Return Expectation Based on Economic Growth and its Implication for Public Pension System

Hiroshi Miyai
The Nikko Research Center Ltd.
Abstract:

The return expectation method based on the equilibrium theory which W. F. Sharpe proposed in CAPM can be used practically, if we apply appropriate market portfolio and reasonable societal risk tolerance (SRT). In this study, I apply J-MIX for the market portfolio and try to introduce some economic factors into SRT calculation. The relationships between macro-economic factors and social wealth are analyzed, which are increasing rate of wage, saving and employment numbers, calculated by the aspect of national economic distribution data. Then, some methods are examined to evaluate the relationship between the risk premium, social wealth and SRT.

The method that I applied to return estimation in this study can provide reasonable expectation corresponding to certain economic changes, because it is constructed by considering the relations of economic fundamentals. As these fundamentals are basic parameters to evaluate pension financing, this method also can be provided relations among economic changes, pension liabilities, and pension asset investments. I applied for this method to assess risk vs. return profiles both of a pay-as-you-go system and a funding system. The results suggests that expected returns of a funding system can be altered by risk levels. However, it is difficult to alter expected returns of a pay-as-you-go system, because it is limited by increasing rate of employees' wage and employment. As these economic fundamentals have fallen recently in Japan, it is necessary to consider a funding system for public pension system in order to enhance expected returns.

This paper is part of my study entitled "Effects of Investing in Markets for Public Pension Assets" which I conducted as a member of the Pension Fund Management Research Institute (PFMRI), where "Asset Liability Management for the Japanese Public Pension Fund" was investigated as a research project in the 1997 fiscal year. I would like to express my gratitude to Dr. Yonezawa, professor at Yokohama National University, for helpful suggestions and comments. My thanks are also due to the research fellows and visiting fellows of PFMRI, especially to Mr. Hishida, Mr. Jimba, Mr. Inoue, and Mr. Uno, who gave me valuable comments. However, responsibility for the text (and with any surviving errors) rests entirely upon the author.
I. Introduction

The most naive method for estimating expected return is to use the average of the realized data without any amendments. This method supposes that future events will be realized at the same probability in the past. As one improvement, there is the so-called "building block method". This method supposes that the future ex-ante risk premium of certain assets is equal to the excess return calculated by means of subtracting a return of risk free asset from returns of risk assets during the same period. It also supposes that the calculated risk premium is stable for many years, and so, expected return on each asset can be estimated by adding the calculated risk premium into the expected risk free return. The assumption of risk premium stability for each asset is one of the problems of this method, because the calculated excess return is changeable depending on the data sample period. And another problem pointed out is that changes in economic structure is not consider in this method, despite clear changes.

Another method generally applied for return estimation is the "scenario approach". In this method, some probable economic scenarios in the future are assumed for the first time, then expected return is estimated using both their probability and the realized return observed under their same scenario in the past. One criticism is that a consistency of return estimation is not guaranteed in this approach, because short-term fluctuation in the economy might affect scenario assumptions and result in arbitrariness of the scenario.

W. F. Sharp (1990) advocated a method based on a return equilibrium mechanism in the market in order to solve these problems. This method is based on his theory introduced as the CAPM (Capital Asset Price Model) in the 1960's and applied in the US market first. His method has been applied to the Japanese market and developed as a practical model. The basic idea of this model is that the most efficient portfolio for the investors is the market portfolio, if market efficiency and its equilibrium are assumed. To put this theory into practice, it is necessary to evaluate societal risk tolerance (SRT) using appropriate indicators of social wealth. Expected return can be calculated by this SRT, expected covariance among asset classes, and proportions of each asset classes in the market portfolio. One of the advantages of this method is that once its methodology has been established, following expected return can be calculated in a same manor. Another advantage is that a fluctuation of expected return depending on the data sampling period could be avoided by using a market portfolio constructed based on by investors' consensus. For the reason given above, this method is useful for investors such as pension plan
sponsors in developing their long-term policy asset mix.

The purpose of this paper is to improve the expected return method based on CAPM, by introduced of macro economic aspects to the relationship. For this reason, SRT is estimated using increasing wage and employment rates as an indicator of social wealth, and is applied to estimate return. This application will not only resolve problems with return expectation, but also provide reasonable long-term expected return for pension plans from the point of view of the national economy.

Also, this application can provide risk vs. return profiles of public pension systems such as a pay-as-you-go system and a funding system under the same assumption of economic conditions. I would like to show a meaning of pension money investment into market by comparing risk vs. return profiles of public pension systems.
II. A study framework

As asset management companies and pension plan sponsors have generally tended to use naïve analysis based on realized return to develop their policy asset mix, little attention has been paid to the long-term view and consistent framework for return expectation in relation to economic growth. As a result, expected returns estimated by money managers have been relatively short-term and influenced by the market situation at that time. Then, some assumptions for their estimation tend to have a short-term bias and change frequently, and also stability is not assured. Due to these points, and because of the difficulty of a reasonable explanation for short-term assumptions without a long-term viewpoint, this kind of estimation framework has not been sufficiently accepted by all investors.

