An Integrated Valuation and Risk Model
for German Fixed-Income Portfolios

Frank Nielsen, Simon Juen

Summary
In this work we present the analytical models, methodologies and features of a state-of-the-art fixed-income valuation and risk analysis system developed at BARRA, Inc. In particular, we discuss the adaptation and application of this system for the German bond market. The following topics are covered.

(1) Valuation: The term structure estimation is based on a modified Cox-Ingersoll-Ross model. Bonds are valued using option-adjusted cash flows and spreads estimated for four categories (Pfandbriefe, DM-Eurobonds, liquidity, coupon effect).

(2) Risk Analysis: Systematic risk is estimated using a factor model with the three principal components of term-structure risk (‘shift’, ‘twist’, and ‘butterfly’), as well as spread risk for specified market segments. Specific risk is estimated from duration and market spreads. We also discuss the extension of the factor model to characterize the highly nonlinear risk of derivatives.

(3) Performance Analysis: Portfolio performance is evaluated by analyzing the components of risk-adjusted returns according to term-structure changes, spreads, and bond-specific factors.

Keywords
Term structure estimation, factor model, GARCH model, nonlinear risk, performance analysis, German bond market

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Introduction

In the past, fixed-income investing in Germany was often a simple buy-and-hold strategy partly due to the illiquidity in the non-government sectors. Any risk management mainly consisted of duration and maybe convexity as risk measures. But with an increase in interest rate volatility and more fixed-income instruments being available, the need for sophisticated valuation and risk measures has grown. More and more institutional investors demand risk and performance reports. They like to understand the different sources of fixed-income risk. Furthermore, insurance companies have started trading more actively. An increased interest on part of the foreign investors for the Pfandbrief sector is an additional criteria that has made the German bond market more volatile. Since mid 1995 there has been an augmenting number of so called Jumbo-Pfandbrief issues which require some minimum standards like the issue size of at least 500 Million DEM. (See, for example, Deutsch et al. [1996].) The liquidity of these issues is much higher due to the commitment of the issuer to quote realistic bid-ask spreads and mandatory price offers. Moreover since April 1995 the Pfandbriefindex PEX has been calculated and has enhanced the transparency and liquidity of this sector. By the end of 1995 the Pfandbrief sector covered about 1/3 of the whole German fixed-income market. In addition Jumbo-Pfandbrief issuer have started to obtain Ratings for their issues which also has led to more interest abroad. (See Brandt [1996].) Market experts have estimated that around 15% of the total Pfandbrief sector is placed abroad now. (See, for example, Buehler and Hies [1996].)

Our work concentrates on the development of a risk and valuation model that captures these new developments as accurately as possible. We will start by describing the valuation of single bonds: How to estimate the term structure, marketwide and sector specific spreads, for example, a liquidity spread and a Pfandbrief sector spread. We will focus on the risk inherent in the German fixed-income market and will estimate a multi-
focus on the risk inherent in the German fixed-income market and will estimate a multi-
factor model that captures the main sources of volatility in the German market. The
model will separate marketwide risk from bond specific risk, show the portions of term
structure and spread risk and combine the different risk factors to a reliable factor
covariance matrix. The valuation of derivatives and their impact on the portfolio profile
will be outlined in the next section. Finally, performance analysis and attribution will be
introduced. We will use our multi-factor model to attribute the portfolio return and risk to
its different sources. The performance attribution enables us to evaluate the success of a
portfolio on a risk-adjusted basis.

Valuation

The price of a bond represents the market’s assessment of the present value of its ex-
pected future cash flows. A model will estimate a fitted price based on marketwide
properties, like the term structure and spreads. (See Kahn in Fabozzi [1991] or Gulrajani
and Chui [1993].) The difference between the market price and the fitted price is the
pricing error and can be interpreted as under- or overvaluation of the particular bond. Our
model estimates bond prices as:

\[ PM_i(t) = \sum_T \frac{c_{f_i}(T) \cdot PDB(t,T)}{\exp[\kappa_i(t) \cdot T]} + \epsilon_i(t) \]

or

\[ PM_i(t) = PF_i(t) + \epsilon_i(t) \]

with:

\[ \kappa_i(t) = \sum_j x_{i,j} \cdot spr_j(t) \]

where:

\[ PM_i(t) \quad \text{Market price of bond } i \text{ at time } t \]

\[ PF_i(t) \quad \text{Fitted price of bond } i \text{ at time } t \]

\[ c_{f_i}(T) \quad \text{(Option adjusted) cash flow of bond } i \text{ at time } T \]
PDB(t,T) = (Default free) pure discount bond price at time t maturing at time T

