THE LONGEVITY RISK ASSOCIATED WITH THE PENSION LIABILITY

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ABSTRACT
The human race's life expectancy has rapidly been extended in the past 100 years. This tendency is especially significant in the advanced country. For instance, the average life span of Japanese men has increased from 71.73 years old to 78.56 years old during the years from 1975 to 2005. Moreover, the average life span of Japanese women increased from 76.89 to 85.52 years old in the same period.

From the viewpoint of risk management of corporate pensions, it is not desirable to have an unexpected increase in life expectancy (longevity risk). Due to recent changes in pension regulations and accounting standards, the longevity risk has become apparent to corporate sponsors and their shareholders.

In this paper, we describe the longevity risk associated with the pension liability from a practical perspective. Focusing on Japanese corporate pension schemes, we illustrate a relationship between the longevity risk and the corporate pension liability. Especially, we refer to a "guaranteed period" which is a characteristic of Japanese corporate pension schemes.

In order to forecast improvements in mortality rates, we use a general example of a stochastic extrapolative model (Lee-Carter model). Assuming a present value of typical pension benefits, we analyze the influence of mortality improvements on the value. At the same time, we compare cases that have different payment terms of the guaranteed period. In addition, we recognize necessity of an extension of the model through a comparison of results estimated using actual data from four countries (Japan, US, England & Wales, and Italy).

KEYWORDS
Longevity risk, Guaranteed period, Lee-Carter model
1. INTRODUCTION

The human race's life expectancy has rapidly been extended in the past 100 years. This tendency is especially significant in the advanced country. For instance, the average life span of Japanese men has increased from 71.73 years old to 78.56 years old during the years from 1975 to 2005. Moreover, the average life span of Japanese women increased from 76.89 to 85.52 years old in the same period.

The improvements in mortality rates of senior citizens in the advanced countries are considered significant. The left side of Figure 1 shows the transition of the average life expectancy at age 65 in four countries, Japan, the United States, England & Wales, and Italy. Numbers were calculated how long subject people older than 65 years old will stay alive based on the actual mortality rate of each year. Therefore, the age for Japanese in 1995, the year of the Great Hanshin-Awaji Earthquake, was temporarily low for this reason. Though there are some fluctuations in each year, it shows a steady increase of the average life expectancy.

Though quality of life is not always related to its length, longer life is welcome in principle because it means more time to enjoy oneself. On the other hand, from the perspective of social security costs such as medical and pension, longevity might not always be good thing.

The improvements in mortality rates until now have relied on the development of medical technology, better diet and adequate funding of social security. Though there is no doubt that the average life expectancy will continue to rise, there are many opinions as to how many years or how it will grow. Nobody knows for sure. This is recognized as a cause of “Longevity Risk.”

The rest of the paper is organized as follows. In section 2, we explain how to measure and manage the longevity risk are increasingly very important in the context of corporate pensions. In section 3, we describe the longevity risk associated with the pension liability from a practical perspective focusing on Japanese corporate pension schemes. In section 4, we forecast the mortality improvements using the Lee-Carter model. In section 5, we conclude the paper.

2. LONGEVITY RISK

2.1. LIFE ANNUITY

From the viewpoint of risk management of corporate pensions, it is not desirable to have an unexpected increase in life expectancy. In principle, the corporate pension is a scheme to provide benefits to support employees’ lives in their old age during their retirement. Therefore, the basic scheme design of the corporate pension can be defined as a procedure to provide a fixed amount of benefit until the retired employee will die; “life annuity.” As one of the variety, there is “joint life annuity”, which is a scheme design to provide fixed benefits.
until both members of a couple will die. In either case, lifetime benefit is considered as a basic scheme for pensions.

If the scheme designs to provide lifetime benefits is a premise, improvements in mortality rates that extend the lifetime means extending the period of benefit payment as well. The extended life will cause an increase in the pension liability, the present value of benefit to be paid in the future, and would tend to increase the burden of the plan’s sponsor. From the corporate accounting point of view, an increase in pension liability means cost increase that suppresses the performance of the company.