The point I want to make in this study is that providing a return estimation method based on asset return and economic growth analysis, and on an analogy of these results will enable recognition of the influence of some economic factors on a long-term basis. Figure-1 describes a framework of this study with some related studies.

1. Recent studies

Several articles have been devoted to the study of the relationship between production and employment, savings and capital market conditions. A typical study in this field is "current condition in the Japanese economy" published by the Economic Planning Agency, in which problems of pension investment are examined from economic and social welfare perspectives (the Economic Planning Agency, 1996). Also, Watanabe (1995) focused on the aspect of distribution in the national economy and tried to analyze the relation between economic growth, inflation rates, increasing rates in real employee's wage and market return.

Meanwhile, W. F. Sharp (1990) measured the national welfare of society and explored the relationship between risk premium implied in the market and its value. He used real security value per capita (R$\text{SVPC}$) and cumulative rate of return (CRR) as indicators of social welfare, and estimated societal risk tolerance, then calculated an expected return practically based on the CAPM he himself advanced in a 1960's. Azuma(1989, 1990) applied Sharp's framework to the Japanese securities market, and showed the availability of return estimation based on CAPM and developed a practical model for asset allocation decisions.

There are not many studies attempting to analyze the relationship between
economic growth and return of pension systems. The Economic Planning Agency (1996) tried to introduce a method to evaluate a return of pension systems in "the macro-economic return in a pay-as-you-go system". However, it did not concern about investment risk of pension systems. I define the risk of a pay-as-you-go system as variability of its returns in order to compare risk vs. return profiles between a pay-as-you-go system and a funding system.

Figure 1. Framework of study
2. A placement of this study

Although a framework of return estimation based on CAPM is examined in this study, the following three new points are taken into consideration.

① To examine the relationship between economic growth rates and increasing rates of employee’s real wages, labor’s relative share, and employment numbers.

② To estimate social welfare using increasing rates of real wages and employment and to analyze the relationship between welfare and risk premium, and to connect to return estimation based on CAPM.

③ To compare not only the expected return but also the risk of a pay-as-you-go system with those estimates related to a funding system based on the same economic fundamentals.

① above describes quantitatively the relationship between macro economic fundamentals related to national economic distribution. ② is an attempt to connect expected return in capital markets with the national economy, also is one of the important points in ensuring consistency of the assumption with pension liability calculations, since these fundamentals, especially the increasing wage rate and employee numbers are the basic variable for liability estimation. ③ is attempt to implicate above method to evaluate risk vs. return profiles of pension systems based on macro economic fundamentals.

3. Data used for this study

① Macro economic data: Japanese economic white paper, annual report of national economic accounting; The Economic Planning Agency.

② Corporate finance data: NSX500; The Nikko Research Center Ltd.

③ Return data: Japan Mix Index (J-MIX); The Nikko Research Center Ltd.
III. Relationship between economic growth and labor productivity and labor power

1. Theoretical background

Labor productivity \( y \) is defined as dividing domestic gross productivity \( Y \) over employment population \( L \). Where, following equations are introduced by calculating increase of \( y \) when \( Y \) increase \( \Delta Y \) and \( L \) increase \( \Delta L \).

\[
\frac{dy}{y} = \frac{dY}{Y} - \frac{dL}{L} \quad \frac{dY}{Y} \approx \frac{dy}{y} + \frac{dL}{L} \tag{1}
\]

This equation means that real growth rate of domestic gross productivity \( \frac{dY}{Y} \) is obtained by summing up the increasing rate of real labor productivity \( \frac{dy}{y} \) and employment population \( \frac{dL}{L} \). Also, labor productivity can be divided as follows by using employees' wages \( W \) and national income \( E \).

\[
y = \frac{Y}{L} = \frac{W * E}{L} = \frac{W}{E} = w * \frac{1}{e} * \frac{1}{k} \tag{2}
\]

Where, \( w \) means employee wage per person, \( e \) means labor's relative share, \( k \) means the proportion of national income against GDP. Increasing rate of \( y \) can be calculated by means of \( \Delta y \) which will be determined by the increase of \( \Delta w \), \( \Delta e \), and \( \Delta k \), as in the same way described above.

\[
\frac{dy}{y} \approx \frac{dw}{w} - \frac{de}{e} - \frac{dk}{k} \tag{3}
\]