\( x_{i,j} \) = Exposure of bond i to yield spread j

\( spr_j(t) \) = Yield spread j at time t

\( \varepsilon_i(t) \) = Pricing error of bond i at time t

\( \kappa_j(t) \) = Total spread of bond i at time t

The market is characterized by the term structure, represented by the pure discount bonds PDB(t,T) and the marketwide and sector spreads (\( spr_j(t) \)). This model clearly identifies the different factors and bond exposures that determine the bond price. The estimated values \([PDB(t,T), spr_j(t), \varepsilon_i(t)]\) result from fitting the model to actual trading prices at time t. We use a modified Cox-Ingersoll-Ross approach to estimate the term structure. (see Cox, Ingersoll and Ross [1985].) The term structure itself is described with spot rates instead of PDB's. The procedure is explained in appendix A. The valuation model for the German Bond market consists of 12 term structure vertices or key rates and four yield spreads. We use the spot rates at the 1, 3, 6 months and 1, 2, 3, 4, 5, 7, 10, 20 and 30 years vertices. Due to the low liquidity of bonds with maturities less than a year we do not estimate the term structure for the 1, 3 and 6 months vertices but use the DEM-LIBOR's (London Interbank Offered Rates) instead.

The spot rate between two vertices is calculated by assuming constant forward rates between the vertices. There is always a trade off between forecasting relative value and closely fitting the market price. If you were to use as many vertices as bonds, the pricing error would be zero but there also would be no added information in the model. Additionally, robust estimation becomes more difficult with an increasing number of factors because the number of bonds per component decreases. On the other hand, a small number of factors will lead to higher pricing errors. We choose 12 term structure factors as a compromise that still covers the whole maturity spectrum. The four yield spreads capture non-term structure related sources of return. In our model we investigate the following spreads:
A. Pfandbrief spread as difference between the Pfandbrief term structure and the default free term structure
B. DEM Eurobond spread above the default free term structure
C. Liquidity spread
D. Current yield spread captures the different tax treatment of coupon and principal cash flows

The Pfandbrief term structure is given by the spot rates of the different subindices of the Pfandbriefindex PEX. The PEX index was developed in conjunction of the “Verband Deutscher Hypothekenbanken”, the “Verband oeffentlicher Banken” and the institute for banks and finance at the University St. Gallen in Switzerland. (See Buehler and Hies [1996].) The index follows the notional-bond concept, which means that the PEX consists of synthetic bonds with fixed terms. The characteristics of the synthetic bonds like the coupons and the time to maturity are kept constant. The index is calculated using private mortgage bonds (“Hyphothekenpfandbriefe”) and municipal bonds (“oeffentliche Pfandbriefe”). The PEX index is divided into 10 subindices with constant maturities of one to ten years. The exact weighting scheme is attached in Appendix B. We calculate the Pfandbrief spread from index data because of the lack of historical market prices for Pfandbriefe.

The DEM Eurobond spread is based on the par-yield swap curve. The swap curve is a set of fixed rates that can be exchanged for the 12 month’s LIBOR. The swap spread as difference between the swap and the sovereign curve captures the premium for the additional credit risk of non-government organizations. The main participants in the swap market are financial institutions with an average bond rating of “double A”. Therefore the swap curve can function as a reasonable proxy for the majority of Eurobond issues. In a second step we convert the par-yields of the 2, 3, 4, 5 and 10 year vertices to spot rates and calculate the swap spread against the default free term structure.
The liquidity spread is estimated based on the government universe of the J.P. Morgan Germany index. Each single bond gets a dummy variable of 1, -1 or 0, depending on the liquidity classification of J.P. Morgan for Government bonds. The classification differentiates the bonds - based on trading volume - into four categories. The liquidity spread is an output parameter of the Cox-Ingersoll-Ross function that minimizes the pricing error. The function is described in Appendix A.

The current yield spread, which takes into account the different tax treatment of coupon and principal payments, is a function of the coupon and the actual price of a bond. It is calculated as the slope in a regression of bonds’ weighted pricing errors against a measure of the tax advantage for certain investors. The weight is the bond’s duration, and the measure for the tax advantage is the difference between current yield and the bond’s equivalent par yield.

Risk

The risk involved in investing in bonds has increased over the last years. The main risk sources for a domestic bond manager are term structure movements, changes in spreads and the credit or default risk.

