Value-at-Risk: A Risk Theoretical Perspective with Focus on Applications in the Insurance Industry

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Abstract
The value-at-risk conception is becoming more and more a global standard for the measurement and control of market risk of banks active in trading, paying special consideration to the use of derivative instruments. Therefore the question is timely, if and under which conditions a corresponding application of this conception is possible within the framework of risk control of the asset portfolio of an insurance company.

To accomplish this the present paper first develops the basic conception of the value-at-risk approach. The structural parallelity to the risk theoretical approach to solvency control is emphasized. Next the applications in the bank case and the bank specific realization of the VaR-conception are discussed. This forms the basis to contrast the insurance case and to draw conclusions on the specific necessities of realising the VaR-conception in the insurance case.

Résumé
La conception "valeur au risque" devient de plus en plus un standard globale pour le mesurage et contrôle du risque des banques qui sont actifs sur le marché financier. Cette approche tient particulièrement compte des engagements dans des produits dérivés. Pour cette raison, il se pose la question si et à quelle condition une application parallele de cette conception soit possible dans le cadre du contrôle du risque d’un portefeuille des actifs financiers d’une compagnie d’assurance.

Pour répondre à cette question, la contribution présente développe premièremen̄t la conception de base du approche "valeur au risque". Nous mettons l’accent sur le parallelisme structur̄al à l’approche risque théorique du contrôle de la solvabilité. Puis, la discussion se concentre sur l’application du cas des banques et sur la réalisation de la conception "valeur au risque" spécifique à la banque. Cela établit les bases de l’opposer le cas de l’assurance et de formuler des conséquences à l’égard des nécessités spécifiques de la réalisation de la conception "valeur au risque" dans le cas de l’assurance.
Keywords
Value-at-risk, solvency control, risk control of financial assets.
1. Introduction

The Value-at-Risk conception is becoming more and more a global standard for the measurement and control of the market risk for banks active in trading, with special consideration of the use of derivative instruments. Therefore the question is timely, if and under which conditions a corresponding application of this conception is possible within the framework of risk control of the asset portfolio of an insurance company.

For this purpose the present paper first develops the basic conception of the Value-at-Risk approach. The structural parallelity to the risk theoretical approach to solvency control is emphasized. Then the applications in the bank case and the bank specific realization of the VaR-conception are discussed. This forms the basis to contrast the insurance case and to draw conclusions to the specific necessities of realizing the VaR-conception in the insurance case.

2. Value-at-Risk: A Risk Theoretical Analysis

2.1 Basic Construction: VaR as a Quantile Value

Let denote \( V_t \) the future (random) value of a financial asset resp. a portfolio of financial assets at time \( t \) and \( v_0 \) the corresponding (known) value at present. Let denote \( F_i(x) \) the distribution function of \( V_t \). We assume for the following that \( F_i \) possesses a density function \( f_i(x) \).

Let in addition denote \( 0 < \alpha < 1 \) a prescribed safety level as well as \( VaR_\alpha(t) \) the Value-at-Risk at confidence level \( \alpha \) for the time horizon \( t \). The basic structural
property of $VaR_\alpha(t)$ then is given by

\[ P(V_t - v_0 \leq -VaR_\alpha(t)) = \alpha \quad (2.1a) \]

resp. equivalently

\[ P(v_0 - V_t \geq VaR_\alpha(t)) = \alpha \quad (2.1b) \]

The quantity to be controlled is the change in value $\Delta V(t) := V_t - v_0$ of the financial asset resp. the portfolio during $[0, t]$. $VaR_\alpha(t)$ is identical to the negative value of the $\alpha$-quantile of the distribution of $V_t - v_0$. Equivalently to this $VaR_\alpha(t)$ is identical to the $(1-\alpha)$-quantile of the distribution of $v_0 - V_t$, i.e. the Value-at-Risk at the confidence level $\alpha$ is that realization of the loss amount $v_0 - V_t$, which will not be exceeded with probability $1 - \alpha$, as illustrated in the following figure.