This equation indicates that increasing rate of labor productivity \( \frac{dy}{y} \) is obtained by subtracting the increasing rate of labor relative share \( \frac{de}{e} \) and the proportion of national income against GDP from the increasing rate of real employees' wage per person. We may combine equation (1) and (3) to obtain following equation (4).

\[
\frac{dY}{Y} \approx \frac{dw}{w} - \frac{de}{e} - \frac{dk}{k} + \frac{dL}{L} \tag{4}
\]

Consequently, the real economic growth rate can be divided into the following increasing rate of fundamentals, those are real wage, labor relative share, national income against GDP, and the employment number.
2. Changes of GDP growth rate and macro-variables

Figure 2 and 3 indicate trends in the above five variables. Although there is a relatively high growth rate of about 10% in real GDP by the first half of 1970’s, it falls to 3-5% during the middle of 1970's to around 1990, then falls to 1-2%. The increasing rate of employees' wage shows a pattern of following the GDP growth rate with 1-2 years delay. On the other hand, the increasing rate of labor's share is characterized by volatility rather than level, shows less volatility after 1980 than in the 1970's.

Figure 2  Trends in real GDP growth rate, increasing rate of employees' real

During the 1955 - 1980 fiscal year period, the increasing rate of national income / GDP is steady at about 5%, except in 1973 and 1974, when extremely high rates of more than 15% are observed. However, the rate falls to about 2% after that. Although employment number increase is about 5% in the last half of 1950's, it falls to 2% by around the 1980 fiscal year. It is steady at about 2-3% from 1980 to the last half of 1980's, this period is remarkable as an economic bubble in Japan, reaches to about 4%, then gradually declines after 1990.

Figure 3  Trends in real GDP growth, increasing rate of both national income / GDP and the employment numbers.
3. Analysis of the relationship between economic growth and labor productivity, labor power.

In this section, investigation using multi-regression analysis on whether the relationship of fundamental factors described in the equation (4) is recognized empirically as introduced theoretically in the section (1) is carried out. We find the 93.7% change in the real growth rate is explained by the four fundamental variables. There is considerable evidence to prove the equation (4) that the coefficient of the equations are significant statistically as indicated by t-value, which shows a significance level of coefficient. Also theoretical mark conditions of coefficient are consistent to the analytical results.

Figure 4. The results of multi-regression analysis concerning real GDP growth.

\[
\text{Real GDP growth rate} = 1.003 \times \text{increasing rate of wage} - 0.855 \times \text{increasing rate of labor's relative share} \\
(18.755) \quad (-14.409) \\
+ 0.833 \times \text{increasing rate of national income / GDP} \\
(-20.93) \\
+ 1.122 \times \text{increasing rate of employment} \\
(15.571)
\]

\[
\text{coefficient of determination}=0.937 \\
\text{parentheses show t-value}
\]

Table 1. Correlation coefficients among variables

<table>
<thead>
<tr>
<th></th>
<th>Real GDP</th>
<th>Real wage</th>
<th>Labor relative share</th>
<th>National income/GDP</th>
<th>employment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real wage</td>
<td>0.545</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor relative share</td>
<td>0.021</td>
<td>0.394</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>National income/GDP</td>
<td>0.203</td>
<td>0.387</td>
<td>0.115</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>employment</td>
<td>0.542</td>
<td>0.041</td>
<td>-0.111</td>
<td>0.146</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The correlation coefficients among these variables are shown in Table 1. Relatively high coefficients are observed between real GDP growth rate and increasing rates of real wage and employment, that are 0.545, 0.542 respectively. There are somewhat high correlation coefficients among explanatory variables, especially between the increasing rate of real wage and labor share, and national income / GDP, which are 0.394, 0.387 respectively. However, it is suggested that there isn't sufficient evidence of multi-co-linearity statistically, because their provability is rejected at the 5% significance level.
IV. Return expectation method based on market equilibrium theory

1. The market equilibrium theory

The following is a summary of the return expectation method based on market equilibrium theory described in the studies of W. F. Sharp (1990) and Azuma (1989, 1990).

Let's consider the combination of the market portfolio (RM) and risk free asset \( r \) for rational investors in an efficient market. Expected return and risk could be calculated in the following equations.

\[
E = X \cdot E(R_M) + (1 - X) \cdot r \tag{6}
\]
\[
S = X \cdot S(R_M) \tag{7}
\]

Where,
- \( X \) : proportion of market portfolio
- \( E(R_M) \) : expected return of market portfolio
- \( S(R_M) \) : standard deviation of market portfolio
- \( r \) : return of risk free asset