The graph displays the end-of-month term structures between 1/88 and 1/96. The chart shows that term structure movements in general are not parallel. In particular the short end of the curves are much more volatile than the long end. Therefore duration as the only risk measure is not sufficient.
The purpose of a risk model is to forecast the volatility of bond prices and their correlations due to marketwide or common factors and bond specific risk. Our marketwide risk forecasts are based on a multi-factor model that uses weekly changes in the key rates and changes in yield spreads. The bond specific risk model captures the risk not explained by term structure movements and yield spread changes. We call it specific risk and by assumption it will be uncorrelated with the marketwide and sector risk factors.

**Term Structure Risk**

We developed a three-factor risk model to forecast changes in the default-free term structure. For the term structure estimation we use the data of Government bonds included in the J.P. Morgan Germany index. This index covers the government bond issues of the German market. Due to thin bond coverage and low trading volume on the short and the long end of the term structure, we restrict the risk model estimation to maturities from 1 to 10 years. As pointed out earlier the term structure is described by default free spot rates estimated at the different vertices. The risk of a bond due to term structure changes is determined by the covariance matrix of weekly changes of spot rates and the exposure of the bond to these changes. Changes in spot rates of different maturities are highly correlated and therefore these spot rate changes can be captured with a small number of factors without losing much explanatory power.

**Correlation Matrix of Spot Rates Changes 1/90 - 1/96**

<table>
<thead>
<tr>
<th></th>
<th>1/90</th>
<th>1/96</th>
<th>1/91</th>
<th>1/92</th>
<th>1/93</th>
<th>1/94</th>
<th>1/95</th>
<th>1/96</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0.532</td>
<td>1.000</td>
<td>0.324</td>
<td>0.908</td>
<td>1.000</td>
<td>0.517</td>
<td>0.924</td>
<td>1.000</td>
</tr>
<tr>
<td>0.532</td>
<td>0.932</td>
<td>0.891</td>
<td>0.912</td>
<td>0.932</td>
<td>1.000</td>
<td>0.528</td>
<td>0.912</td>
<td>0.932</td>
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<td>0.324</td>
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<td>1.000</td>
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<tr>
<td>0.908</td>
<td>0.912</td>
<td>0.932</td>
<td>1.000</td>
<td>0.400</td>
<td>0.723</td>
<td>0.827</td>
<td>0.879</td>
<td>0.888</td>
</tr>
<tr>
<td>1.000</td>
<td>0.912</td>
<td>0.932</td>
<td>1.000</td>
<td>0.400</td>
<td>0.723</td>
<td>0.827</td>
<td>0.879</td>
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<tr>
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</tr>
</tbody>
</table>

The matrix shows the high correlation between the spot rates changes at the different vertices. It also indicates that the correlation decreases with increasing time differences, e.g. the correlation of changes of the one year and the ten year spot rate is only 0.436 whereas the correlation of 4 and 5 year spot rate changes is 0.932.
We perform a principal component analysis and use the first three factors. The three principal components explain 95.6% of the dynamics of the term structure. They are a linear combination of the weekly changes of the spot rates. Moreover, these three factors have very intuitive shapes. The first factor explains movements of the term structure in the same direction. This shift factor is similar to duration but additionally captures the different volatilities of short and long term rates. The second factor, called twist, captures dynamics of the term structure, where long and short term rates move in opposite directions. The third factor explains changes of the term structure, where long and short term rates move in the same and medium term rates in the opposite direction. We name this principle component the butterfly factor.

**Term Structure Risk Factors**

The graph shows the three term structure risk factors shift (S), twist (T) and butterfly (B). From the 1-year to the 10-year vertex. The STB factors were calculated using a principal component analysis of weekly spot-rate changes of government term structures with data from 1/90 until 1/96. The volatility of spot-rate changes at a given vertex is proportional to the geometric average of the STB factors at this vertex, i.e. the volatility of the 1-year spot-rate change has almost equal contributions from shift and twist factors and a much smaller butterfly contribution.

In a second step we compute the sensitivity of a single bond to the shift, twist and butterfly factors. These exposures are determined through “shocking” the term structure by a standard deviation shift, twist or butterfly and by calculating the fitted prices based on the resulting term structure. The percentage price change is used as the exposure of a single bond to the three common factors. Finally we calculate the weekly factor returns by regressing the bond excess returns against their exposures to the three factors:
\( R_i = X_{S,i}R_S + X_{T,i}R_T + X_{B,i}R_B + r_{sp,i} \)

with

- \( R_i \) = Weekly excess return of government bond \( i \)
- \( R_S \) = Weekly shift factor return
- \( R_T \) = Weekly twist factor return
- \( R_B \) = Weekly butterfly factor return
- \( X_{S,i} \) = Exposure of bond \( i \) to the shift factor
- \( X_{T,i} \) = Exposure of bond \( i \) to the twist factor
- \( X_{B,i} \) = Exposure of bond \( i \) to the butterfly factor
- \( r_{sp,i} \) = Weekly specific return of government bond \( i \)

Based on the regression we generate a history of factor returns for the default free market sector which enables us to build a three-factor covariance matrix.