2.2. RECENT CHANGES IN FUNDING REGULATIONS AND ACCOUNTING STANDARDS

The alteration of the accounting standards and the pension funding regulations are recent topics in regards to corporate pensions. During of the process to converge accounting standards internationally, accounting standards regarding retirement benefits are also to be revised. On the other hand, there have been more strict funding regulations in the United Kingdom, the United States and Netherlands.

One example of the changes in the accounting standards was an elimination of the deferred recognition process that smoothed the actual gains and losses[^1], allowed until the amendment. After the elimination of smoothing, changes to the pension liability will be reflected in the income statement or the balance sheet immediately. It is considered to enhance the transparency of pension liability because the amount of the pension liability will be reflected immediately without going through the smoothing process. This enhanced transparency of the pension liability will reveal hidden risks.

The exposure to risks means that the measurement method of the pension liability and its result will be exposed to more attention as well. Deliberations as to how to measure risks such as longevity risk will be even more necessary because those risks will be more visible to corporate sponsors and stakeholders.

2.3. LDI: INVESTMENT BASED ON LIABILITY

The enhanced transparency of the pension liability accelerates prevalence of LDI, Liability Driven Investment. As long as smoothing was allowed, there was not much of a financial advantage to including a position to hedge the pension liabilities in the pension assets. If expanding investments to long term bonds or introducing interest rate swaps in order to match the cash flow of pension benefits will have only a limited effect on the current earnings or the balance sheet, there will be little motivation to implement a LDI strategy. However, immediate reflection of the pension liability on the balance sheet will increase the

[^1]: The difference of pension liability caused by the alteration of actuarial assumption, and the difference between the estimation based on the actuarial assumption and the actual experience.
necessity of LDI and of risk management, including longevity risk.

2.4. BUY-OUT OF CORPORATE PENSION SCHEME

Buy-out businesses in pension are increasingly centered in the UK. This is also related to the changes in accounting standards. Those buy-out firms will receive assets corresponding to the calculated amount of pension liabilities; as a consideration for the take over of a corporate pension scheme. The pension liabilities calculated include margins for risk so liabilities are 10 to 30% more expensive compared to the pension liabilities based on IFRS. In the case of a buy-out, there will be latent conflicts of interests between a company who wants to transfer the pension liabilities with a minimum cost and a buy-out firm who wants to assume the pension liabilities with a maximum price. In this case, the value of pension liabilities is used to settle conflicts of interests rather than just for the purpose of disclosure. The amount of pension liabilities used to reconcile the conflicts needs to have a more sophisticated and cogent valuation. This means that more importance will be placed on the measurement methods for the risks associated with the pension liability such as the longevity risk.

2.5. RISKS ASSOCIATED WITH THE PENSION LIABILITIES

There are three major risks\textsuperscript{2} associated with pension liabilities: interest rate risk; inflation risk; and longevity risk.

Generally, interest rate risk is recognized as a fluctuation of the discount rate used to calculate the pension liabilities. In the case of a scheme where its benefits are linked with interest rate levels, a change in interest rate might change the benefit itself.

Inflation risk is a characteristic of corporate pension schemes. The scheme that is not linked with price level directly, but proportional with the final salary, also bears inflation risks because the final salary is influenced by price level.

Regarding interest rate risk and inflation risk, we have financial instruments to hedge them in the capital market. For example, there are interest rate swap and index-linked bonds. Plan sponsors and stakeholders are familiar with the concepts of interest rate risk and inflation risk so it is easier for them to understand those risks.

Conversely, the risk concerning the fluctuation of mortality tables is not familiar to plan sponsors and stakeholders. In Cass Business School (2007), it was shown that mortality rates used for the valuation of pension liability are different in each country. The comparison with the fluctuation of pension liability caused by changing the discount rate in order to explain the influence in the difference of mortality rates gives the amount of pension liability. There are few people who understand the effects of mortality improvements on the pension

\textsuperscript{2} Other than the three risks described here, there are risks such as a risk of unexpected benefits because of early retirements.
liabilities. There have been some challenges to using Mortality-Linked Securities (the EIB/BNP longevity bond, etc. Refer to Blake (2006)) as a financial instrument to hedge the longevity risk in the capital markets. Indexes regarding mortality tables have also been developed and disclosed (Such as Lifemetrics by JP Morgan Securities and QxX.L.S. by Goldman Sachs, etc). In any case, how to measure and manage the longevity risk are very important subjects in the area of corporate pensions.