The objective is to choose \( X \) to maximize the following utility function \( U \). To find the optimal value of \( X \), first order condition of this equation relative to \( X \) is set equal to zero, and re-arranged to \( X \) as follows:

\[
U = E - S^2 \frac{t}{t}
\]
\[
= X \cdot E(R_M) + (1 - X) \cdot r - X^2 \cdot S(R_M)^2 \frac{t}{t}
\]

\[
\frac{\partial U}{\partial X} = 0
\]

\[
0 = E(R_M) - r - 2X \cdot S(R_M)^2 \frac{t}{t}
\]

\[
X = \frac{E(R_M) - r \cdot t}{2S(R_M)^2 \cdot t} \tag{8}
\]

\( t \) : risk tolerance

If each side of the equation (8) for a given investor is multiplied by his or her portion of total invested wealth, then a sum is indicated for the average weight of investment for all the investors in society. Because this risk tolerance shows the risk tolerance of a typical investor in the society, it is called societal risk tolerance \( (\tau) \). Suppose that the market is efficient and risk free assets can be borrowed and lent freely, borrowing and lending must cancel out to remain only risk assets. Since
that is equivalent to holding the available securities, for markets to clear, X must equal 1. Thus equilibrium requires the following:

\[
X = \frac{E(R_M) - r}{2S(R_M)^2} \\
1 = \frac{E(R_M) - r}{2S(R_M)^2} \\
E(R_M) - r = \frac{2S(R_M)^2}{\tau} \tag{9}
\]

Suppose that SRT is a linear function of investor's wealth, equation (9) is rearranged as follows:

\[
\tau = k \cdot Wh \\
E(R_M) - r = \frac{2S(R_M)^2}{k \cdot Wh} \tag{10}
\]

Equation (11) is obtained from equation (10). If we apply realized excess return multiply investor's wealth as a denominator in equation (11), and variance of market return as a numerator, we can calculate k. Moreover, we can calculate expected return for each asset based on the following equation. That is obtained from the same process explained above to set the first order condition of equation (12) relative to hi equal to zero.

\[
k = \frac{2S(R_M)^2}{(E(R_M) - r) \cdot Wh} \tag{11}
\]

\[
U = (1 - \sum h_i) \cdot r + \sum h_i E(R_i) - \frac{1}{\tau} \sum \sum h_i h_j \sigma_{i,j} \tag{12}
\]

\[
\frac{\partial U}{\partial h_i} = 0 \\
0 = -r + E(R_i) - \frac{2}{\tau} \sum h_i \sigma_{i,j} \\
E(R_i) - r = \frac{2}{\tau} \sum h_i \sigma_{i,j} \tag{13}
\]

If we could apply appropriate SRT and the variance covariance matrix, expected return could be estimated in following equation.

\[
E(R_{i,t+1}) - r = \frac{2}{\tau} \sum h_{i,t} \sigma_{i,j,t} \tag{14}
\]
2. Verification

We can only use realized excess return, whereas the risk premium in equation (10) is expected to be ex-ante risk premium, that is:

Average excess return = risk premium + unexpected return

The following equation will be a verified substitute for the above equation.

\[ R_m - r = \beta \cdot \left( \frac{1}{Wh} \right) + \epsilon \]  

(15)

At first, RSVPC (real security value per capita) and CRR (cumulative rate of return) are examined as a candidate of Wh in the equation (15), which W. F. Sharp(1990) and Azuma(1990) applied for their study.

Because both indicators have obvious trends shown in Figure-5, trend modification is applied to the same method as Azuma(1990) did in his study. Investors can anticipate that this trend line would show the increase of wealth. It is necessary to remove this trend in order to evaluate the real influence for investor's risk tolerance. Then, assuming that this trend line shows the base value of wealth at any given time and unexpected separation from this line shows real change of societal risk tolerance. That is

Indicator of social wealth = social wealth / trend modification term

Figure-5  Trend lines of social wealth
Table-2 shows the results of analysis based on equation (15). It is confirmed that the relations of the realized excess return and CRR, RSVPC are significant statistically same as the study of Azuma (1990). This result suggests that average excess return is influenced by social wealth. Furthermore, as $\beta$ is a positive value, it is suggested that the higher the value of wealth, the lower the risk premium (the lower the value of wealth, the higher risk premium).

Table-2 Results of the relation between realized excess return and wealth indicators

<table>
<thead>
<tr>
<th>Indicator of wealth</th>
<th>t-value for $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRR</td>
<td>2.43</td>
</tr>
<tr>
<td>RSVPC</td>
<td>2.52</td>
</tr>
</tbody>
</table>