**Spread Risk**

We compute spread risk for the Pfandbrief- and the Eurobond sector. These two estimated risk factors will be added to the factor covariance matrix. As shown in the chart below, these two spreads are the major sources of spread risk.
Spread Risk

The chart shows the annualized volatilities of weekly spread changes for the Pfandbrief spread (PEX), the Eurobond swap spread (Euro), the current yield spread (Coupon), and the liquidity spread (Liquidity). The volatilities were calculated using weekly data from 1/90 until 1/96 (for the current yield spread from 9/94 until 1/96 only).

For the Pfandbrief sector we analyze the changes of the spread given by the difference of the PEX spot-rate curve and the default-free term structure at the 2, 3, 4, 5 and 10 year vertices. We exclude the one year vertex because the one year PEX spot-rate is equal to the one year FIBOR (Frankfurt inter banking offered rate). Maturities above 10 years are not used for spread risk estimations because the asset coverage is very thin in that range.

We calculate Eurobond spreads from the difference of the swap spot-rate curve and the default-free spot-curve at the same vertices as the Pfandbrief spreads. From weekly historical data over the last six years we find that spread changes at different maturities are highly correlated and that the spreads increase proportional to duration. This allows us to reduce the number of risk factors by performing a principal component analysis as for the default-free term structure. We can define a Pfandbrief spread factor and a Eurobond spread factor by taking only the dominant principal components, respectively. The exposure or factor loading of a Pfandbrief or a Eurobond to its spread factor is determined by its duration.
This chart shows the shift and twist factors obtained from a principal component analysis of weekly changes of the Pfandbrief spread over the default-free term structure. The Pfandbrief shift and twist factors explain 57% and 20% of the observed spread covariance structure, respectively, across the given vertices.

The first two principal components of the Pfandbrief spread are shown in the graph above. These components can be interpreted as shift and twist factors of the spread in the same way as those for the default-free term structure. The Pfandbrief spread shift factor explains 57% and the twist factor explains 20% of the total covariance of spread changes, respectively. In the risk model we use only the shift factor to estimate Pfandbrief spread risk as a first step. The addition of a twist factor is straightforward for applications that require more forecasting accuracy in this area.

The graph shows the annualized Pfandbrief spread volatility at the 5-year and the 10-year vertex calculated each week from the preceding 26 weeks. The curves reveal the heteroscedasticity of these spreads that we model with GARCH processes.
As an alternative to the principal component model of spread risk we also investigated GARCH models, which allow to capture the observed time-varying volatility (heteroscedasticity) and the mean-reversal of spread changes at higher vertices. We found strong indication for heteroscedasticity of the Pfandbrief spread changes using statistical tests (Q statistics, Lagrange multiplier tests).

As shown in the graph above, the volatilities of the 5-year and 10-year Pfandbrief spreads have changed substantially over the last five years. We use the following GARCH process to calculate the conditional expected variance of weekly spread changes:

\[ h_i^2(t) = \alpha_i + \beta_i h_i^2(t-1) + \gamma_i \varepsilon_i^2(t-1) \]

where

- \( h_i^2(t) \) is the variance forecast for the spread change at vertex \( i \) in week \( t \),
- \( \alpha_i, \beta_i, \gamma_i \) are the GARCH parameters for spread changes at vertex \( i \),
- \( \varepsilon_i(t-1) \) is the mean-corrected spread change at vertex \( i \) in week \( t-1 \).

The parameters \( \alpha, \beta \) and \( \gamma \) are estimated from historical weekly spread changes. The unconditional variance \( \Theta_i^2 \) at vertex \( i \), is given by

\[ \Theta_i^2 = \alpha_i \gamma_i / (1 - \beta_i \gamma_i) \]

The table below summarized GARCH(1,1) parameter estimates. The GARCH half-life of the Pfandbrief spread increases from 1.44 weeks at the 5-year vertex and 14.4 weeks at the 10-year vertex. Since the weekly 5-year and 10-year spread changes are substantially correlated (average correlation \( \rho = 0.49 \) from 1/90 until 2/96), we also estimated the GARCH parameters for the average of the 5-year and 10-year spread changes to obtain a single estimate for spread risk with maximum information.
The table lists the GARCH(1,1) parameters estimated for the 5-year and 10-year Pfandbrief spread changes, as well as for the average of the 5-year and 10-year spread change. The unconditional volatility $\Theta$ (annualized, in basis points) is the long-term average volatility. The estimated decay time of a volatility jump to half of the initial increase is given by the half life in weeks. This decay time increases with longer maturity up to 14.4 weeks at the 10-year vertex.