3. PRACTICAL PERSPECTIVE
3.1. DEFINED BENEFIT CORPORATE PENSION SCHEME AND MORTALITY RATE

In this section, the longevity risk associated with the pension liabilities will be described from a practical perspective. Japanese corporate pension scheme will be focused on to discuss the relationships between the longevity risk and the pension liabilities. The total defined benefit pension assets of 3,371 listed Japanese companies on March 2007 are 61.6 trillions yen. (Nikkei Media Marketing, Inc.)

Relatively among the small schemes, the schemes exposed to the longevity risk are not necessarily common. This is because providing life annuity is not necessarily a requirement in the rules of scheme design. There are three defined benefit corporate pension schemes approved in Japan: TQPP, Tax Qualified Pension Plan; EPF, Employees’ Pension Fund; and DBCPP, Defined Benefit Corporate Pension Plan. There is a requirement for EPF plans to provide a certain level of life annuity. However, since it is possible to switch an EPF to a DBCPP (which doesn’t need to provide life annuity) (“elimination of the substitutional portion” since 2001), the providing of life annuity can be deemed, in fact, as not obligatory.

Historically, the Japanese corporate pension system was developed from a retirement lump sum allowance system. This scheme design is to provide a lump sum allowance at retirement and there are usually no funding assets outside of the company. Therefore, in the average corporate pension schemes, the annual payment of the pension benefits is decided that the present value of the payments will be equivalent to this lump sum amount.

In the context of Japanese corporate pension schemes, providing life annuity normally means to include guaranteed installments. For example, if the guaranteed period is 10 years, and the pensioner died after 7 years, the bereaved family will receive the remaining 3 years of benefits. On the other hand, if the pensioner dies after the 10-year guaranteed period has expired, the bereaved family would not receive any further benefits. In this case, however, the annual payment of the pension benefits is decided that the present value of the payments within a guaranteed period will be equivalent to this lump sum amount. This means the rest of the period beyond the guaranteed period is considered a bonus from the company to their employees. This is one of the characteristics of the Japanese corporate pension scheme and a reason why big companies and some foreign companies, which provide high level benefits, select the lifetime pension.
3.2. REASSESSMENT OF THE MORTALITY RATE

Funding valuation is performed annually as a financial settlement to verify if the pension assets meet the funding standards. There are two kinds of funding standards, the standard for continuation based on the going concern of the scheme and the standard for discontinuation based on the termination of the scheme.

The key actuarial assumptions used here are usually reassessed at the time of the actuarial valuation, performed once every 5 years. The withdrawal rates and the salary increase rates are scheme specific and they are computed based on actual recent experience. On the other hand, the mortality rates used for the funding valuation of Japanese corporate pension schemes are uniform, not scheme specific. The latest mortality table, referred to as the “19th”, is decided based on actual domestic mortality in 2000.

The mortality rates used for the accounting valuation are practically the same as the ones used for the actuarial funding valuation. In reality, there should be a specific mortality rate for the retired employees of a relevant company. Though it would not necessarily correspond with the National Mortality Table, this difference is not usually considered in the accounting liability valuation.

3.3. RECOGNITION OF DIFFERENCE

How is the difference of pension liability, due to the difference between estimated and actual mortality and the difference caused by reassessment of the mortality rates, recognized? The difference caused by reassessment of the mortality rates will be reflected at the time of actuarial valuation so that a certain amount of losses will be recognized. The difference between estimated and actual mortality will be recognized at the annual financial settlement. However, these gains/losses seem to be invisible because it is quite small compared with the gains/losses in the pension assets (difference between expected investments return and actual performance.)