3. Relationship between increasing rate of employees' wage and social wealth

Assume that social wealth at time $t$ is equivalent to social wealth at time $t-1$ plus national savings at time $t$ multiplied by investment return of $r$ as shown in equation (16). Where, a rate of saving is calculated that saving divide by disposable income ($YD$) as shown in equation (17). Assume that disposable income is nearly equal to employees' wage ($YD=W$), and take the differentiation of equation (17) to get equation (18). Because it is defined that employees' wage are equal to wage per

\[ Wh_t = (1 + r)(Wh_{t-1} + S_t) \]  
\[ s = \frac{S}{YD} \]  
\[ \frac{ds}{s} = \frac{dS}{S} - \frac{dYD}{YD} \approx \frac{dS}{S} - \frac{dW}{W} \]  
\[ \frac{dS}{s} \approx \frac{dw}{w} + \frac{dL}{L} \]  
\[ \Theta \quad W = wL \]  
\[ \frac{dW}{W} \approx \frac{dw}{w} + \frac{dL}{L} \]  
\[ \frac{S_t - 1}{S_{t-1}} = \frac{ds}{s} + \frac{dw}{w} + \frac{dL}{L} \]  
\[ S_t = S_{t-1}(\frac{ds}{s} + \frac{dw}{w} + \frac{dL}{L} + 1) \]  
\[ Wh_t = (1 + r)(Wh_{t-1} + S_{t-1}(\frac{ds}{s} + \frac{dw}{w} + \frac{dL}{L} + 1)) \]
employee, multiply the employment number \((W=wL)\), take the differentiation to get the equation (19) and re-arranged to equation (20). Finally, equation (21) is introduced from equation (16). I would like to calculate an indicator of social wealth based on equation (21). Where, accumulated wealth is calculated that setting \(Wh0=1\), \(S0=0.1\) at beginning and using the increasing rates of fiscal years for saving, wage, and employment.

Figure-6 indicates changes in household's savings and wages. It is recognized that the rate of household savings lags increasing wages. Figure-7 describes the correlation between them with a 7 years lag. In this figure, a straight line shows regression line and the value in parentheses shows t-value. Because a significant relationship between these variables is shown, this regression line may apply for estimating the rate of household savings to introduce a long-term expected return in chapter 5.

If accumulated wealth might not be invested in the market, it would describe as \((1+r)=1\) in equation (21). Suppose that we might invest into short-term instruments as risk-free assets, it would be applied for risk free return as \(r\) in equation (21). If we might invest into the market, it would be applied for its return. CRR corresponds to a case in which we ignore saving but invest into the market.

Figure-8 shows cumulative return on those indexes as setting 1 at the beginning of the calculation at 1955 fiscal year and their trend lines. The higher the
investment return, the higher the cumulative return, as a matter of course. Also, it is observed that the cumulative return calculated from the value of household savings and wealth shows a smooth curve, while it shows relatively sharp movement when investment is taken into account.

Figure-8  Cumulative return of wealth indexes and their trend lines

![Cumulative return of wealth indexes and their trend lines](image)

The regression analysis is applied for these three indexes as a proxy for wealth (Wh') as shown in equation (22) instead of equation (15). For this analysis, trend modification is applied in CRR and RSVPC. Figure-3 shows the results. It is clear that wealth indexes calculated from the increasing wage and employment rate can apply for the proxy of social wealth, as every t-value for \( \beta_w \) shows greater than 2. In addition, it shows the same level of t-value against CRR when invest into the market. As the coefficient shows a positive value, it is suggested that the higher the value of wealth, the lower the risk premium (the lower the value of wealth, the higher risk premium), as shown in the analysis for CRR and RSVPC.

Because CRR is only calculated from accumulating the return of market portfolio, the relationship between economic condition and social wealth is not clear. On the other hand, this method described above can account for the relationship between national economy and social wealth. Then, I intend to apply this approach to estimate security return.
\[ R_M - r = \beta_w \cdot \left( \frac{1}{W_{t-1}} \right) + e \]  \hspace{1cm} (22)

Table 3: Results of the relationship between realized excess return and wealth indicators

<table>
<thead>
<tr>
<th>Indicator of wealth</th>
<th>t-value for ( \beta_w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not invest into market (( r = 0 ))</td>
<td>2.07</td>
</tr>
<tr>
<td>Invest into risk free asset (( r = \text{risk free asset return} ))</td>
<td>2.17</td>
</tr>
<tr>
<td>Invest into market portfolio (( r = \text{market portfolio return} ))</td>
<td>2.51</td>
</tr>
</tbody>
</table>
V. Return estimation based on national distribution

In this section, expected return based on CAPM is estimated using the theoretical background and results as described before, that is, the relationship between economic growth and labor productivity, labor power, as well as the market equilibrium theory and its analysis in a practical world. Figure 9 shows this process.

It is proved that economic growth is divided into fundamentals such as increases in wage, labor's relative share, and employment numbers. I intend to use these fundamentals in order to make the connection between market return and economic growth. As I mentioned above, it is confirmed from analysis that the following indicator of wealth is sufficiently capable to use, that is constructed with the increasing wage and employment number and those invested into the market.

