Risk measures and efficient use of capital

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• Intuition on FX risk
• Simple example of capital choice
• Generalisation to currency invariance
• Difficulties with TailVaR
Currency risk

• Business in euros and pounds: (buffer-) capital better be in euros and pounds
• Same cost, same value, of extra basket of capital, both in Francfort and in London?
Example of P/L - FX correlation

• Date 0  FX rate 1/1
• Date 1 random FX rate : # of foreign to get one domestic :

  (1/2, 1, 3)

Date 1 net worth domestic (-2, 6, -1)
required date 0 capital domestic : +2
(worst-case risk measure)
required date 0 capital foreign : +3
Efficient choice of capital

Deal with \((-2, 6, -1)\) and rate *domestic* to *foreign* of \((1/2, 1, 3)\)

solve: \(\min (d + f)\) such that (zero int. rates)

\[
\begin{align*}
    d + 2f - 2 &> 0 \\
    d + 1/3f - 1 &> 0
\end{align*}
\]

and get \(d = 4/5, f = 3/5\), at a date 0 cost of 7/5 (domestic or foreign)!
Generalisation

Exchange rates $e_0$ and $e_1$, P/L $X$ domestic

$$\min\{ m; d + f / e_1 + X \in A, d + f / e_0 = m \}$$

and with numéraires $r$ and $s$ :

$$\min\{ m; rd + sf / e_1 + rX \in A, d + f / e_0 = m \}$$
Results

With a coherent acceptance set $A$ in future domestic currency the optimisation above leads to an extended acceptance set $B$ such that the coherent risk measures $\rho$ and $\sigma$ defined by $B$ and by $e_1B$ satisfy for each $X$

$$\sigma( e_1 r/s X ) = e_0 \rho( X )$$

London    Francfort
About distribution invariance

Back to FX rate \((1/2, 1, 3)\), and a permuted P/L:
\((-1, 6, -2)\) instead of \((-2, 6, -1)\)
solve for \(\min (d + f)\) such that:
\[
\begin{align*}
d + 2f - 1 &> 0 \\
d + 1/3f - 2 &> 0
\end{align*}
\]
and get \(d = 2, f = 0\) at a cost of 2: the derived measure is not distribution invariant!

TailVaR in Francfort and London do not fit via \(e_0\)!