The longevity risk is not yet recognized as an emergency issue because there are not many corporate pension schemes providing life annuity in Japan and their effects will only be gradually revealed. Furthermore, the longevity risk is, by nature, not very noteworthy so not many corporate sponsors actually take the longevity risk seriously. However, if the implementation of a LDI strategy becomes more widespread, the existence of the longevity risk will gain much more attention. Even if the adoption of LDI is able to hedge the interest rate risk and the inflation risk, the longevity risk will attract attention as a remaining risk.

3.4. ACTIVE MEMBERS

What kind of strategy can be created to deal with the longevity risk from the aspect of scheme design?
Under Japanese regulations, there is flexibility of changing the scheme design for the active members. If an agreement is made as a result of negotiations with a labor union, it is possible to change the scheme design. This scheme design change is sometimes accompanied with a decrease of benefits. For example, it is possible to change the original guaranteed life annuity to the annuity certain not to change the amount of benefit corresponding to the amount of retirement lump sum allowance. This means that the annual payment won’t change but the perpetual annuity part beyond the guaranteed period, a bonus portion, will be eliminated.

As described before, the present value of benefit payments for a guaranteed period will be defined to be equivalent to the amount of the retirement lump sum allowance. Therefore, an extension of the guaranteed period will cause a decrease of a yearly benefit payment. However, if the guaranteed period is extended, the present value of the total liabilities of a life annuity is decreased because it is originally a lifetime pension. For example, extending the guaranteed period from 15 years to 20 years, the start of the lifetime benefit will be postponed from the 16th to the 21st year, so that the time span of the longevity risk exposure will be narrowed.

As just described, a scheme change (decrement of benefit) is possible, as far as active members are concerned, if only certain requirements are fulfilled. If the sponsors of a pension schemes take the longevity risks seriously, it is possible to mitigate the risk by changing the scheme design for the active members. Therefore, the sponsors have an option to decrease the longevity risks without any support from capital markets, such as derivatives. Of course it is not desirable that the sponsors arbitrarily decrease benefits since corporate pension schemes have a very important role providing financial support for the retired employees. However, the option remains.

3.5. PENSIONERS AND DEFERRED VESTED

It is not practical measure to change the benefits design for the pensioners and the deferred vesteds, unlike for the active members. Under Japanese regulations, it is required to meet some strict conditions such as the financial difficulty of the sponsoring company. Because the life annuity provided to these peoples is recognized as a vested right, it is difficult to change it afterward.

Many big companies, who used to sponsor the Employees’ Pension Fund, have switched to the Defined Benefit Corporate Pension Plan. It intended to increase profits related to the transfer to the government of the substitutional portion of the Employees’ Pension Fund. The Defined Benefit Corporate Pension Plan was enacted in 2001 – switch from the EPF to the DBCPP (elimination of the substitutional portion) began to be allowed. After 2001,  

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3 In the present situation in Japan, the main purpose of a scheme change is cost reduction. Implementing scheme change just for risk management is rare.
there have been many schemes switched from the EPF to the DBCPP.

As described before, though the sponsor of the Employees’ Pension Fund is required to provide a certain level of life annuity, the Defined Benefit Corporate Pension Plan does not have such a requirement. Therefore, there are many Japanese corporate pension schemes with longevity risks at least for those who were the pensioners or the deferred vesteds when the scheme have switched from the EPF to the DBCPP. Those risks can not be eliminated by changing the benefit design because it is difficult to change it for the pensioners and deferred vesteds. A strategy with support from the capital markets, such as the longevity bonds, might be helpful for a scheme with many of such vested holders.

4. FORECASTING IMPROVEMENTS IN MORTALITY RATES

4.1. LEE-CARTER MODEL

In this section, a forecast regarding the improvements of future mortality rates will be tried using the Lee-Carter model. Assuming a present value of typical pension benefits, we analyze the influence of mortality improvements on the value. At the same time, we will examine how the amount of pension liabilities will be influenced by the existence and the length of a guaranteed period.