Value-at-Risk as the $(1-\alpha)$-quantile of the distribution of $v_0 - V_t$

With respect to the distribution function $F_t(x)$ of $V_t$ $VaR_\alpha(t)$ can in addition be characterized as follows. Let denote $V_\alpha(t) := F_t^{-1}(\alpha)$ the $\alpha$-quantile of the distribution of $V_t$, i.e. we have $P(V_t \leq V_\alpha(t)) = \alpha$. From this we get $P(v_0 - V_t \geq v_0 - V_\alpha(t)) = \alpha$. 
and therefore

\[ \text{VaR}_a(t) = v_0 - V_a(t) . \]  \quad (2.2)

To prepare the further analysis we split up \( V_a(t) \) into the expected value part \( E(V)_a \) as well as into the excess part \( Z_a(V) := E(V) - V_a(t) \), which depends on the assumed distribution of \( V \). Expression (2.2) then changes to

\[ \text{VaR}_a(t) = [v_0 - E(V)] + Z_a(t) . \]  \quad (2.3)

In case we make the assumption \( E(V) = v_0 \), which is likely to hold approximately for very short time horizons (a day, a week), cf. Beckström/Campbell (1995b, p. 41), we obtain

\[ \text{VaR}_a(t) \approx Z_a(t) , \]  \quad (2.4)

i.e. the Value-at-Risk corresponds to a measure for the deviation of the distribution of \( V \) from its expected value. At the same time this way of proceeding reduces the model risk of assuming unjustly an expected increase in value, which would result in a reduction of VaR according to (2.3). The assumption \( E(V) = v_0 \) implies a determination of the Value-at-Risk which is on the safe side. At the same time in this case we can interpret the Value-at-Risk directly as a measure of risk.

**Example 1:**

Suppose \( V \) is following a normal distribution, \( V^*_a := [V - E(V)] / \sigma(V) \) is following a standard normal distribution. In this case we have \( N_a = -N_{1-a} \), and obtain \( P(V^*_a \leq -N_{1-a}) = \alpha \). From this we derive \( \alpha = P(V_a \leq E(V) - N_{1-a} \sigma(V)) = P(v_0 - V_a \geq \)
\( v_0 - E(V) + N_{1-\alpha} \sigma(V), \) i.e. the Value-at-Risk is given by

\[
\text{VaR}_\alpha(t) = v_0 - E(V) + N_{1-\alpha} \sigma(V).
\]  

(2.5)

If we are willing to assume \( E(V) = v_0 \), we obtain

\[
\text{VaR}_\alpha(t) = N_{1-\alpha} \sigma(V).
\]  

(2.6)

i.e. the Value-at-Risk is proportional to the risk measure standard deviation.

An alternative way of presenting the VaR-conception can be obtained by introducing the quantity Return-on-Value-at-Risk \( \text{RoVaR}_\alpha(t) \) defined by

\[
\text{RoVaR}_\alpha(t) := \frac{V - v_0}{\text{VaR}_\alpha(t)}.
\]  

(2.7)

The structural property (2.1a) then equivalently reads

\[
P[\text{RoVaR}_\alpha(t) \leq -1] = \alpha \]  

(2.8a)

resp.

\[
P[\text{RoVaR}_\alpha(t) > -1] = 1 - \alpha.
\]  

(2.8b)

i.e. the probability of not exhausting the Value-at-Risk by the return (related to the VaR) of capital investment is under control and high.

2.2 How to realize the VaR-conception

To calculate a VaR the decision maker needs to obtain a forecast of the probability distribution of the future value of \( V \). This means that the decision maker must create a finance model that describes value. A standard way of proceeding in this connection is to relate the values \( V^i \) of the financial assets to relatively few fundamental factors, e.g. on the basis of linear factor models - cf. in general e.g. Al-
brecht/Maurer/Mayser (1995) - for $\Delta V_i^t := V_i^t - v_i^0$ or for $\ln \Delta V_i^t$. In the first case the modelling assumption reads (assuming m factors, $j = 1, \ldots, m$)

$$\Delta V_i^t = a_i(t) + \sum_{j=1}^{m} b_j(t) F_j(t) + \varepsilon_i(t)$$  \quad (2.9)

with the standard assumptions $E[\varepsilon_i(t)] = 0$, $i=1, \ldots, I$; $\text{Cov}[\varepsilon_i(t), \varepsilon_i(t)] = 0$ for $i \neq i'$, and $\text{Cov}[F_j(t), \varepsilon_i(t)] = 0$ for all $i, j$.

