VaR please make reserves reducing results and try not to distributes earnings not realized
Market VaR for Fixed Rent Portfolio

\[ P - P' \text{ is the net variation of portfolio value due the incidence of Duration and Convexity} \]

\[ P - P'' \text{ is the variation of portfolio value due the incidence of only Duration} \]
The problem is to estimate \( \delta i \)

\[
-MD = - \frac{1}{i} \sum_{t=1}^{n} \frac{tCF_t}{P} (1 + i)^t
\]

\[
CV = \frac{1}{P} \sum_{t=1}^{n} \frac{t(t + 1)CF_t}{(1 + i)^{t+2}}
\]

\[
\Delta \% P = -MD\, di + 0.5\, CV\, di^2
\]
Static VaR Model for interest rate to estimate $\delta_i$

1. Determine the best probability distribution
2. Simulation process
3. Apply the test of K/S
The best distribution fit
The test of goodness of fit
Analyzed by the K/S model

<table>
<thead>
<tr>
<th>Distribution</th>
<th>A-D</th>
<th>Chi-Square</th>
<th>K-S</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logistic</td>
<td>1.3710</td>
<td>326.6128</td>
<td>.0942</td>
<td>Mean=.06,Scale=0.99</td>
</tr>
<tr>
<td>Gamma</td>
<td>4.1333</td>
<td>303.8663</td>
<td>.0982</td>
<td>Location=.705,Scale=0.52,Shape=13,56702</td>
</tr>
<tr>
<td>Student's t</td>
<td>1.9466</td>
<td>278.0459</td>
<td>.1029</td>
<td>Midpoint=.02,Scale=1.63,Deg. Freedom=6.65</td>
</tr>
<tr>
<td>Normal</td>
<td>2.5494</td>
<td>322.3094</td>
<td>.1033</td>
<td>Mean=.02,Std. Dev.=1.81</td>
</tr>
</tbody>
</table>
Simulation using logistic distribution
The dynamic estimation of $\delta i$

Use the Garch (1,1) model

Back Testing Process

Estimate the volatility for day $t+1$ and for $t+\tau$ days
The conditions to use Garch (1,1)

1) The presence of black noise in the autocorrelation and partial autocorrelation of the residuals produced by the regression on a constant of the daily interest rate variations

2) The presence of heteroskedasticity in the model

The Garch (1,1) model

\[ \sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \]
The properties of Garch (1,1)

If \((\alpha + \beta) < 1\), we can estimate the volatility for a period beginning at time \(t\) for the next \(\tau\) days

\[
\sigma_{t+\tau}^2 = \frac{\omega}{1-(\alpha + \beta)} \left\{ (\epsilon - 1) \left[ \alpha + \beta \frac{1 - (\alpha + \beta)^{\tau-1}}{1 - (\alpha + \beta)} \right] + \frac{1 - (\alpha + \beta)^\tau}{1 - (\alpha + \beta)} \sigma_t^2 \right\}
\]

Also:

\[
\begin{align*}
\frac{1}{1-(\alpha + \beta)} \\
\frac{\omega}{1-(\alpha + \beta)}
\end{align*}
\]

Estimate days of persistence

Estimate traditional volatility
\[ \sigma_t^2 = 0.222 + 0.179 \varepsilon_{t-1}^2 + 0.798 \sigma_{t-1}^2 \]

\[ (0.04) \quad (0.019) \quad (0.016) \]

Back testing for Garch(1,1) model
The volatility and $\delta_i$

The Normal and logistic distribution

To estimate the maximum variation we use the positive branch of an interval of confidence:

$$\delta_i = r + z\sigma$$

- Normal distribution and rule of $t^{1/2}$
- Logistic distribution and rule of $t^{1/2}$
- Normal distribution and Garch (1,1)
- Logistic distribution and Garch (1,1)
Excess of $\delta_i$ using rule of $t^{\frac{1}{2}}$ vs. Garch (1,1) model

Maximum daily loss with a 2.5% of probability for the following series

<table>
<thead>
<tr>
<th>Model</th>
<th>Gold</th>
<th>Euro/USD</th>
<th>D. Jones</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t^{\frac{1}{2}}$</td>
<td>-2.219%</td>
<td>-1.326%</td>
<td>-2.40%</td>
</tr>
<tr>
<td>Garch(1,1)</td>
<td>-1.910%</td>
<td>-1.279%</td>
<td>-1.470%</td>
</tr>
</tbody>
</table>
The Credit Risk for fixed income
Transition matrix Vs. excess of interest rate

<table>
<thead>
<tr>
<th>Initial Qualification</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>Def.</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBB</td>
<td>0.02</td>
<td>0.33</td>
<td>5.95</td>
<td>86.93</td>
<td>5.30</td>
<td>1.17</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>BB</td>
<td>0.03</td>
<td>0.14</td>
<td>0.67</td>
<td>7.73</td>
<td>80.53</td>
<td>8.84</td>
<td>1.00</td>
<td>1.06</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0.11</td>
<td>0.24</td>
<td>0.43</td>
<td>6.48</td>
<td>83.46</td>
<td>4.07</td>
<td>5.20</td>
</tr>
<tr>
<td>CCC</td>
<td>0</td>
<td>0</td>
<td>0.22</td>
<td>1.30</td>
<td>2.38</td>
<td>11.24</td>
<td>64.86</td>
<td>19.79</td>
</tr>
</tbody>
</table>