The Lee-Carter model is the most well known stochastic and extrapolative model.\(^4\) When \(x\) denotes age, \(t\) denotes a time series, the central death rate for age \(x\) at time \(t\) would be

\[
\ln(\mu_{x,t}) = a_x + b_x k_t + e_{x,t} \tag{1}
\]

\(a_x\) is an age-specific parameter that indicates the average level of mortality rates irrespective of time series. \(b_x\) is another age-specific parameter that characterizes the sensitivity of \(\ln(\mu_{x,t})\) to change in the mortality index \(k_t\), which indicates the change of mortality respective to time series. \(e_{x,t}\) is the error term. The data used were the actual mortality rates of the last 30 years in Japan, the United States, England & Wales, and Italy extracted from the Human Mortality Database.

4.2. PARAMETER ESTIMATION

Because all the parameters on the right side of formula (1) are not observed variables, the values can not be estimated by simple regression process. Therefore, singular value decomposition is used for solving this problem (refer to Lee and Carter (1992)). Constraint conditions shown below are added in order to obtain unique values for parameter estimation.

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\(^4\) Refer to Cairns (2007) regarding stochastic models other than Lee-Carter Model. It shows a comparison with another model considering the cohort effect.
As a result, the estimated value for $a_x$ is

$$\hat{a}_x = \frac{1}{T} \sum_{i=1}^{T} \ln(m_{x,i})$$  \hspace{1cm} (4)

$\hat{a}_x$ is defined as the average mortality of each age group. In addition, a singular value decomposition brings

$$\ln(m_{x,i}) - \hat{a}_x = \sum_i u_{x,i} q_{v_{ri}}$$  \hspace{1cm} (5)

It gives estimated values for $\hat{b}_x$ and $\hat{k}_t$. Table 1 shows the results of parameter estimation. By applying those estimated values to the time series, a forecast of future mortality improvement can be obtained (see Figure 1.)

Figure 2 shows the transition of the survival rates. From 1975 to 2005, the survival rates are based on the actual mortality experience. From 2010 to 2045, they are based on the forecasts of the future mortality improvement obtained using the Lee-Carter model. It shows the rates of how many survivors will be remaining at each age in 5-year increments when the number of survivors at age 60 is set as 1. The sum of the survival rate of a transition year from age 60 to the oldest group gives an average life expectancy at the transition year. Moreover, the sum of the survival rate which is discounted to the age 60 using an interest rate is equal to the present value of life annuity payments at age 60.

### 4.3. COMPARISON OF RISKS OF EACH SCHEME DESIGN

In this section, the pension liabilities are calculated using a forecast of mortality improvements obtained in the preceding section. Equating the present value of life annuity payments as a pension liability, the pension liability considering mortality improvements and the pension liability not considering mortality improvements are compared. Hereafter, the pension liability considering future mortality improvements will be referred to as the dynamic liability. $L_{x,t}^{(d)}$ shows the dynamic liability on age $x$ after $t$ years.

$$L_{x,t}^{(d)} = \sum_{j=1}^{\omega} \left( \frac{1}{1+r} \right)^j \prod_{j=0}^{j-1} (1-q_{x+j,j+j})$$  \hspace{1cm} (6)

$\omega$ represents the age at the end point in the mortality table, $r$ represents the discount rate. In the same practice, the pension liability not considering mortality improvements in the future is referred to as the static liability. $L_{x,t}^{(s)}$ shows the static liability on age $x$ after $t$
years.

\[ L_{x,t}^{(s)} = \sum_{i=1}^{n} \frac{1}{(1 + r)^i} \cdot \prod_{j=0}^{i-1} (1 - q_{x+j,j}) \]  

(7)

\( n \) is a guaranteed period. Given a set pension liability of a pension scheme with a guaranteed period as \( L_{x,t}(n) \), the dynamic and the static liability are shown as,

\[ L_{x,t}^{(d)}(n) = \sum_{i=1}^{n} \frac{1}{(1 + r)^i} + \sum_{i=n+1}^{\infty} \frac{1}{(1 + r)^i} \cdot \prod_{j=0}^{i-1} (1 - q_{x+j,j}) \]  

(8)

\[ L_{x,t}^{(e)}(n) = \sum_{i=1}^{n} \frac{1}{(1 + r)^i} + \sum_{i=n+1}^{\infty} \frac{1}{(1 + r)^i} \cdot \prod_{j=0}^{i-1} (1 - q_{x+j,j}) \]  

(9)

Formula (6) and (7) can be considered as formula (8) and (9) with \( n=0 \) respectively.