To calculate the portfolio-VaR in this case (ignoring the time parameter) only an identification of the sensitivities $a_i$, resp. $b_j$, the expected values $f_j = E(F_j)$, the covariances $f_{ij} = \text{Cov}(F_i, F_j)$ as well as the variances $s_i^2 = \text{Var}(\varepsilon_i)$ is necessary.

In case of non-linear pay-offs and respective valuations the usual approach is to linearize the non-linear pricing models through the use of the Taylor expansion (this leads to the sensitivity models for the VaR). In case of option and option-like securities, however, one works directly with (non-linear) option pricing models.

To determine the Value-at-Risk of a portfolio $V_a^t(t)$ on the basis of one of the preceding models there are the following possibilities:

- analytical determination on the basis of a distributional assumption, usually in conjunction with variance-covariance-estimations,
- Monte Carlo-simulation,
- historical simulation.

As for the purpose of this contribution primarily the conceptual framework of the VaR-approach is of relevance, we refer to Beckström/Campbell (1995b) and Wilson (1994) for more details of realizing the VaR-conception.
2.3 Portfolio Effects

In case of a homogeneous class of financial assets VaR is proportional to the number $k$ of assets,

$$\text{VaR}_\alpha^P(t) = k \text{VaR}_\alpha(t) .$$  \hfill (2.10)

This is easily seen on the basis of (2.1a) using the property $P(V - v_0 \leq -\text{VaR}_\alpha(t)) = P(k(V - v_0) \leq -k \text{VaR}_\alpha(t))$.

We now take a look at a heterogeneous portfolio consisting of $n$ asset classes. The value of class $i$ at time $t$ is denoted by $V_i$, similarly $V_i^p$ denotes the corresponding portfolio value. Obviously for $\Delta V_i(t) = V_i^p - v_i$ we have

$$\Delta V_i(t) = \sum_{i=1}^{n} \Delta V_i(t)$$  \hfill (2.11)

and the Value-at-Risk of the portfolio $\text{VaR}_\alpha^P(t)$ is defined by

$$P(\Delta V_i(t) \leq -\text{VaR}_\alpha^P(t)) = \alpha .$$  \hfill (2.12)

Example 2:

Carrying on with example 1, we assume that all $V_i$ follow a normal distribution. We define $\mu_i(t) = E(V_i)$, $\sigma_i(t) = \sigma(V_i)$ and $\rho_{ij}(t) = \rho(V_i, V_j)$. We then have $V_i^p \sim N(\mu_p(t), \sigma_p(t))$ where $\mu_p(t) = \sum_{i=1}^{n} \mu_i(t)$ and $\sigma_p(t) = \left[ \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij}(t) \sigma_i(t) \sigma_j(t) \right]^{1/2}$. 
For the Value-at-Risk of the portfolio we therefore obtain

\[
\text{VaR}_a(\rho; t) = \sum_{i=1}^{n} [v_i^t - \mu_i(t)] + N_{1-\alpha} \left[ \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{ij}(t) \sigma_i(t) \sigma_j(t) \right]^{\frac{1}{2}}.
\]

This means that for a heterogeneous portfolio consisting of different asset classes the Value-at-Risk is not necessarily additive. In case of example 2 for instance the sum of the Value-at-Risk amounts of each single asset class is given by

\[
\sum_{i=1}^{n} \text{VaR}_a(t) = \sum_{i=1}^{n} [v_i^t - \mu_i(t)] + N_{1-\alpha} \sum_{i=1}^{n} \sigma_i(t).
\]

With the exception of the case of a perfect positive correlation between all asset classes, i.e. \( \rho_{ij} = 1 \) for all \( i \neq j \), the collective Value-at-Risk according to (2.13) is always lower than the sum of the Value-at-Risk amounts of the asset classes, i.e.

\[
\text{VaR}_a(\rho; t) < \sum_{i=1}^{n} \text{VaR}_a(t).
\]

In case (still keeping the assumption of a normal distribution) of not all the asset classes being perfectly positively correlated, we consequently have a collective reduction effect when calculating the portfolio-VaR. This clarifies the effect that the VaR always has to be understood relative to the portfolio of assets to be analyzed. This especially gains importance, when we interpret the VaR as a required risk based capital, cf. part 3.1 of this paper.