In the return estimation process, the first step is to assume the increasing rate of wage and employment under reasonable economic growth. The second step is to determine SRT based on equation (11). The third step is to evaluate the variance covariance matrix for each asset class, in which the idea of conditional probability is applied for in this study.

Consequently, expected return for each asset class is estimated from equation (14) based on the above results, where the weight of each asset class in the market is applied to the weight of J-MIX.

Figure 9 Estimation process for equilibrium return

<table>
<thead>
<tr>
<th>Evaluation of variance-covariance Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>To apply for conditional probability</td>
</tr>
<tr>
<td>To determine decreasing weight using J-MIX return data</td>
</tr>
<tr>
<td>To evaluate variance-covariance matrix from using the decreasing weight of return</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Determination of SRT</th>
</tr>
</thead>
<tbody>
<tr>
<td>To consider for the increasing real wage and employment rate as a indicator of wealth (Wh)</td>
</tr>
<tr>
<td>To calculate k based on equation (11) and determine SRT at reasonable Wh</td>
</tr>
</tbody>
</table>

Calculation of expected return for each asset class
Compute variance-covariance for each asset class, recent weight of J-MIX, and SRT into equation (14)
The advantages of this method are following:

1. "Consistency" in the return estimation process might be maintained
2. "Stability" of the estimated return might be maintained
3. "Market consensus" of expected return based on market pricing mechanism might be reflected

(1) Assumption of economic fundamentals

The increasing wage rate is assumed to be 0~2%, as it has a tendency to decrease since the last half of 1970's as shown in Figure 2 and this trend might continue in the future. The increasing rate of labor's relative share is assumed to be 0~2%, this level is observed as an average after fiscal year 1976, and the trend of variability has changed since fiscal year 1975. The increasing rate of national income / GDP is negligible, because its coefficient in the equation (5) is a relatively small number and its increasing rate is relatively low about 2%. The increasing of employment number in the future is assumed to be 0~3% based on the trends in past data. The GDP growth rate might be calculated from equation (5), if we adopt above data. The bound areas are calculated as follows:

\[ \text{The real GDP growth rate} = 1.003 \times (\text{increasing wage rate}) - 0.855 \times (\text{increasing rate of labor's relative share}) - 0.083 \times (\text{increasing rate of national income / GDP}) + 1.122 \times (\text{increasing rate of employment}) \]

\[ = 1.003 \times (0~2\%) - 0.855 \times (0~2\%) - 0 + 1.122 \times (0~2\%) \]

\[ = -1.7~4.3 \text{ (average=1.3\%)} \]

The real GDP growth rate is estimated between -1.7~4.3% as described above, however, this range shows potential economic growth. Table-4 indicates assumptions of fundamentals in estimating expected return under this potential growth. Supposing that real GDP growth rate is 1.3%, increasing wage rate is 0.377, and employment rate increase is 1.0%, the real wage rate increase may be estimated 0.5% in the equation (5). Increasing rate of household's savings is calculated as real increasing wage rate is supposed to be 0.5% in the Figure 7.
Table 4  Macro-economic assumptions

<table>
<thead>
<tr>
<th>Real GDP growth rate</th>
<th>Incr. rate of labor relative share</th>
<th>Incr. rate of employment</th>
<th>Real incr. wage rate</th>
<th>Incr. rate of household's savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3%</td>
<td>0.377%</td>
<td>1.0%</td>
<td>0.5%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

(Note 1) Real GDP growth rate is assumed to be the average of the range.
(Note 2) Increasing rate of labor relative share is assumed to be the average during 1976-1995 fiscal year.
(Note 3) Increasing rate of employment is assumed as the trend from the past data.
(Note 4) Real increasing wage rate is estimated as described above.
(Note 5) Increasing rate of household's savings is calculated at 13.2% based on Figure 7. As this value is in the same level as an average over the past 10 years, that is 13.4%, increasing rate is assumed to be 0.0%.

(2) Equilibrium expected return

First, variance and average excess return are calculated from J-MIX data based on the process used for Figure 9. Figure 10 shows these results used for 20 years data at each time since March 1990-March 1996. Average excess return is stable at about 3% after 1991. Although variance shows decrease during 1991-1993, it is stable at about 65%² after 1993.

Figure 10  Variance and average excess return of J-MIX

Figure 11 shows SRT change based on the estimate for average wealth of society from equation (21). As the indicator of wealth in Figure 8 which is calculated by assuming market investment, is lower than the trend line after fiscal year 1992,
the wealth which is evaluated by its reciprocal shows an increase. On the other hand, SRT estimated by both the average excess return and variance shows an increase during 1990-1991, but a decrease after that. The level of SRT is stable at about 40 after 1994.

Figure-11 Changes of average social wealth (estimated from wage rate) and SRT.