Default Process

\[
P^* = \frac{100}{(1+i^*)} = \left[ \frac{100}{(1+i)} \right] (1 - \pi) + \left[ \frac{f 100}{(1+i)} \right] \pi
\]

\[
\left(\frac{1+i}{1+i^*}\right) = 1 - \pi + f\pi
\]

\[
1 - \left(\frac{1+i}{1+i^*}\right) = \pi - f\pi
\]

\[
\pi = \frac{(i^* - i)}{(1+i^*)(1-f)}
\]
How to cover the risks

The derivative market with different products is used to cover the different risks

**CDS**  
Credit default Swaps to cover the Credit Risk in Fixed income investment

**Options**  
On equities, interest rate and commodities, to reduce the market risk or to assure a maximum price to buy

**Future Market**  
On equities, interest rate and commodities to fix a price
The mathematics of derivatives

The formulas of Black and Sholes and Black

\[ C = SoN(d1) - \frac{E}{e^{nt}} N(d2) \]

\[ d1 = \frac{\ln\left(\frac{So}{E}\right) + \left(i + \frac{s^2}{2}\right)\frac{t}{n}}{s\sqrt{\frac{t}{n}}} \]

\[ d2 = d1 - s\sqrt{\frac{t}{n}} \]

This formula must be correct by the following reasons:

- The returns don't follow the rule of \( t^{1/2} \)
- The returns aren't \( nid \)
- The returns are correlated
- 4) There are a strong presence of heteroskedasticity in the returns
The changes proposed in the Black and Scholes formula

Replacing the volatility for \( t \) days estimated with the rule of \( t^{1/2} \) by the estimated with Garch for the next \( \tau \) days

\[
d1 = \frac{\ln\left(\frac{S_0}{E}\right) + \left(i + \frac{s^2}{2}\right) \frac{t}{n}}{s\sqrt{\frac{t}{n}}} + \frac{i}{n} + \frac{1}{2} \frac{s^2 \cdot t}{n} \]

\[
d1 = \frac{\ln\left(\frac{S_0}{E}\right) + \frac{t}{n} + \frac{s^2 \cdot t}{n}}{s\sqrt{\frac{t}{n}}} + \frac{s^{2\text{Garch}}}{s^{\text{Garch}}} \]

\[
d1 = \frac{\ln\left(\frac{S_0}{E}\right) + \frac{t}{n} + \frac{s^{2\text{Garch}}}{s^{\text{Garch}}}}{s^{\text{Garch}}} \]
The variation of prime options due the volatility variation

The first derived from the price relates to the volatility

\[ \Lambda = \frac{\partial C}{\partial s} = S_0 \frac{1}{\sqrt{2\pi}} e \left( -\frac{d_1^2}{2} \right) \sqrt{\frac{t}{365}} \]

In the Black and Scholes model the volatility is constant during the option life. In consequence we can modify the Vega value using:

\[ \Lambda^{Garch} = \frac{\partial C}{\partial s} = S_0 \frac{1}{\sqrt{2\pi}} e \left( -\frac{d_1^{2GARCH}}{2} \right) \sqrt{\frac{t}{365}} \]
The formula of Black applied to options of interest rate and foreign exchange

\[ F = (S_0 + s) e^{(r-c)t} \]

\[ C = e^{-rt} \left[ F N(d1) - E N(d2) \right] \]

\[ d1 = \frac{\ln \left( \frac{F}{E} \right) + \frac{s^2}{2} t}{s t} \]

\[ d2 = d1 - s t \]

Also change the volatility using Garch (1,1) value

\[ d1 = \frac{\ln \left( \frac{F}{E} \right) + s^{2Garch}}{s^{Garch} t} \]

\[ d2 = d1 - s^{Garch} t \]
A trivial example

<table>
<thead>
<tr>
<th>Assets</th>
<th>Quantity (units)</th>
<th>Price per unit</th>
<th>Portfolio value</th>
<th>% VaR for next 30 days</th>
<th>VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euros</td>
<td>100000</td>
<td>1.4271</td>
<td>142.710</td>
<td>7.38</td>
<td>10532</td>
</tr>
<tr>
<td>Gold</td>
<td>100</td>
<td>1505</td>
<td>150.500</td>
<td>13.3</td>
<td>20016</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>293.210</td>
<td></td>
<td>30548</td>
</tr>
<tr>
<td>-VaR</td>
<td></td>
<td></td>
<td>-30.548</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net</td>
<td></td>
<td></td>
<td>262.662</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Strategic result

<table>
<thead>
<tr>
<th>Concept</th>
<th>cash</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prime collection</td>
<td>5.847</td>
<td>Prime for sell an option on 100000 € @ 0.0584</td>
</tr>
<tr>
<td>Option Exercise</td>
<td>137.000</td>
<td>Collect the option 100000 € @ 1.37</td>
</tr>
<tr>
<td>Gold sell</td>
<td>150.680</td>
<td>Collect to sell of 100 oz of gold @ 1506.80</td>
</tr>
<tr>
<td>Portfolio cash</td>
<td>293.527</td>
<td>Compare with the exposure of 262.662 net of VaR</td>
</tr>
</tbody>
</table>
Conclusions

To solve the problem of to hedge or not to hedge

• A good forecast of Market VaR
• A good forecast of Credit VaR
• A good estimation of prime options
• A good strategic design
Questions