The implementation of GARCH volatility estimates into an integrated risk model requires a separate calculation of correlations between spread changes and the STB factor returns for the default-free term structure. The forecasting accuracy of a GARCH risk model in comparison to a principal component model can be estimated using bias tests.

**Specific Risk**

The final part of the risk estimation is the bond specific risk. The specific risk of a single bond is that part of the total risk not explained by the shift, twist, butterfly or one of the two spread risk factors. It is defined as the volatility of the specific return:

$$r_{sp,i(t)} = r_{i(t)} - r_{i(t,TS)}$$

with

- $r_{sp,i(t)}$ = Specific return of bond $i$ in time $t$
- $r_{i(t)}$ = Total return of bond $i$ in time $t$
- $r_{i(t,TS)}$ = Return of bond $i$ in time $t$ due to term structure movements
  $$= X_{t(sh,i)} r_{t(sh)} + X_{t(tw, i)} r_{t(tw)} + X_{t(ba, i)} r_{t(ba)} + D_{t(PF,i)} r_{t(PF)} + D_{t(EB,i)} r_{t(EB)}$$
- $X_{t(j,i)}$ = Exposure of Factor $j$ to bond $i$ in time $t$
- $r_{i(t)}$ = Return of factor $j$ in time $t$
By construction of the factor covariance matrix we assured that the specific return of a single bond is uncorrelated with the other five factors. We also assume that the specific risk of a single bond is uncorrelated to any other bonds’ specific risk. Looking for a simpler way of estimating the specific risk we observed that specific risk is proportional to duration. It is also obvious that the specific return or pricing error for Eurobonds is much bigger than for government bonds due to the broad range of issuers including a wide range of qualities. This diversity led us to the decision to use the swap curve as a proxy for the Eurobond sector.

The Pfandbrief sector on the other hand shows a higher specific volatility compared to the Government sector mainly due to the different grade of liquidity. We therefore define the specific risk model as

\[ s_{(i)} = b \cdot D_{(i)} + c \cdot \kappa_{(EB,i)} \cdot D_{(EB,i)} + d \cdot \kappa_{(PF,i)} \cdot D_{(PF,i)} \]

with

- \( D_{(i)} \) = Duration of bond \( i \) at time \( t \)
- \( D_{(PF,i)} \) = Duration of Pfandbrief \( i \) at time \( t \)
- \( D_{(EB,i)} \) = Duration of Eurobond \( i \) at time \( t \)

where \( b, c, \) and \( d \) are constants, \( s_{(i)} \) is the specific risk forecast. \( \kappa_{(EB,i)} \) is the quality spread of Eurobond \( i \) in time \( t \) above or below the swap term structure and \( \kappa_{(PF,i)} \) is the spread above or below the Pfandbrief term structure not captured by the coupon and liquidity spread. We differentiate between the Eurobond and Pfandbrief sector because of the big differences in terms of quality, market participants and homogeneity of these two sectors.

Putting it all together, the estimated variance of a portfolio's return is given by:
VAR (R_p) = xFx^T + \omega

with

\[ x = \text{Exposure vector of the bond } i \text{ or portfolio } p \]

The exposure of the portfolio \( p \) to the factor \( j \) is equal to

\[ x_{pj} = \sum h_{(i)} * x_{(i,j)}, \text{ with } x_{(i,j)} \text{ equal to the exposure of bond } i \text{ to factor } j \]

\[ F = 5 \times 5 \text{ factor covariance matrix that captures term structure and spread risk} \]

\[ \omega = \text{Specific variance} \]

\[ = \sum (h_{(i)} * s_{(i)})^2 \]

\[ h_{(i)} = \text{Weight of bond } i \text{ in the portfolio } p \]

\[ s_{(i)} = \text{specific risk of bond } i \]

The model enables the bond portfolio manager to identify the different sources of portfolio risk. Additionally, the manager can analyze the portfolio against a benchmark like the REX index to get a better feel of the relative bets in the portfolio. The bond manager can view the impact of different trades on the portfolio risk and the expected return based on the managers interest rate and spread scenarios.

**Derivatives**

The valuation of interest rate derivatives is based on the Cox-Ingersoll-Ross term structure model. Bonds with embedded options are valued by adjusting their nominal cash-flows. Embedded options include puts and calls, mandatory sinking funds with and without purchase options, mandatory serial repayments, and extendible bonds.