Because the weight of short-term instrument, domestic bond, and domestic equities in the J-MIX and their expected variance-covariance are estimated as shown in Table 5, expected return for each asset is calculated based on equation (14). Where, wealth is estimated by setting the real increasing wage rate, employment, household's savings, and temporary market return, for 0.5%, 1.0%, 0.0%, and 5.2% respectively. And, SRT is calculated by the deviation from the trend line for the wealth and k in the equation (11). Although SRT=57.19 is obtained under the above conditions, this level is higher than the level observed in the past. The reason for this result is that the indicator of wealth is much lower than the trend line obtained under the above condition. After all, the expected return for short-term instruments, domestic bonds, and domestic equities is estimated as 0.38, 10.2, and 6.46% respectively.
Table-5 Expected return for each asset class

<table>
<thead>
<tr>
<th></th>
<th>Short-term Instrument</th>
<th>Domestic Bond</th>
<th>Domestic Stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of J-MIX (%)</td>
<td>0.12</td>
<td>0.38</td>
<td>0.35</td>
</tr>
<tr>
<td>Expected Covariance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short-term Instruments</td>
<td>0.41</td>
<td>0.32</td>
<td>1.04</td>
</tr>
<tr>
<td>Domestic Bonds</td>
<td>0.32</td>
<td>17.21</td>
<td>5.98</td>
</tr>
<tr>
<td>Domestic CBs</td>
<td>0.40</td>
<td>12.26</td>
<td>119.21</td>
</tr>
<tr>
<td>Domestic Stocks</td>
<td>1.04</td>
<td>5.98</td>
<td>309.60</td>
</tr>
<tr>
<td>International Bonds</td>
<td>-0.02</td>
<td>5.25</td>
<td>1.29</td>
</tr>
<tr>
<td>International Stocks</td>
<td>-0.72</td>
<td>7.20</td>
<td>46.10</td>
</tr>
<tr>
<td>$\Sigma (Weight of J-MIX \times expected Covariance)$</td>
<td>0.18</td>
<td>18.38</td>
<td>173.72</td>
</tr>
<tr>
<td>Expected Return</td>
<td>0.38</td>
<td>1.02</td>
<td>6.46</td>
</tr>
<tr>
<td>Expected Risk</td>
<td>0.64</td>
<td>4.15</td>
<td>17.60</td>
</tr>
<tr>
<td>SRT</td>
<td>57.19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Note 1) The weight of J-MIX is as of the end of March 1996.

(Note 2) Expected covariance is estimated by using decreasing weight on the return data based on the idea of conditional probability.

(Note 3) SRT may change for the k. Value of k here is applied for the average of the values during March 1990 - March 1996.

(Note 4) As the equation (21) is applied to evaluate wealth, it is necessary for expected market return. Here, 5.2% as a market return is applied, which is obtained from the temporary expected return for each asset and recent market weight for them.
VI A meaning of pension money investment into market
1. Estimation of investment risk and return of a pay-as-you-go system

(1) Return

The following estimation method is based on the approach that the Economic Planning Agency (1996) tried to introduce in "the macro-economic return in a pay-as-you-go system", which was a part of its annual publication entitled "the present situation of the Japanese economy". I would like to discuss this based on a method using simplified examples, in which there would be three different generations 0, 1, 2, where generation 1 follows generation 0, and generation 2 follows generation 1.

We assume that the number of people at generation 0 is A. If the number of people at generation 0 would increase at a growth rate of n, the number of people who would pay their premiums at generation 1 would be calculated (1+n)A. And the number of people at generation 2 is calculated (1+n)^2A. On the other hand, if people died at a mortality of \( \delta \), the number of people who would survive at the generation 1 would be calculated (1-\( \delta \))A. And, generation 2, it would be calculated (1-\( \delta \))(1+n)A.

Premiums of working generations and pension benefit to retirement generations can be evaluated as shown in figure 12. Where, \( w_{0.2} \) means employees' wage per person, \( a_{0.2} \) means premiums required in proportion to employees' wages, and \( b_{0.2} \) means pensioners' benefits.

Figure 12. The number of people, premiums, and benefits in a different generation.