Table 2 and Figure 3 show the results of formula (8) and (9) with the forecast of future mortality improvements based on the actual mortality data of Japan. Here \( x=60, t=0, r=2.5\% \) whereas \( n \) is in cohorts of 0, 10, 15, 20 years. It is clear that given a longer guaranteed period, there is a smaller difference between the static liability and the dynamic liability. The reason is that a longer guaranteed period mitigates the influence of improvements in mortality rates, because the benefit payments during the guaranteed period are irrespective of the death of the pensioner.

On the other hand, it can be said that improvements in mortality rates exert influence on the pension liability with a guaranteed period, even though it is small compared to a case with 0-guaranteed period. It could be because improvements in mortality rates have a great influence on survival rates right after termination of a guaranteed period but not on the present value during the guaranteed period. The conclusion is that though the existence of a guaranteed period has surely an effect to reduce some of the influence of improvements in mortality rates, it is not enough to erase this influence entirely.

4.4. CROSS-NATIONAL COMPARISON

Table 3 and Figure 4 show cross-national comparison of four countries, Japan, the United States, England & Wales and Italy. In the tables \( x=65, n=0 \) exclusively. The largest difference between the static and the dynamic liability is observed in Japan. The reason is that the level of improvement in Japanese mortality rates in past was relatively fast compared to the other three countries. This is a natural consequence of using the extrapolative model, which obtains a forecast of the future from actual records of the past.

On the other hand, there is the thought that the human life has a potentially limited span and we are approaching this limitation. From this viewpoint, the results that in Japan, with the highest life expectancy, the life expectancy will be extended the most in the future as well, are not consistent with the intuition.
5. CONCLUSIONS

As the human race’s life expectancy around the advanced country rise, the existence of the longevity risks in corporate pension schemes is gaining attention. The recent tightening of the funding regulations and the changing in the accounting standards has accelerated this trend. It is becoming a very important issue from the aspect of risk management of corporate pension schemes as to how to measure and manage the longevity risk.

Focusing on the current practice in Japan, it can be said that the existence of a guaranteed period is a characteristic of corporate pension schemes in Japan. Forecasts of the mortality improvements were performed using the Lee-carter model. Influences of the existence and the length of a guaranteed period on the amount of the pension liabilities were examined as well. A cross-national comparison, using the actual mortality rates of four countries, was also performed. It showed a not intuitive result, given that there are limitations in the life expectancy of the human race. Considering this result, expansion of the model will be an issue for future work.

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Figure 1: The extension of the average life expectancy at age 65 (The data until 2005 are based on the actual mortality rates. The data after 2006 are based on the forecasts using the Lee-carter model.)
Table 1: Estimated values of parameters (Japan, male)

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Figure 2: Transition of the survival rates (Japan, male)
Table 2: Alteration of the pension liabilities with a difference of guaranteed period (Japan, male, x=60, r=2.5%)

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<td>rate((d-s)/s)</td>
<td>9.19%</td>
<td>8.86%</td>
<td>8.20%</td>
<td>6.87%</td>
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</table>

95% | 21.61961 | 21.93874 | 22.25535 | 22.64175 |
5%  | 13.69887 | 14.30869 | 15.23834 | 16.72878 |

Figure 3: Alteration of the pension liabilities with a difference of guaranteed period (Japan, male, x=60, r=2.5%)
Table 3: Cross-national comparison (male, \(x=65\), \(r=2.5\%\))

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<thead>
<tr>
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<th>EW</th>
<th>Italy</th>
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Figure 4: Cross-national comparison (male, \(x=65\), \(r=2.5\%\))