Risk modeling of bond portfolios containing interest rate derivatives like options on bond futures is complicated by the nonlinear relationship between option prices and the under-
lying bond prices. For standard puts or calls this nonlinearity is most pronounced for options at the money. Our objective is to find a linear factor decomposition for options returns that can be integrated with the factor model for bond returns. One approach that has been implemented already successfully for equity options is based on a decomposition of an option’s return into a component with constant implied volatility at the beginning of the period and a component resulting from the change of the implied volatility using Black Scholes option valuation. (see Sheikh, [1995].) Each component can be represented by a factor model. For interest rate derivatives the first component is given by the product of the option’s elasticity (with respect to the bond price) with the underlying bond’s model return (bond exposures times STB factor returns). The second component contains the option’s return due to the changes of the state variables of the term-structure model. Once this component has been represented as a linear combination of STB factor returns, multiplied with the corresponding sensitivities or exposures, the factor covariance matrix and portfolio variances can be calculated. The development of this model is in progress.

Performance Analysis and Attribution

The multi-factor approach allows to attribute the performance of a portfolio to the different sources of return over time. This attribution applies to active and total return analysis, where active return is defined as difference between portfolio and benchmark return. The central goal of performance analysis is to understand how the performance was achieved and to evaluate if the bond portfolio manager was skillful. The multi-factor attribution approach is able to answer questions like: How did the bond portfolio manager perform compared to the market or among other portfolio managers? What factors contribute and explain the superior or inferior bond performance? We therefore attribute the portfolio return to the marketwide and sector specific factors of the model. Changes of the difference between the market and the fitted price are bond specific and indicate the ability to successfully pick undervalued bonds.
In particular the multi-factor attribution identifies the following sources of bond returns (see Kahn, [Fall, 1991]):

1. The bonds have shortened, interest has accrued and coupons were paid
2. The term structure has moved and the bond values have changed accordingly
3. Marketwide and sector spreads have changed and as a result bond prices will change
4. Unexpected changes in bond ratings lead to bond value changes
5. Unexpected cash flows due to embedded options will change the portfolio value
6. Bond specific return generated by the move closer to or further away from the fair market value

The multi-factor approach explicitly models the effect of the first three sources of return, whereas the latter three effects generate bond specific returns. The sum of the six return sources explains the portfolio value movement from time $t_1$ to $t_2$. We can successively apply the different effects to move from the market price at time $t_1$ to the market price at time $t_2$.

**Price Change Over Time**

<table>
<thead>
<tr>
<th>Fitted price at time $t_1$</th>
<th>$-\varepsilon_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll down term structure and reinvest scheduled cash flows</td>
<td>$\Delta P_{RD}$</td>
</tr>
<tr>
<td>Rolled down term structure</td>
<td>$\Delta P_{TS}$</td>
</tr>
<tr>
<td>=&gt; new term structure</td>
<td></td>
</tr>
<tr>
<td>old Spreads =&gt; new spreads</td>
<td>$\Delta P_S$</td>
</tr>
<tr>
<td>old cash flows =&gt; new cash flows</td>
<td>$\Delta P_{CF}$</td>
</tr>
<tr>
<td>old ratings =&gt; new ratings</td>
<td>$\Delta P_R$</td>
</tr>
<tr>
<td>Fitted price at time $t_2$</td>
<td>$\varepsilon_2$</td>
</tr>
</tbody>
</table>

Market price at time $t_1$

Market price at time $t_2$
This price chain demonstrates one possibility to add the different bond value effects from time t₁ to time t₂. Simply rolling down the term structure generates a price change ΔP_{RD}. The actual change of the term structure leads to ΔP_{TS} and the marketwide and sector specific spread changes are responsible for the price change ΔP_{S}. The price change ΔP_{CF} is due to unexpected principal flow, maybe because of an unanticipated call of an embedded option. Finally a change in bond ratings generate a price change ΔP_{R}. The difference between the market and the fitted price at time t₁ and time t₂ is covered by the difference of the mispricing at time t₁ and time t₂ (ε₂ - ε₁). The return attributed to each single effect is the price change divided by the market price at t₁. The attributed returns’ sum to the total observed return over the single period.