2.4 Time Effects

To be able to identify possible time effects on the VaR we have to specify a specific type of stochastic process \( \{V_i; t \geq 0\} \) for the development of the value of a
financial asset or a portfolio of such assets. To be in accordance with examples 1 and 2 we, in the following analysis, assume a Brownian motion (Wiener process with drift \( \mu \) and diffusion \( \sigma \)), the standard model of a time continuous random walk (with drift). The starting point of the random walk is \( V_0 = v_0 \). The general development of the Brownian motion can be put in the following form

\[
V_t = v_0 + \mu t + \sigma \sqrt{t} Z
\]  

(2.15)

where \( Z \) is a standard normally distributed random variable, i.e. \( Z \sim N(0, 1) \).

From (2.15) we have

\[
E(V_t) = v_0 + \mu t \quad \text{and} \quad \sigma(V_t) = \sigma \sqrt{t} \quad .
\]

(2.16)

The corresponding Value-at-Risk according to (2.5) now is

\[
VaR_{\alpha}(t) = N_{1-\alpha} \sigma \sqrt{t} - \mu t \quad .
\]

(2.17)

In this case the development of the VaR possesses a linear as well as a degressive part. If one is willing to assume a random walk without drift (\( \mu = 0 \)), then the term (2.17) reduces to

\[
VaR_{\alpha}(t) = N_{1-\alpha} \sigma \sqrt{t} \quad ,
\]

(2.18)

which means that the VaR is degressively dependent on the length of the time horizon.

2.5 Value-at-Risk and Shortfall-Probability

The shortfall-probability of a general random variable \( X \) with respect to a target \( z \) is given by

\[
LPM_0(z; X) = P(X \leq z) \quad .
\]

(2.19)

If we denote \( Q_\alpha \) as the \( \alpha \)-quantile of the distribution of \( X \) we generally have \( Q_\alpha = \)
sup \( \{ z; LPM_d(z; X) \leq \alpha \} \). With respect to the Value-at-Risk the following relations are valid

\[
VaR_\alpha(t) = -\sup \{ v; LPM_0(v; V_t - v_0) \leq \alpha \} \quad (2.20a)
\]

\[
= \inf \{ -v; LPM_0(v; V_t - v_0) \leq \alpha \}
\]

resp.

\[
VaR_\alpha(t) = v_0 - \sup \{ v; LPM_0(v; V_t) \leq \alpha \} \quad (2.20b)
\]

In case \( X \) possesses a density function we are able to obtain more specific relations, particularly \( Q_\alpha \) is the solution of the equation \( LPM_0(Q_\alpha; X) = \alpha \); because \( LPM_0(v; X) \) is strictly monotonous in \( v \) we therefore obtain \( Q_\alpha = LPM_0^{-1}(\alpha; X) \) \((\alpha)\). According to that (2.20) now reads

\[
VaR_\alpha(t) = -LPM_0^{-1}(\alpha; V_t - v_0) \quad (2.21a)
\]

resp.

\[
VaR_\alpha(t) = v_0 - LPM_0^{-1}(\alpha; V_t) \quad (2.21b)
\]

2.6 The Risk Theoretical Dimension of the VaR: Solvency Control

The interpretation of VaR as a reference quantity to determine the required risk based capital, cf. part 3.1, clarifies the connection between risk control of a portfolio of assets on the one hand and a portfolio of insurance risks on the other. Let denote \( S \) the accumulated claim amount of a portfolio of insurance risks for one period (typically one year) and \( \pi \) the collective (risk) premiums earned at the beginning of the period. Now the standard risk theoretical way to determine the necessary solvency capital \( SC \) with respect to the collective of insured risks is to keep the loss probability under control, i.e. to maintain a tolerance level \( \epsilon \). The control
criterion chosen in this case reads:

$$P(S \geq SC + \pi) = \varepsilon$$  \hspace{1cm} (2.22)$$

The version of (2.22), which is equivalent to (2.1a), obviously is $P(\pi - S \leq -SC) = \varepsilon$, the version equivalent to (2.1b) is $P(S - \pi \geq SC) = \varepsilon$. The change in value $\Delta V_p(1)$, which in case of the Value-at-Risk reflects the one-period profit on investment, corresponds to the technical profit/loss position $\pi - S$ in case of the risk theoretical solvency control. Note in addition that the determination of the required solvency capital is always related to the entire collective of insured risks and not to the sub-collectives (which would correspond to asset classes) and usually is not done for shorter periods $[0, t]$ with $t < 1$. This is indicating some main distinctions between the insurance and the bank case, which we are going to deal with in part 4.1.