<table>
<thead>
<tr>
<th>Generation</th>
<th>the number of working people</th>
<th>the number of beneficiaries</th>
<th>Premiums</th>
<th>Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>A</td>
<td>0</td>
<td>( a_0 w_0 A )</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>(1+n)A</td>
<td>(1-( \delta ))A</td>
<td>( a_1 w_1 (1+n) A )</td>
<td>( b_1 w_1 (1-( \delta )) A )</td>
</tr>
<tr>
<td>2</td>
<td>(1+n)^2A</td>
<td>(1+n)(1-( \delta ))A</td>
<td>( a_2 w_2 (1+n)^2 A )</td>
<td>( b_2 w_2 (1+n)(1-( \delta )) A )</td>
</tr>
</tbody>
</table>

In a pay-as-you-go system, premiums and benefits need to balance at each generation. So, this condition can be described by the following equation.

\[
a_2 w_2 (1 + n)^2 A = b_2 w_2 (1 + n)(1 - \delta) A
\]

\[\therefore \quad b_2 = \frac{(1 + n)a_2}{(1 - \delta)} (23)\]
Because macro economic return for this pension system is defined as dividing pension benefits by premiums, we can obtain equation (24).

\[ 1 + g = \frac{b_2 w_2 (1 + n)(1 - \delta) A}{a_1 w_1 (1 + n) A} \]
\[ = \frac{b_2 w_2}{a_1 w_1} (1 - \delta) \]
\[ = \frac{b_2}{a_1} (1 + \gamma)(1 - \delta) \]  \hspace{1cm} (24)

Where, \( \gamma \) shows increasing wage rate, and \( (1 + \gamma) = \frac{w_2}{w_1} \).

We can get equation (25) by putting equation (23) into equation (24), and by the assumption that the premiums are equal among generations.

\[ g = (1 + n)(1 + \gamma) - 1 \]
\[ = n + \gamma + n\gamma \]
\[ \approx n + \gamma \]  \hspace{1cm} (25)

We can estimate the return of a pay-as-you-go system by generally applying the above approach.

(2) risk

I defined the risk of a pay-as-you-go system as variability (standard deviation) of returns, as well as the funding system. We can describe the risk as follows,

\[ \sigma^2_g = \text{var}(n) + \text{var}(\gamma) + 2 \text{cov}(n, \gamma) \]  \hspace{1cm} (26)

estimation of risk and return

We can estimate risk and return based on equation (25) and (26) by using the fundamental data shown in Table-4.

<table>
<thead>
<tr>
<th>Real GDP growth rate</th>
<th>Incr. rate of labor relative share</th>
<th>Incr. rate of employment</th>
<th>Real incr. wage rate</th>
<th>Expected return</th>
<th>Expected risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3%</td>
<td>0.377%</td>
<td>1.0%</td>
<td>0.5%</td>
<td>1.51%</td>
<td>1.24%</td>
</tr>
</tbody>
</table>

(Note 1) Macro-economic data are some in Table-4.
(Note 2) Expected return is calculated using equation (25)
(Note 3) Expected risk is calculated using equation (26)
2. Comparison of risk and return between a pay-as-you-go system and a funding system

Figure 13 shows the risk and return characteristics of a pay-as-you-go and a funding system on a risk vs. return plane. The risk vs. returns of pension funding system is estimated by a standard portfolio which is constructed of domestic bonds, equities, and short-term instruments at a proportion of 6:3:1. Also, these three underling assets are plotted in this figure.

Because both pension systems is estimated based on macro-economic fundamentals such as the increasing rate of employees' wage and the employed population, these data can be compared.

The expected return of a pay-as-you-go system is estimated at about 1.5% and is quite a low level. The risk which is defined by the variability of return remains at a reasonably low level, while a standard portfolio which is a typical for funding system shows a relatively high risk and high return profile.

The significant difference between a pay-as-you-go system and a funding system is that the expected return in the former could never increase unless employees' wage or the employed population increased, while the expected return in the latter could increase by taking more risk on a efficient frontier. Both employees' wage and employment population in Japan shows a trend of fall recently and could not be expected to increase in the future. This suggests that we could not obtain a relatively high expected return from a pay-as-you-go system. It is necessary to enhance the return of the public pension system by means of a funding system.

Figure 13 the risk and return characteristics of a pay-as-you-go and a funding system
VII. Conclusion

It is necessary for pension sponsors to estimate long-term expected return when they try to establish asset mix policy. A great deal of effort has been made on analysis for realized return to estimate expected return. What seems to be lacking, however, is a connection with economic fundamentals. Therefore, few estimations for long-term expected return take into account economic conditions.

In this study, I focused on aspects of national economic distribution and confirmed empirically the relationship between economic growth and fundamentals such as increasing wage rate and employment. In addition, I intend to connect to estimations based on CAPM established by develop a indicator of social wealth using these macro-variables, and examine the relation with SRT. As a result, changes in economic conditions can be analyzed.

Increasing wage rates and employment are one of the basic factors in pension financing and are necessary for estimating pension liability. In this study, I worked to connect macro-economic growth to return expectation for assets, while, providing a consistent method for evaluating pension liability. I examined risk vs. return profiles of both a pay-as-you-go system and funding system based on above method. It is difficult to enhance expected return of a pay-as-you-go system, because both increasing wage rate and employment are limited in a low level. It is necessary to consider a funding system to enhance expected return.
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