Furthermore the multi-factor approach enables us to calculate the contribution of each single valuation factor to the attributed returns. We are able to contribute the performance due to the term structure movement to each of the twelve key rates. We can look at the performance contribution of each of the different marketwide and sector specific spreads. On the other hand fixed-income managers may be interested in a broader analysis of their performance that focuses more on the aggregated bets. They like to see the impact of a positive shift in interest rates or a decrease in the difference of long and short term rates. Fixed-income managers usually do not bet explicitly on changes of single vertices. Therefore it appears to be useful to have an aggregated level of performance attribution in addition to the very specific and detailed vertex by vertex analysis. One approach to build such aggregated levels for the term structure movements involves defining shift, twist and butterfly factors. As we have seen earlier these three movements explain more than 95% of term structure volatility. The term structure movement can be attributed to a shift, a twist, a butterfly plus a residual movement, which in part can be explained by spread changes. To understand the performance attribution based on aggregated factors we will look at a sample portfolio managed against a benchmark of Government bonds in December 1995. The portfolio manager tried to outperform his benchmark by investing in Pfandbriefe and Eurobonds. Additionally, he bet on a twist of the term structure, in par-
ticular he forecasted that the term structure will flatten. The table below displays the performance attribution over this month. He underperformed the benchmark by 26 basis points. A look at the active return column shows that he lost performance in his Eurobond investment and on his term structure bet. The decision to invest in Pfandbriefe was successful. On a finer level of detail, e.g., on a key rate level, the main information that the twist bet was responsible for almost all term structure related underperformance, would have been less obvious.

### Return Attribution to aggregated Factors

<table>
<thead>
<tr>
<th>Source</th>
<th>Return in %</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Portfolio</td>
<td>Benchmark</td>
<td>Active</td>
</tr>
<tr>
<td>Term Structure Roll down</td>
<td>0.57</td>
<td>0.55</td>
<td>0.02</td>
</tr>
<tr>
<td>Term Structure Shift</td>
<td>0.30</td>
<td>0.28</td>
<td>0.02</td>
</tr>
<tr>
<td>Term Structure Twist</td>
<td>0.12</td>
<td>0.30</td>
<td>-0.18</td>
</tr>
<tr>
<td>Term Structure Butterfly</td>
<td>0.04</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Term Structure Residual</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Pfandbrief Sector</td>
<td>0.10</td>
<td>0.00</td>
<td>0.10</td>
</tr>
<tr>
<td>Eurobond Sector</td>
<td>-0.23</td>
<td>0.00</td>
<td>-0.23</td>
</tr>
<tr>
<td>Specific Return</td>
<td>0.05</td>
<td>0.07</td>
<td>-0.02</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.97</strong></td>
<td><strong>1.23</strong></td>
<td><strong>-0.26</strong></td>
</tr>
</tbody>
</table>

So far we have analyzed the return side, but have neglected looking at the risk inherent in the achieved performance. Furthermore we have not analyzed multi-period performance, yet. Only the combined analysis of return and risk over time allows to differentiate between luck and skill of a fixed-income manager. Statistics can help determine investment skill. In particular, we will look at the t-stat and the information ratio. Both ratios can be used to analyze the factor-by-factor risk and return profile, e.g., total returns, active returns or even the returns on a much finer level, like the shift, twist or butterfly return / risk profile.
The t-stat is defined as

\[ t - \text{stat} = \frac{R}{\sigma(R)} \cdot \sqrt{N} \]

where

- \( R \) = Observed return
- \( \sigma(R) \) = Standard deviation of return \( R \)
- \( N \) = Number of periods

The t-stat measures whether the observed mean return differs significantly from zero. If returns are normally distributed and the t-stat exceeds a value of 1.96, then there is a 95% chance that the performance is due to investment skill. A small example will clarify the usefulness of this ratio. Assume an active bond portfolio manager who on average outperformed his benchmark by 175 basis points per year. His active risk - the standard deviation of the portfolio returns around the benchmark return - is 200 basis points. How many years does he have to keep this record before he can claim to be a skillful bond portfolio manager?

\[
\begin{align*}
t - \text{stat} &= \frac{R}{\sigma(R)} \cdot \sqrt{N} \\
1.96 &= \frac{175}{200} \cdot \sqrt{N} \\
N &= 5 \text{ years}
\end{align*}
\]

It is even harder to become statistically significantly successful on a finer level, e.g., on a bond picking level.

The second measure, the information ratio (IR) is defined as annualized return divided by annualized risk. This could be total return divided by total risk, active return divided by
active risk or even Pfandbrief sector return divided by the sector specific Pfandbrief risk. The higher the ratio, the higher the added value of the strategy. An information ratio of 0.5 means the portfolio manager is able to gain 50 basis points by adding one unit of risk, measured as standard deviation. In our small example the bond portfolio manager has an information ratio of

$$IR = \frac{175}{200} = 0.875$$

There is a close link between these two ratios. If the length of the N periods corresponds to T years, then the information ratio is just

$$IR = \frac{t - \text{stat}}{\sqrt{N}}$$

To summarize: The t-stat measures the statistical significance of the return, whereas the information ratio captures the risk-return trade-off of the strategy. We are able to apply these measures on a factor-by-factor analysis and therefore can identify if and where the manager adds value. We can determine if the manager has skills in forecasting term structure movements, anticipating spread changes or picking the most successful issues.