2.7 A Generalized VaR-Conception on the Basis of the Shortfall-Expectation

The VaR-conception based on the relationship (2.1) is criticized because it only considers loss probabilities and not e.g. the average loss amount. A corresponding generalization is proposed by Schröder (1996) for a VaR on return basis. A similar proposal is presented in Albrecht/König/Maurer/Schradin (1996), who control the expected excess-value of the random variable (discounted) loss amount in the framework of determining the required risk based capital in case of default risk. In the following we will discuss this approach in order to obtain a generalized VaR-conception under more general aspects.

To do this we use the conception of the Return-on-Value-at-Risk according to (2.7). The property (2.8a), being a version of (2.1a), then can be equivalently stated in a form that the shortfall-probability of RoVaR with respect to the target return $r_T = -1$ is under control. As shown in Albrecht (1994, S. 93) a general class of
measures of shortfall-risk of a return quantity \( R \) with respect to the target return \( r_T \) is given by

\[
LPM_n(r_T) = E\left[ \max (r_T - R, 0)^n \right]
\]

(2.23)
i.e. it corresponds to the lower partial moments of order \( n \) of \( R \) with regard to \( r_T \).

The case \( n = 0 \) corresponds to the shortfall-probability, being the control criterion underlying (2.8a). The case \( n = 1 \) corresponds to the shortfall-expectation, which will be the basis of our further analysis. The control of the generalized Value-at-Risk \( VaR^*_s(t) \) is now done on the basis of limiting the shortfall-expectation with respect to \( r_T = -1 \) to a small quantity \( \alpha \) (e.g. 1% or 5%). Formally the property is

\[
E\left[ \max (-1 - R, 0) \right] = \alpha
\]

(2.24a)
resp. equivalently

\[
E\left[ \max \left( \frac{v_0 - V_t}{VaR^*_s(t)} - 1, 0 \right) \right] = \alpha
\]

(2.24b)

As in general we have \( E[\max (kX, 0)] = k E[\max (X, 0)] \), this consequently is equivalent to

\[
E\left[ \max \left( v_0 - V_t - VaR^*_s(t), 0 \right) \right] = \alpha VaR^*_s(t)
\]

(2.24c)

Obviously the relation (2.24c) has the following interpretation. The expected value of loss amount \( v_0 - V_t \) in excess of the (generalized) VaR is limited to a defined fraction of the VaR.

2.8 Value-at-Risk and Required Risk Based Capital in Case of Default Risk

If one succeeds in specifying a probability distribution of the credit loss\(^7\) \( CL \), a de-
termination of the required risk based capital again is possible on the basis of a VaR-conception. Examples of such probability distributions can be found in Albrecht/König/Maurer/Schradin (1996) or in Iben/Brosterton-Ratcliffe (1994). The VaR$_{CL}$ for a specific position subject to default risk is then determined according to the relation

$$ P(CL > VaR_{CL}) = \varepsilon \quad . \tag{2.25} $$

The generalized VaR-conception of part 2.7 in the default risk-framework is discussed in Albrecht/König/Maurer/Schradin (1996).

3. Applications of the VaR-Conception for Risk Control and Risk Based Profit Control (Bank Case)

3.1 Risk Based Capital Requirements

The application of the VaR-conception to determine the capital requirements for the trading operations of a bank with respect to market risk within the framework of a bank internal risk measurement model is supported by several institutions, e.g. The Group of Thirty, The Derivatives Policy Group and in Europe the Basle Committee on Banking Supervision. The required capital on a daily basis for trading operations proposed by the Basle Committee is the VaR$^0$ multiplied by a factor three or more (to take account for the inherent "model risk"). The time horizon over which the VaR is calculated amounts to $t = 10$ days.

For the following analysis we assume that the probability distribution of $V_t$ is correctly specified, i.e. there is no model risk. If we now denote $RAC_{\text{in}}(t)$ to be the required risk based capital for a time horizon of length $t$, then the following
relation has to be fulfilled

\[ RAC^m(t) \geq \text{VaR}_a^p(t) \quad (3.1) \]

From (2.12) we therefore obtain

\[ P(RAC^m(t) + \Delta V_i^p < 0) \leq \alpha \quad (3.2) \]

This means that the probability that the change in value of the considered portfolio will exhaust the required capital over a time horizon of length \( t \) is limited by \( \alpha \).

Because of the collective effects described in part 2.3 one obtains capital requirements which are unnecessarily high in case one uses the isolated Value-at-Risk amounts for the single asset classes and not for the entire collective of assets.

### 3.2. Prescription of Risk Limits

In contrast to part 3.1, where one takes the positions in financial assets as given and tries to determine the required capital, the situation is different, when constructing risk limits to be prescribed. Here one assumes an amount of risk based capital \( RAC^u(t) \) as given for a time interval \([0, t]\). The possible investment strategies are now limited by the requirement that with confidence level \( \alpha \) a change in value of the resulting portfolio will not exhaust the given \( RAC \). If we denote formally \( \vartheta \) as a decision parameter, which determines the distribution of \( V_i^p \), i.e. \( V_i^p = V_i^p(\vartheta) \), and therefore as well determines the Value-at-Risk at confidence level \( \alpha \), i.e. \( \text{VaR}_a^p(t) = \text{VaR}^p_a(t, \vartheta) \), then the following relation has to be fulfilled:

\[ \text{VaR}_a^p(t, \vartheta) \leq RAC^u(t) \quad (3.3) \]

In case that \( V_i^p \) is normally distributed the possible investment strategies \( \vartheta \) would determine the accessible \((\mu, \sigma)\)-combinations \((E(V_i^p), \sigma(V_i^p)) = (E_{\vartheta}(V_i^p), \sigma_{\vartheta}(V_i^p))\)
and, because of that, one would have the restriction:

\[ \nu_0 - \mu(V,') + N_{1-a} \sigma(V,') \leq RAC''(t) \]  \hspace{1cm} (3.4)

The setting of risk limits described so far is relating to the portfolio level. To be able to control sub-positions, too, risk limits for sub-positions (e.g. for asset classes) are as well of interest. This leads to the non-trivial tasks of splitting up the entire VaR and of distributing it risk adequately over the sub-positions considered.

### 3.3 Risk Adjusted Performance Control

So far only the control of risk was the subject of our analysis. Beyond that the return (performance) of financial operations is of central interest.

As a simultaneous optimization of the activities under the restrictions (3.3) resp. (3.4) is practically infeasible one has to take more simple approaches into account. As a starting point we use the Return-on-Value-at-Risk according to (2.7), and we assume that a minimum return of \( r_0(t) \) is the target for the desired (risk adjusted) return. We can formalize that, either by requiring that the target return has to be achieved in expectation only, i.e.

\[ E[RoVaR_\alpha^p(t)] \geq r_0(t) \]  \hspace{1cm} (3.5)

or alternatively with a high confidence level, i.e.

\[ P[RoVaR_\alpha^p(t) \geq r_0(t)] \geq 1 - \varepsilon \]  \hspace{1cm} (3.6)
4. VaR for Insurance Companies: Possibilities and Problems

4.1 Preliminary Remarks on the Peculiarities of the Insurance Case

The analysis of part 3 pertains to the specific realization of the general VaR-conception of part 2 to the bank case being characterized by

- a focus on market risk
- a focus on short-term horizons and
- a focus on trading operations.

The insurance case is significantly different. First of all in the insurance case the purpose of holding financial positions is different. Financial instruments are not held for trading purposes but for investment purposes accompanied by risk management activities. Investment is not done for its own sake but to be able to fulfill the liabilities of the insurance company, i.e. it has to be seen in an asset/liability-framework. As insurance liabilities are of middle or of long term, this has consequences for the relevant time horizon. As the investment in financial assets has to be seen under middle-to-long term aspects the exclusive considerations of market values loses its importance and other categories of value (e.g. accounting values, book values) have to be considered, too. This especially goes for countries (e.g. Germany), where accounting is not done on a market value basis. Summing up, we have the following important differences compared to the bank case:

- the purpose of holding financial positions,
- the relevant term of the time horizon,
- the categories of value to be considered as well as
- the general importance of the liabilities when controlling the investment process.