Summary

This article has presented BARRA’s approach to model the German bond market and its different sectors. We built a valuation model based on term structure and spread estimation procedures. We demonstrate how to capture the main forces that drive term structure movements. Additionally, we present a sophisticated method to forecast the different volatilities of the Pfandbrief and the Eurobond sector. The GARCH model also captures the tendency of these sector spreads to revert to their mean over time. We addressed the impact of derivatives in the portfolio context and how to evaluate them. Last but not least,
we described our approach to attribute risk and return to its various sources. This attribution can be done on a very detailed level or alternatively on a more intuitive aggregated level. We cover multi-period performance analysis and introduce two statistical measures that allow to distinguish between skill and luck on a risk-adjusted basis. This analysis helps to identify the strengths and weaknesses of an investment strategy over time.

REFERENCES


Appendix A

Term Structure Estimation

Our term structure estimation routine produces forwards, spreads and two Cox-Ingersoll-Ross parameters. In particular, the input parameters for the objective function are

\[ \Rightarrow \quad \text{Bond data, like market price, coupon, etc} \]
\[ \Rightarrow \quad \text{The model vertices} \]
\[ \Rightarrow \quad \text{LIBOR rates for the 1, 3, and 6 month vertices} \]
\[ \Rightarrow \quad \text{Forwards rates, spreads and Cox-Ingersoll-Ross parameters from the previous term structure estimation} \]

The output parameters that minimize the objective function are

\[ \Rightarrow \quad \text{A length } m \text{ vector of forward rates } f_r(j) \]
\[ \Rightarrow \quad \text{A length } k \text{ vector of spreads } s_r(k). \]
\[ \Rightarrow \quad \text{The short term rate } r_0 \]
\[ \Rightarrow \quad \text{The Cox-Ingersoll-Ross parameter } \kappa \text{ and } \theta, \text{ where} \]
\[ \kappa = \text{Cox-Ingersoll-Ross mean reversion rate and} \]
\[ \theta = \text{Cox-Ingersoll-Ross long term spot rate} \]

These parameters are used to calculate the forward priors.

The forward rates can easily be transformed into spot rates and PDB’s. The parameters will be estimated so that they minimizes the following objective function:

\[
E = \sum_{i=0}^{n-1} \left( \frac{P(i) - Q(i)}{w(i) \cdot P(i)} \right)^2 + \sum_{j=0}^{n-1} \left( \frac{(fp(j) - f_r(j))}{\mu(j)} \right)^2 + \sum_{k=1}^{l-1} \left( \frac{C(k)}{v(k)} \right)^2
\]

with

\[ n = \text{number of bonds} \]
\[ m = \text{number of forward rates} \]
\[ l = \text{number of spreads} \]
\[ P(i) = \text{market price of bond } i \]
\[ Q(i) = \text{fitted price of bond } i \]
\[ f_r(j) = \text{forward rate between vertices } j \text{ and } j+1 \]
\[ fp(j) = \text{Cox-Ingersoll-Ross forward prior between vertex } j \text{ and } j+1 \]
\[ w(i) = \text{weight of bond } i \]
\[ \mu(j) = \text{weight of forward prior } j \]
\( v(k) = \) weight of spread \( k \)

\( \text{spr}(k) = k^{th} \) spread

\( C(k) = k^{th} \) spread penalty function

The first term in the above equation is a penalty for large relative pricing errors \( (P(i) - Q(i)) \). The weight \( w(i) \) depends on the kind of a bond, in this case on the liquidity, the duration and the type of embedded options.

The second term in the equation eliminates kinks of the term structure by pulling it toward a smooth “Cox-Ingersoll-Ross” term structure. We minimize the squared differences of forward rates less the forward rates priors predicted by the Cox-Ingersoll-Ross model.

The third term in the equation is a penalty to prevent the benchmark spread to become positive and all other spreads from becoming negative.
Appendix B

The weight matrix of the PEX index

<table>
<thead>
<tr>
<th>Maturity</th>
<th>6%</th>
<th>7.50%</th>
<th>9%</th>
<th>Sum</th>
<th>Weighted Coupon</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Year</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>2 Year</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>3 Year</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>4 Year</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>5 Year</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>6 Year</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>7 Year</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>8 Year</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>9 Year</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>10 Year</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Sum 38.50 26.80 34.70 100.00 7.44

Source: Buehler and Hies [1996].