This does, however, not imply that the VaR-conception will be not of use for insu-
rance companies. We see the possible applications primarily in risk control and we will discuss that in the following sections. But, the general VaR-conception has to be realized in the insurance case in a different way. This, however, as well means that VaR-systems specifically designed for the bank case are not necessarily suitable for the insurance case. At least they have to be modified to be able to meet the special requirements of the insurance case. Because of the longer term and the asset/liability-context we believe, that actuarial stochastic investment models will be of decisive importance in such a modified VaR-system.

4.2 Risk Based Capital Requirements

A regulation of capital requirements in the insurance field which explicitly takes into account investment risk are the US-RBC (risk based capital)-standards proposed by the NAIC. A study of the RBC-standards again reveals the important differences between the insurance and the bank case consisting of

- the use of balance sheet values,
- the one year time horizon and
- the embedding of the Asset-RBC-requirements in a general system of risk categories, containing especially underwriting risk.

But there are further important differences to the VaR-conception. There is no consideration of the possible application of derivative instruments. Basically the approach in calculating the Asset-RBC is only very loosely connected to the models and results of modern capital market theory, which in contrast play a fundamental role in the VaR-conception. In addition the RBC-standard requires the insurers to use a single, rigid approach whereas the Basle Committee allows the banks to use their own internal models to calculate the VaR (subject to the provision that certain
"quality" requirements are fulfilled). 10)

4.3 Control of Loss- and Depreciation-Potential

A further possible application of the VaR-conception in the insurance case is loss control with respect to the asset side. The Value-at-Risk \( \text{VaR}_\alpha(t) \) is a measure for the maximum loss in value \( v_0 - V_t \) at confidence level \( \alpha \), i.e. only in 100 \( \alpha \)% of the cases over a time horizon of length \( t \) a larger loss than \( \text{VaR}_\alpha(t) \) will have to be realized. With regard to loss control one has to distinguish between loss on a market value basis and loss on a book value basis (depreciation) depending on the relevant accounting rules.

With respect to the control of loss and/or depreciation it is recommendable to complete Value-at-Risk considerations by using stress-tests, i.e. the consequences of worst-case-scenarios on the loss/depreciation potential. This takes into account the consideration that Value-at-Risk calculations are typically based on average changes in value on a statistical basis and are not adequately able to cope with the consequences of exceptional developments on the capital markets.

An extension of stress-tests to include the liability side of the insurance operations are the Resilience Tests applied in the United Kingdom.

5. Conclusions

The VaR-conception discussed in part 2 of the present paper is to be understood as a general approach to measure and to control the risk potential of a portfolio of financial assets. So far this general conception has only been specifically realized for the requirements of the trading operations of banks. The insurance case is different
in many instances and therefore a genuine realization of the VaR-conception has to be adapted to be able to meet adequately the specific requirements of insurance companies. Actuarial stochastic investment models should prove to be the key in accomplishing this task.
Endnotes

1) Cf. e.g. in general Beckström/Campbell (1995a).
2) Cf. e.g. the latest publication of the Basle Committee on Banking Supervision (1996).
3) Cf. e.g. in addition Ufer (1996).
4) Cf. e.g. Boudoukh et al. (1995) or Schröder (1996).
5) The development of a VaR-conception in case of default risk is done in part 2.8.
6) Boudoukh et al. (1995) propose in contrast to that to control the expected "worst-case-scenario-risk", formally measured by E[\min(Z_1, \ldots, Z_n)] , where the Z_i correspond to the changes in value of successive periods.
7) One correspondingly has to work with a discounted credit loss, when the risk of default pertains over several periods of time.
8) More precisely the higher value of the VaR of the last day and the average of the daily VaR values over the last 60 days has to be taken.
9) We believe that the applications in the domain of risk adjusted performance control discussed in part 3.3 are primarily of relevance for trading operations only.
10) Styblo Roder (1995, p. 23) emphasizes the difficult trade-offs between a single, rigid approach and internal VaR-models.
References


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